A PUZZLE RHYME FROM 1782

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ABSTRACT

A puzzle of thirty birds, known in many ancient cultures, is found in an Icelandic spelling book for children from 1782, transposed into Icelandic context so well that it seemed genuinely domestic. Its form is a three-verse rhyme governed by complex Old-Germanic rules of prosody. The birds are sold for units typical for Icelandic middle-age trade commodities, used up to recent times. The composition is completely adjusted to the Icelandic culture and may therefore be considered as Icelandic ethnomathematics of early modern times.

1 Introduction

In an Icelandic spelling textbook for children, dated in 1782, a three-verse rhyme is found with three mathematical attributes:

- The form of the verses is cleverly composed in accord with mathematical rules.
- The content of the verses is a puzzle about thirty birds: ducks, swans and buntings, to be sold for 30 ells of woollen cloth, which leads to a consideration of ancient trade units used in Iceland from medieval times up to the twentieth century.
- The puzzle has its parallel in *Liber Abaci* (1202) by Leonardo Pisano, with roots in many different cultures, the oldest source being Chinese, from the 5th century AD.

These three aspects of the puzzle-poem will be discussed, and the intercultural influences that have brought it into its present form.

In his book *Ethnomathematics – Link between Traditions and Modernity*, D’Ambrosio (2001) states:

> The great motivator for the research program known as Ethnomathematics is to seek to understand mathematical knowing/doing throughout the history of humanity, in the context of interest groups, communities, peoples and nations (D’Ambrosio, 2001: 8).
>
> Culture, which is the set of compatibilized behaviours and shared knowledge, includes values. In the same culture, individuals provide the same explanations and use the same material and intellectual instruments in their everyday activities. The set of these instruments is manifested in the manners, modes, abilities, arts, techniques – in the **tics** of dealing with the environment, of understanding and explaining facts and phenomena, of teaching and sharing all this, which is the **mathema** of the group, of the community, of the **ethno**. That is, it is their **ethnomathematics** (2001:24).

In the light of these quotations, reasons for allowing both the composition of the verses and the specific Icelandic trade units to be considered as Icelandic ethnomathematics will be proposed. Finally the absorption of an international mathematical heritage into different cultures, its preservation and assimilation will be discussed.

2 Icelandic Trade Units

Iceland, an island in the North Atlantic, was settled by Norwegian Vikings around 900 A.D. It submitted to the Norwegian King in the thirteenth century, and late in the fourteenth century it became a part of the Danish realm. There were cultural interchanges with Europe: from the seventeenth century mainly through Copenhagen, where Icelandic students had some priority for admission to the University's halls of residence.

Icelandic society remained relatively stagnant until the late nineteenth century; characterized by subsistence farming and little internal trade. Foreign trade was mainly in the form of barter, also due to a lack of proper monetary units. From the 1740s official regulations prescribed public education in reading and Christian religious instruction, provided by the families, while no law existed on instruction in arithmetic until 1880.

The ancient trade units, called *landaurar*/landpennies, were a *hundrað*/hundred, equivalent to 120 *alin*/ells (an ell was originally approx. 60 cm of woollen cloth), 240 (valid) fish or one cow. The ell was therefore equal in value to two fish. A unit equivalent to 120 (ten dozen) ells was called the "large hundred." The large hundred existed in medieval times as an alternative to the regular hundred, as did a "large thousand" or 1200 (Karker, 1974, 116). These are treated in several arithmetic textbooks from the 18th century, printed (Hatton) and handwritten (Lbs. 1694, 8vo).

A hundred was the equivalent of a cow, i.e. a mature cow without defect in spring, or six ewes, woolly with lambs, in spring. The monetary value of *landaurar* was variable, and up to and beyond the 18th century there were differences of opinion on how to compute it. Farms were also measured by hundreds. An average farm, valued at 20 hundreds, was supposed to support livestock of 20 cows or 120 sheep (*Statistics Iceland*, 1997). Each farm had its assessment from medieval times. The value was only re-assessed if the land was damaged by sand, floods etc. The area of land was only measured when estimating time for mowing it.

This Icelandic trade unit system existed from medieval times up to the 20th century. Its main use was in barter. Proper monetary units gradually took over from the 19th century. For the modern reader it is confusing that ells were also used as regular units of measurement for length before the adoption of the metric system in 1907. Ells as trade units are found in an arithmetic textbook dated as late as in 1938 (Daníelsson, 1938, 149), and as a measure of length for the primary level in the 1960s (Bjarnason, vol. 4, 12).

