THE IDEA OF SPACE: FROM EUCLID TO VIRTUAL ARCHITECTURE

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ABSTRACT

The discovery (or invention) of non-Euclidean geometry and of the higher dimensions (from the fourth on), the new idea of space to summarize, is one of the most interesting examples of the profound repercussions that mathematical ideas will have on humanistic culture and on architecture. The paper will discuss the elements necessary to give sense to the word *Space*.

1 Escherian Preface

In October 1964 the Dutch graphic artist Maurits C. Escher was to give a series of lectures in the United States and Canada, where his son Gorge lived. Shortly after his arrival in Canada, Escher was hospitalized in Toronto and underwent an urgent operation.

The series of lectures was canceled. However, Escher was quite meticulous and had written the complete text of the lectures. This text was conserved and published in 1986 in the book *Escher on Escher: Exploring the Infinite* [9]. The chapter that contains the text is entitled “Lectures that were never given". Escher had planned to give a lecture on the theme of the periodic covering of a surface, an art in which he had become an expert. At the end of the lecture he had planned to show the famous 1939/1940 print *Metamorphose II* in the unusual format of 195 by 4000 cm.

Here is how Escher describes his work: “I propose to round off this talk by showing you a woodcut strip with a length of thirteen feed. It's much too long to display in one or ever in two slides, so I had it photographed in six parts, which I can present in three successive pairs and which you are invited to look at as if were one uninterrupted piece of paper. It's a picture story consisting of many successive stages of transformation. The word *Metamorphose* itself serves as a point of departure. Placed horizontally and vertically in the plane, with the letters O and M as points of intersection, the words are gradually transformed into a mosaic of black and white squares, which, in turn, develop into reptiles. If a comparison with music is allowed, one might say that, up to this point, the melody was written in two-quarter measure.
Now the rhythm changes: bluish elements are added to the white and black, and it turns into a three-quarter measure. By and by each figure simplifies into a regular hexagon. At this point an association of ideas occurs: hexagons are reminiscent of the cells of a honeycomb, and no sooner has this thought occurred than a bee larva begins to stir in every cell. In a flash every adult larva has developed into a mature bee, and soon these insects fly out into space.

The life span of my bees is short , for their black silhouettes soon merge to serve another function, namely, to provide a background for white fishes. These also, in turn, merge into each other, and the interspacing take on the shape of black birds. Then, in the distance, against a white background, appear little red-bird silhouettes. Constantly gaining in size, their contours soon touch those of their black fellow birds. What then remains of the white also takes a bird shape, so that three bird motifs, each with its own specific form and color, now entirely fill the surface in a rhythmic pattern.

Again simplification follows: each bird is transformed into a rhomb, and this gives rise to a second association of ideas: a hexagon made up of three rhombs gives a plastic effect, appearing prospectively as a cube. From cube to house is but one step, and from the house a town is built up. It's a typical little town of southern Italy on the Mediterranean, with, as commonly seen on the Amalfi coast, a Saracen tower standing in the water and linked to shore by a bridge. [*It is the town of Atrani*.]

Now emerges the third association of ideas: town and see are left behind, and interest is now centered on the tower: the rook and the other pieces on a chessboard. Meanwhile, the strip of paper on which *Metamorphose* is portrayed has grown to some twelve feet in length. It's time to finish the story, and this opportunity is offered by the chessboard, by the white and black squares, which at the start emerge from the letters and which now return to that same word *Metamorphose*."

So ends Escher's lecture.

Escher was intrigued by the idea of metamorphosis, and his series entitled *Metamorphose* can be considered his artistic legacy. It is clear from the text of the lecture he was supposed to give but never did that most of all Escher was interested in the possibility of inserting the idea of transformation, a sense of uncertainty and continuous mutation, into his works. It was that endless process of a shape that transforms into another and yet another that fascinated him. In fact, he was very interested in decoration and in Islamic art, but in his opinion these perfect geometrical shapes were missing the sense of endless, continuous transformation, into infinity. Escher was an artist of infinity. Infinity and metamorphosis.