The *landaurar* trade-unit system was exclusively Icelandic, but was also familiar to the Danes who ran the trade with Iceland in the period 1602–1787 as a monopoly enterprise. A Danish textbook, *Compendium Arithmetic*, has a special section with the heading *Islandske Taxt*/Icelandic Rate (Matthisen, 1689, 33-34). It states that one thousand fish equal 1 *lest*/ton, while 1 *lest* equals 1200 fish, again a mixture of the decimal and duodecimal system.

The unique *landaurar* system arose from Iceland's special circumstances, in that it had two main trading commodities woollen cloth measured in ells, and from the fifteenth century onwards also fish while having no access to other resources, such as metals for coinage.

The *landaurar* may therefore be considered as ethnomathematics of the recent past of Icelanders, as they persisted in Iceland long after barter trade dwindled in the other Nordic countries.

3 Icelandic Rhymes as Ethnomathematics

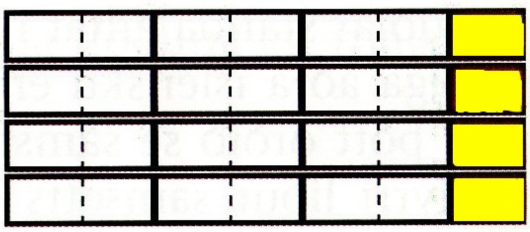
During the 19th century the population of Iceland rose from 47 to 74 thousand. The country was extremely poor and every effort was needed to protect the vernacular from influences from Danish, the language of the colonial authorities. Printed books were relatively rare but, along with the widespread custom of writing up all kinds of knowledge in manuscripts together with another widespread custom, to tie the knowledge in rhymes, they provided a way to preserve and develop the language. Up to the late 19th century ordinary people in Iceland had few opportunities, or reasons, to study mathematics and apply it, except for the barter trade with foreign merchants.

Leisure time was limited for artistic activities with a mathematical aspect, such as wood carving or embroidery, not least due to the lack of natural light in winter. People lived in houses of turf and stone, where small semi-opaque membranes were used as window-panes. At 64-66ºN latitude in the middle of winter the sun is only aloft for a couple of hours a day. Therefore, important outlets for artistic inclinations were phonetic. These conditions supported the maintenance of composing verses, governed by complex rules of prosody, an old Germanic tradition which has survived up to present day.

A four-line stanza, the quatrain, is composed of four lines, where each line comprises 3–5 two-syllable sets, often ending in a half-set. The two-syllable sets have alternately high and low stresses. The first and third lines of each verse and the second and fourth must have the same number of syllables. Exceptions may be made with unstressed syllables.

The rules of the poetry in the four-lined verse form are for instance *alliteration*, requiring that two words in the first line begin with a vowel or the same consonant, placed in the line according to rules on stress of the syllables. The first stressed syllable in the second line must also begin with a vowel or the same consonant. This alliterative pattern is repeated in the third and fourth lines. Of the three alliterative syllables, only one may be placed in a low-stress two-syllable set. The lines also have *end-rhyme*, and in more sophisticated versions they may also have internal rhyme.

The three-verse poem in question has three and a half two-syllable sets in each line and looks as follows. The end rhyme is sequential and is marked in yellow. Alliteration is allowed in the 1st and 3rd syllable set, the 2nd and 3rd and the 3rd and the half 4th in the first line and the 1st syllable set in the second line, and likewise in the third and fourth line.



(Einarsson and Björgvinsson, 2001)

Below, the alliteration of the puzzle-rhyme is marked in red, the high-stressed syllables are underlined and the rhymes are highlighted yellow. For clarification an attempt is made to translate the middle rhyme into English under the same alliterative restrictions:

Vinnumaður vildi fá
verkalaunin bónda hjá,
Eg sá fljúga fugla þrjá,
flýtum oss að veiða þá.

Andir fyrir alin tvær, álptin jöfn við fjórar þær, Titlingana tíu nær, tók ég fyrir alin í gjær.	Two are ducks to equal ell even is swan to four, I tell A bunch of buntings, free to sell bade I yest'day ten an ell.

Af fuglakyni þessu þá,
til þrjátíu álna reikna má,
Þó vil eg ekki fleiri fá,
en fugl og alin standist á.
(Lbs. 2397).