2 Space is Mathematics

“I seem to detect a firm belief that, in philosophising, it is necessary to depend on the opinions of some famous author, as if our minds should remain completely sterile and barren, when not wedded to the reasoning of someone else. Perhaps he thinks that philosophy is a book of fiction written by some man, like the *Iliad*, or *Orlando Furioso* - books in which the least important thing is whether what is written there is true. This is not how the matter stands. Philosophy is written in this vast book, which continuously lies upon before our eyes (I mean the universe). But it cannot be understood unless you have first learned to understand the language and recognise the characters in which it is written. It is written in the language of mathematics, and the characters are triangles, circles, and other geometrical figures. Without such means, it is impossible for us humans to understand a word of it, and to be without them is to wander around in vain through a dark labyrinth." Galileo Galilei in *The Assayer* (*Il Saggiatore*), published in Rome in 1623.

Thus, without mathematical structures we cannot understand nature. Mathematics is the language of nature. Now let's jump forward a few centuries. In 1904 a famous painter wrote to

Emile Bernard :

“Traiter la nature par le cylindre, la sphère, le cône, le tous mis en perspective, soit que chaque cote d'un objet, d'un plan, se dirige vers un point central. Les lignes parallèles à l'horizon donnent l'étendue, soit une section de la nature. Les lignes perpendiculaires à cet horizon donnent la profondeur. Or, la nature, pour nous hommes, est plus en profondeur qu'en surface, d'oμu la nécessite d'introduire dans nos vibrations de lumière, représentée par les rouges et le jaunes, une somme de bleutes, pour faire sentir l'air." [15]

The art historian Lionello Venturi commented that in Cezanne's (the artist in question) paintings there are no cylinders, spheres and cones, so the artist's quote represents nothing but an ideal aspiration to an organization of shapes transcending nature.

During the period when Cezanne was painting, and even a few years earlier, the panorama of geometry had changed since Galileo's time. In the second half of the 19th century geometry had mutated significantly. In a letter of December 1799 Gauss wrote to Farkas Bolyai on his tentative to prove the Fifth Postulate of the Elements of Euclid, starting from a demonstration by absurd: “My works are very advanced but the way in which I am moving is not conducing to the aim I am looking for, and that you say to have reached. It rather seems to put in doubt the exactness of geometry.” Gauss never published his results on this particular topics in his lifetime. In 1827 he published the “Disquisitiones generales circa superficies curves” in which he introduce the idea of studying the geometry of a surface in a “local way” without minding its immersion in a three dimensional space, studying the invariant properties of the surfaces. He also introduces the idea of the curvature of the surface. Between 1830 and 1850 Lobacevskij and Bolyai built the first examples of non-Euclidean geometry, in which the famous fifth postulate by Euclid was not valid. Not without doubt and conflicts, Lobacevskij would later call his geometry (which today is called non-Euclidean hyperbolic geometry) imaginary geometry, because it was in such strong contrast with common sense. For some years non-Euclidean geometry remained marginal to the field, a sort of unusual and curious form, until it was incorporated into and became an integral part of mathematics through the general ideas of G.F.B. Riemann (1826-1866). In 1854 Riemann held his famous dissertation entitled *Ueber die Hypothesen welche der Geometrie zur Grunde liegen* (On the hypotheses which lie at the foundation of geometry) before the faculty of the University of Göttingen (it was not published until 1867). In his presentation Riemann held a global vision of geometry as the study of varieties of any dimension in any kind of space. According to Riemann, geometry didn't necessarily need to deal with points or space in the traditional sense, but with sets of ordered n-ples. [2]

In 1872 in his inauguration speech after becoming professor at Erlangen (known as the *Erlangen Program*), Felix Klein (1849-1925) described geometry as the study of the properties of figures with invariant character in respect to a particular group of transformations. Consequently each classification of the groups of transformations became a codification of the different types of geometry. For example, Euclidean plane geometry is the study of the properties of the figures that remain invariant in respect to the group of rigid transformations of the plane, which is formed by translations and rotations.

Jules Henri Poincaré held that “the geometrical axioms are neither synthetic a priori intuitions nor experimental facts. They are conventions. Our choice among all possible conventions is guided by experimental facts; but it remains free, and is only limited by the necessity of avoiding every contradiction, and thus it is that postulates may remain rigorously true even when the experimental laws which have determined their adoption are only approximate.

In other words the axioms of geometry are only definitions in disguise. What then are we to think of the question: Is Euclidean geometry true? It has no meaning. We might as well ask if the metric system is true and if the old weights and measures are false; if Cartesian coordinates are true and polar coordinates are false. One geometry cannot be more true than another; it can only be more convenient. Euclidean geometry is and will remain the most convenient." [2]

Poincaré, in *Analysis Situs* (Latin translation of the Greek), published in 1895, is also responsible for the official birth of the sector of mathematics which today is called *Topology*:
“As far as I am concerned, all of the various research that I have performed has brought me to *Analysis Situs* (literally analysis of place)".

Poincaré defined topology as the science that introduces us to the qualitative properties of geometric figures not only in ordinary space, but also in more than 3-dimensional space. Adding the geometry of complex systems, fractal geometry, chaos theory and all of the “mathematical” images discovered (or invented) by mathematicians in the last thirty years using computer graphics, it is easy to see how mathematics has contributed to changing our concept of space - the space in which we live and the idea of space itself. Because mathematics is not merely a means of measurement in recipes, but has contributed, if not determined, the way in which we understand space on earth and in the universe. Specifically in regards to topology, the science of transformations, and the science of invariants. An example is Frank O. Gehry's project for the new Guggenheim Museum in Manhattan, an even more stimulating and more topological project than that of the Guggenheim in Bilbao.

There is certainly a remarkable cultural leap: construction using techniques and materials that allow for the realization of an almost continuous transformation, a sort of contradiction between the finished product and its distortion.

It is interesting to note that the study of contemporary architecture begins with the instruments that mathematics and science make possible; more than technical instruments, cultural instruments. It is important to mention that the discovery (or invention) of non-Euclidean geometry and of the higher dimensions (from the fourth on), the new idea of space to summarize, is one of the most interesting examples of the profound repercussions that mathematical ideas will have on humanistic culture and on art.
Here are the elements necessary to give sense to the word *Space*.

3 Fundamental Elements

Without a doubt the first element is the space that Euclid outlined, with definitions, axioms, the properties of objects that must find room in this space. Perfect space, Platonic space. Man as the blueprint and measure of the universe, an idea which spans centuries. Mathematics and geometry that explain everything, even the form of human beings: *The Curves of Life*, title of the famous 20th century book by Cook, who could not have fathomed how true it could be to find mathematical curves in forms in nature, including those which make up the beginnings of life. From the famous 1914 book by D'Arcy Thompson *On Growth and Form*, to Rene Thom's catastrophe theory, to complexity and the Lorentz effect, to non-linear dynamic systems.

The second element is freedom; mathematics and geometry seem to create an arid realm. For those who are not interested in mathematics, and never studied it with interest in school, it is difficult to understand the intense emotion that mathematics can provoke, and to realize that mathematics is an extremely creative field. It is not only a realm of great freedom where inventions (or discoveries) of new subjects, theories, and fields of research are made, but also where problems are invented. And because mathematicians generally does not need considerable financial resources, it is not only a realm of freedom but of imagination. And of course rigour. Rigorous reasoning.

The third element to consider is how all of these ideas are transmitted and assimilated, perhaps not completely understood, or only heard in passing by various sectors of society. In her book *New Bidimensionality* [12] in the chapter “Digital Technologies and New Surfaces " the architect Alicia Imperiale writes:

“Architects freely appropriate specific methodologies from other disciplines. This can be attributed to the fact that ample cultural changes take place more quickly in contexts other than architecture". She adds: “Architecture reflects the changes that take place in culture, and according to many people, at a painfully slow speed. Architects, constantly trying to be on the forefront, believe that the information borrowed from other disciplines can be rapidly assimilated into architectural design. Nevertheless the translatability, the transfer from one language to another, is a problem. Architects look more and more frequently to other disciplines and other industrial processes for inspiration, and use an increasing amount of computer design and software for industrial production that was originally developed for other sectors".

Later she reminds us that: “It's interesting to note that in the information era disciplines that were once separate are bound to each other through an international language: the digital binary code". Do computers solve all of our problems?

The fourth element is computer, the graphic computer, the ultimate logical and geometrical machine, the fulfilment of an idea of an intelligent machine that is able to resolve various, diverse problems if we are able to make it understand the language we use. The genius idea of a mathematician, Alan Turing, brought to completion thanks to the stimulus of war. A machine built by man, using a logic built by man, created by man. A sophisticated instrument, irreplaceable not only in architecture. An instrument.

The fifth element is progress, the word progress. Can we speak of progress considering non-Euclidean geometry, new dimensions, topology, the explosion of geometry and mathematics in the 20th century? We can speak of knowledge, but not in the sense that new results take the place of previous ones.

Mathematicians often say (referring to an Italian saying): “Mathematics is like a pig, nothing is wasted, eventually even the most abstract and senseless things become useful". Imperiale writes that topology is an integral part of Euclidean geometry. That what escaped the author of this article is what the word space means in geometry. Words. That changing geometry is necessary in order to confront problems that are different because of the difference in the structure of space. Space is the properties, not the objects contained. Words.

The sixth element is words. One of the great gifts of humans is the ability to name things. Often in naming" we use words that are already in use. This habit is problematic because hearing these words we have the impression that we understand, or at least can guess, the meaning. In mathematics this has happened frequently in recent years with words like fractal, catastrophe, complex, hyperspace. Symbolic words, metaphors. Even topology, dimensionality, and seriality have become part of a common lexicon, at least among architects.

One word will have great importance for the idea of space: topology. For more details see *Mathland: from Flatland to Hypersurfaces* [6].

4 Topology

“In the middle of the nineteenth century there began a completely new development in geometry that was soon to become one of the great forces of modern mathematics." Courant and Robbins in the famous book *What is Mathematics?*[3] “The new subject, called analysis situs or topology, has as its object the study of the properties of geometrical figures that persist even when the figures are subjected to deformations so drastic that all their metric and projective properties are lost."

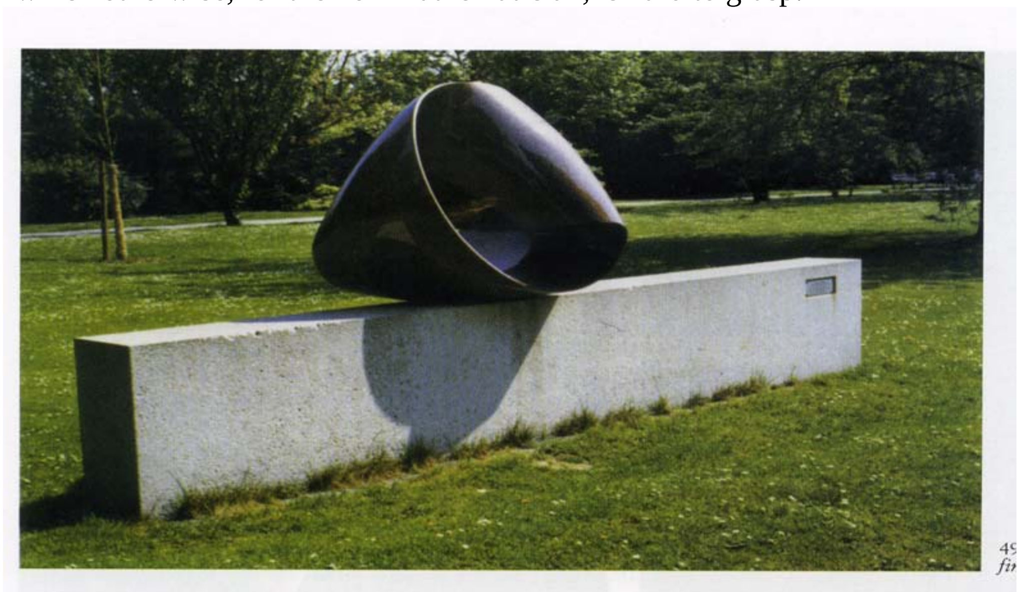
Poincaré defined topology as *the science that introduces us to the qualitative properties of the geometric figures not only in ordinary space but in more than 3-dimensional space*.
Thus topology is the study of the properties of geometric figures that, undergoing intense distortions which cause them to lose all of their metric and projective qualities, for example shape and size, are still invariant. In other words, the geometrical figures maintain their qualitative properties. We can consider figures that are made of materials that can be arbitrarily deformed, that cannot be lacerated or welded: there are properties that these figures conserve even when they are deformed.

In 1858 the German mathematician and astronomer August Ferdinand Moebius (1790-1868) described for the first time in a work presented to the Academy of Sciences in Paris a new surface of three-dimensional space, a surface which is known today as the *Moebius Strip*. In his work Moebius described how to build (quite simply) the surface which bears his name. Among other things, the Moebius Strip is the first example of a non-orientable surface – it is impossible to distinguish between two faces.

Courant and Robbins write: “At first, the novelty of the methods in the new field left mathematicians no time to present their results in the traditional postulational form of elementary geometry. Instead, the pioneers, such as Poincaré, were forced to rely largely upon geometrical intuition. Even today [*Courant and Robbins' book is from 1941*] a student of topology will find that by too much insistence on a rigorous form of presentation he may easily lose sight of the essential geometrical content in a mass of formal detail."

The key word is geometrical intuition. Obviously over the years mathematicians have tried to bring topology into the realm of more rigorous mathematics, but there is still a strong sense of intuition involved. These two aspects, the distortions which maintain some of the geometrical properties of the figure, and intuition play an important role in the idea of space and shape from the 19th century to today. Some of the topological ideas were *sensed* by artists and architects in the past decades, first by artists, then much later by architects.

These shapes that so interested Max Bill in the 1930s [1] could not go unnoticed in architecture, although it took some time: until the diffusion of computer graphics, which allows the visualization of the mathematical objects discussed, thus giving concrete support to the intuition which otherwise, for the non-mathematician, is hard to grasp.



Max Bill, *Endless Ribbon*, marble, 1936.

5 International Venice Architecture Exhibition 2004

In 2004 I attended the International Architecture Exhibition in Venice.

The theme of the exhibition was *Metamorph*.

“Many of the great creative acts in art and science can be seen as fundamentally metamorphic, in the sense that they involve the conceptual re-shaping of ordering principles from one realm of human activity to another visual analogy. Seeing something as essentially similar to something else has served as a key tool in the fluid transformation of mental frameworks in every field of human endeavour. I used the expression `structural intuitions' to try to capture what I felt about the way in which such conceptual metamorphoses operate in the visual arts and the sciences. Is there anything that creators of artefacts and scientists share in their impulses, in their curiosity, in their desire to make communicative and functional images of what they see and strive to understand?

The expression `structural intuitions' attempts to capture what I tried to say in one phrase, namely that sculptors, architects, engineers, designers and scientists often share a deep involvement with the profound sense of involvement with the beguiling structures apparent in the configurations and processes of nature - both complex and simple. I think we gain a deep satisfaction from the perception of order within apparent chaos, a satisfaction that depends on the way that our brains have evolved mechanisms for the intuitive extraction of the underlying patterns, static and dynamic."

These are the words of Martin Kemp, an art historian specialized in the relationship between art and science in the article - *Intuizioni strutturali e pensiero metamorfico nell'arte, architettura e scienze*, in *Focus* [11], one of the volumes that make up the catalogue of the 2004 Venice International Architecture Exhibition.

In his article Kemp writes mainly about architecture. The image accompanying Kemp's article is a project by Frank O. Gehry, an architect who obviously cannot be overlooked when discussing modern architecture, continuous transformation, unfinished architecture, infinite architecture.

Kurt W. Forster, curator of the exhibit, discusses the great complexity, the enormous number of variations developed through essential technological innovations, the continuous surfaces in transformation. He cites the mathematician Ian Stewart's article entitled *Nature's numbers: discovering order and Pattern in the Universe* (1995). Some key words: pattern, structure, motif, order, metamorphosis, variations, transformations, mathematics [10].

Forster writes: “Recent buildings predicated upon continuous surfaces make clear that they depend in conception and realization on the use of computer technology. The single hinge upon which they turn is the computer. Any number of hybrid transformations and exchanges between traditional methods and rapidly developed software have multiplied and modified the process of elaboration and realization of projects. Hardly a method that cannot be integrated within the `loop' of numeric calculations, but more consequential than the flexibility of elaboration and the constant back-and-forth between image and object, is the fact of architecture's migration to the realm of the virtual and simulated."

Forster continues regarding Gehry: “What really interests Gehry is the process, in the sense of dynamic process used to achieve a structural and aesthetic result."

These words, projects and ideas at the 2004 Exhibition were visually closely connected to the ties between mathematics, architecture, topology and transformation that I writing about. The layout of the pavilion of the Venice Exhibition, which caused quite a stir, was assigned to two famous architects: Hani Rashid and Lise Anne Couture. In an article for the catalogue entitled *Asymptote, the Architecture of Metamorph*, they summarized their project as follows:

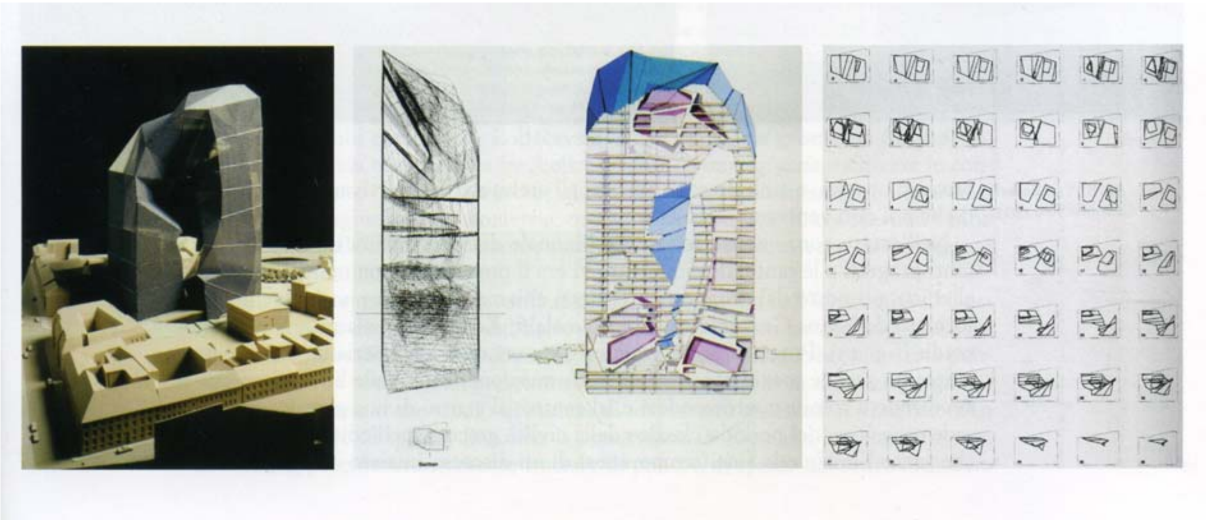
“Asymptote's transformation of the *Corderie* emerged from computer-generated morphing animation sequences derived from utilizing rules of perspective geometry with the actions and dynamics of torquing and `stringing' the space of the *Corderie*. The experience of *Metamorph* is spatial in that it is itself an architectural terrain of movement and flow. The exhibition architecture - from installation and exhibition design to graphic identity and catalogue design - provides for a seamless experience that fuses the *Arsenale*, *Giardini* and Venice, making explicit a contemporary reading of architecture where affinities and disparities co-mingle to produce the effects of flux and metamorphoses of form and thinking."

One of the studies of the layout was described quite significantly as follows:

“Study of the topological surface that develops in the space of the *Corderie* and determines the movements and the curvatures used in designing levels."

Let's backtrack a bit, to the early Nineties. In 1992 the architect Eisenmann (who won the *Leone d'Oro* for his architectures at the 2004 exhibition) and his collaborators projected a skyscraper in Berlin, the *Max Reinhardt Haus*. The structure of the enormous building is based on the topological surface, the Moebius strip. In 1993 Ben van Berkel planned and built the

Moebius House. So these two projects held the place of honour in the large hall of the *Corderie*, as if a reminder of an important step in contemporary architecture, in the idea of transformation, of metamorphosis. An explicit reference to topology.



P. Eiseman, *Max Reinhardt House*, project, Berlin, 1992.

Until a few years ago these were utopic projects - and many still are; architects enjoyed creating projects that were never carried out.

6 Toward a Virtual Architecture

In the chapter *Topological Surfaces* Alicia Imperiale writes [12]:

“The architects Ben van Berkel and Caroline Bos of UN Studio discuss the impact of new scientific discoveries on architecture. The scientific discoveries have radically changed the definition of the word `Space', attributing a topological shape to it. Rather than a static model of constitutive elements, space is perceived as something malleable, mutating, and its organization, its division, its appropriation become elastic".

And the role of topology, from the architect's perspective:

“Topology is the study of the behaviour of a structure of surfaces which undergo deformations. The surface registers the changes in the differential space time leaps in a continuous deformation. This entails further potential for architectural deformation.

Continuous deformation of a surface can lead to the intersection of external and internal planes in a continuous morphological mutation, exactly like in the Moebius Strip. Architects use this topological form to design houses, inserting differential fields of space and time into an otherwise static structure".

Naturally some words and ideas are changed in switching from a strictly scientific field to an artistic and architectonic one. But this is not a problem, nor a criticism. Ideas move freely and each person has the right to interpret and attempt, as with topology, to capture the essence. The role of computer graphics in all of this is essential, it allows the insertion of that deformation-time variable that would otherwise be unthinkable, not to mention unattainable.

Imperiale continues regarding the Moebius Strip:

"Van Berkel's house, inspired by the Moebius Strip (*Moebius House*), was designed as a programmatically continuous structure, that combines the continuous mutation of the dialectic sliding couples that flow into each other, from the interior to the exterior, from the activity of work to that of free time, from the fundamental to the non-fundamental structure."

During the same period Peiter Eisenman was designing the *Max Reinhardt Haus* in Berlin [5].

"The building, composed of arches, made up of intersecting and overlapping forms, presents a unified structure that separates, that compresses, transforms and finally comes back together on the horizontal plane at the height of the attic. The origin of the form is represented in the Moebius Strip, a three-dimensional geometric form characterized by a unique, unending surface that undergoes three iterative operations. In the first, the planes are generated from the extension of the vectors and triangulations of the surfaces... The second iteration overturns the strip, causing an operation similar to that in the first phase, and then appends these surfaces on top of the original form, thus creating a ghost form. The third phase applies an element of Berlinese history to the form itself, wrapping up vast public spaces between the grilled and the base of the ground floor of an already folded surface. Just as the Moebius Strip folds two sides into one surface by folding on itself, the Max Reinhardt Haus denies the dialectic tradition between internal and external and confuses the distinction between public and private."

Both van Berkel's house and Eisenman's project were at the Venice 2004 Biennial to represent the archetypes of topological architecture.

Van Berkel writes that another famous topological object, the Klein bottle, "can be translated into a canal system that incorporates all of the elements that it encounters and causes them to precipitate into a new type of internally connected integral organization." Note that the words integral and internally connected have precise meanings in mathematics.

But this is not a problem because "the diagrams of these topological surfaces are not used in architecture in a rigorously mathematically way, but constitute abstract diagrams, three-dimensional models that consent the architects to incorporate differential ideas of space and time into architecture." [12]

As I mentioned before, architects became aware (albeit rather late) of the new scientific discoveries in the field of topology. And not only did they begin to design and build, but also to reflect.

In her 1999 doctoral thesis *Architettura e Topologia: per una teoria spaziale della architettura*, Giuseppa Di Cristina writes:

"Architecture's final conquest is space: this is generated through a sort of positional logic of the elements, that is through the arrangement that spatial relationships generate; the formal value is thus substituted by the spatial value of the configuration: the external aspect of the form is not as important as the spatial quality. And thus topological geometry, without `measure' and characteristic of non-rigid figures, is not something purely abstract that comes before architecture, but a trace left by that modality of action in the spatial concretization of architecture".

In 2001 Di Cristina edited a book on *Architecture and Science* [4]. In her introduction *The Topological Tendency in Architecture* Di Cristina clarifies that

"The articles that are included here bear witness to the interweaving of this architectural neo-avant-garde with scientific mathematical thought, in particular topological thought: although no proper theory of topological architecture has yet been formulated, one could nevertheless speak of a topological tendency in architects at both the theoretical and operative levels. In particular, developments in modern geometry or mathematics, perceptual psychology and computer graphics have an influence on the present formal renewal of architecture and on the evolution of architectural thought. What mainly interests architects theorizing the logic of curvability and pliability is the significance of the “event", of “evolution", of “process", or the innate dynamism in the fluid and flexible configurations of what is now called *topological architecture*.

Topological architecture means that *dynamic variation* of form, facilitated by information technology, by computer-assisted design, by animation software. The topologification of architectonic forms according to dynamic and complex configurations leads architectural design to a new and often spectacular plasticity, in the footsteps of the Baroque or organic Expressionism."

Stephen Perrella, one of the most interesting *virtual* architects, describes *Architectural Topology* as follows: [13]

“Architectural topology is the mutation of form, structure, context and programme into interwoven patterns and complex dynamics. Over the past several years, a design sensibility has unfolded whereby architectural surfaces and the topologising of form are being systematically explored and unfolded into various architectural programmes. Influenced by the inherent temporalities of animation software, augmented reality, computer-aided manufactured and informatics in general, topological “space" differs from Cartesian space in that it imbricates temporal events-within form. Space then, is no longer a vacuum within which subjects and objects are contained, space is instead transformed into an interconnected, dense web of particularities and singularities better understood as substance or filled space.

This nexus also entails more specifically the pervasive deployment of teletechnology within praxis, leading to an usurping of the real (material) and an unintentional dependency on simulation." Observations in which ideas about geometry, topology, computer graphics, and space time merge. Over the years the cultural nexus has been successful: new words, new meanings, new connections.

7 Mathematics Education: Some Comments

In this article I have tried to discuss some important moments that brought about a mutation in our perception of space. I have attempted to help grasp not only the technical and formal aspects that are essential to mathematics, but also the cultural aspect - using the idea of space in relation to some important aspects of contemporary architecture. I have tried to analyze the way in which the idea of space together with the development of new computer technologies have modified the way not only of working but also of thinking of modern architects. Shapes that were unthinkable can now even be realized. Without the cultural change of the idea of space, without the appearance of the new ideas in geometry, it would have been impossible not only to develop the new computer tools but also, most importantly, to perceive in a different way the space around us, to imagine a world full of shapes, shapes that Humanity never thought of before.

One of the problems of mathematics education is that sometimes it risks being self-centered, losing contact with all the other aspects of human culture, and closing itself up in a sort of ghetto which is difficult to get out of. This also leads to loss of contact between students and teachers. The basic idea behind the projects “Mathematics and Culture”, started in 1975, is to move in exactly the opposite direction. Let people understand that mathematics plays a major role in culture, art, philosophy, literature, the cinema, the theatre, and in the new technologies. In a way, the idea is to move in the direction of a mathematical education that incorporates many of the basic ideas that are at the root of mathematical knowledge. Through conferences, books, exhibitions, films, and with the collaboration of mathematicians from all over the world, it has been possible to build up a vast database of mathematical culture over the last few years. An important part of the “Mathematics and Culture” project is devoted to the visual aspect of mathematics; a book with the title “The Visual Mind 2” has been published by MIT Press, following the publication of “The Visual Mind” MIT Press, in 1993. [7] There is considerable interest in the series of conferences “Mathematics and Culture” held in Venice every year, attended by about three hundred university and high school students. Proceedings are published first in Italian and one year later in English. [8]

This major long-term project is open to anyone who has the ability and the desire to learn more about the cultural aspects of mathematics. But it is also a short-term investment given that many of the initiatives have already become educational programs, and that the projects have already been used in schools, universities, and other cultural institutions. And it’s not by chance that in recent years a number of degree courses focused on mathematical culture have been set up in Italy. But there’s no doubt that at times there has been a lack of conviction on the part of students when their teachers have suggested a deeper awareness of mathematical culture. The goal of these projects is to provide the basic knowledge for a more effective distribution of mathematical education by supplying the knowledge and the tools to better understand the deep links between mathematics and culture.

What all this means is that we are investing in culture which is the prerequisite for any type of didactic or training activity. The aim is to provide the ideas and the stimuli that will then be worked on and modified by the various users in different contexts. The books in the series “Mathematics and Culture” are also becoming useful tools for acquiring information, suggestions, ideas, bibliographic references, thereby creating interest and stimulation. And the projects do not only concern European culture; gradually they are expanding to include the main civilizations in other parts of the world, to counteract the idea that we Europeans have of being at the centre of the world, the only source of true culture.

At the university of Rome “La Sapienza”, starting from the academic year 2005, I started a course for students in mathematics and students in design at the graduate level by the title “Space and Form”. The course is centered on the cultural aspects of mathematics in connection with the arts, architecture, literature, theatre and cinema starting from the second part of the Nineteenth century. The number of students is usually 30 for mathematics and 40 for design. All details on the topics and the references of the course can be found at the website http://www.mat.uniroma1.it/people/emmer, clicking on “students” and then “Spazio e forma”, academic year 2007/2008.

I’d like to end with something that the famous German mathematician David Hilbert wrote in 1928:

“Mathematics knows no races... since it, and the whole world of culture, constitutes a single country.”

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