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The constraints put on the language by the rules of the poetry often make it difficult to comprehend. This is, however, counteracted by the fact that Icelandic has preserved declensions of nouns, pronouns and adjectives, which offer some freedom in word order.

Reading carefully, one gathers that there are three kinds of birds that the worker wants to catch for his wages: Two ducks for an ell, i.e. a duck costs ½ an ell; a swan is equal to four ducks, two ells each; and ten buntings are equal to an ell, so each bunting costs 1/10 ell. The worker wants to catch 30 birds equivalent to 30 ells.

This story looks completely Icelandic. The birds are Icelandic, as is the alliteration of the rhyme, unknown elsewhere, and the ells are medieval barter trade units, long obsolete in other European countries. The composition is completely adjusted to the Icelandic culture and may therefore be considered as Icelandic ethnomathematics of early modern times. It is deeply rooted in Icelandic culture and its intercultural content has assimilated that culture.

4 The Thirty Bird Puzzle

It is well known in many cultures that foreign content is dressed up and seamlessly adapted to a domestic context (V. Ólason, 1967). In spite of its Icelandic look, the thirty-bird puzzle existed long before the settlement of Iceland. It is found in various versions in many different cultures. J. Tropfke mentioned a hundred-bird problem in his *Geschichte der Elementarmathematik* (1980). Its history goes back to 485 A.D. in China, where the commodities to be purchased are 100 fowls for 100 sapeks. A cock costs 5 sapeks, a hen 3 sapeks and three chickens 1 sapek. That puzzle has three solutions.

A problem of the same kind is found by Arabic Abu Kamil (850–930), in India with Bhaskara II (1114–1185) where the birds are of four types, in Persia, and in the Byzantine Empire, from whence it arrived in the West to be taken up by Leonardo Pisano, who presents many versions of different levels of difficulty (Tropfke, 1980: 613-616).

In *Liber Abaci* (1202) by Leonardo da Pisa, a problem comparable to the puzzle in the 1782 Icelandic spelling book is found:

> A certain man buys 30 birds which are partridges, pigeons and sparrows, for 30 denari. A partridge he buys for 3 denari, a pigeon for 2 denari, and 2 sparrows for 1 denaro, namely 1 sparrow for $\frac{1}{2}$ denaro. It is sought how many birds he buys of each kind; you divide the 30 denari by the 30 birds; the quotient will be 1 denaro. You therefore say, I have money with $\frac{1}{2}$, and money with 2, and money with 3, and I wish to make money with 1 (Siegler, 2002: 256).

As was the custom in arithmetic textbooks up to the 19th century, the solution is explained. Leonardo then counts together sets of birds worth the same number of denari:

4 sparrows and 1 partridge make 5 birds for 5 denari,

2 sparrows and 1 pigeon make 3 birds for 3 denari.

Leonardo multiplies each of the two sets by a number such that the sum equals 30, and finds that $3 \cdot 5 + 5 \cdot 3 = 30$

A triple set of 4 sparrows and 1 partridge makes 12 sparrows and 3 partridges, a total of 15 birds.

A fivefold set of 2 sparrows and 1 pigeon makes 10 sparrows and 5 pigeons, also 15 birds.

The answer is thus 22 sparrows, 3 partridges and 5 pigeons, a total of 30 birds for 30 denari.

Leonardo provides more examples. Next he tells a story of a partridge which costs 2 denari, a pigeon for 1/2 denaro and a sparrow for 1/4 denaro. A total of 12 birds are needed for 12 denari. Then:

1 partridge and 2 pigeons make 3 birds for 3 denari and

3 partridges and 4 sparrows add up to 7 birds for 7 denari.

Leonardo finds no numbers to multiply 3 and 7 to make 12, so he doubles the number of birds and denari which gives $1 \cdot 3 + 3 \cdot 7 = 24$.

The first set of 1 partridge and 2 pigeons makes 3 birds for 3 denari

The second set tripled makes 9 partridges and 12 sparrows, 21 bird for 21 denari.

The total is thus 10 partridges, 2 pigeons and 12 sparrows for 24 denari.

Now the solution is easily found to be 5 partridges, 1 pigeon and 6 sparrows for 12 denari.

As the reader may already have gathered, these are simple examples of Diophantine equations, attributed to the Greek Diophantus of Alexandria of the 2nd century A.D.

5 Solution to the Icelandic Puzzle

Applying the same method for solving the Icelandic puzzle, a duck costing ½ an ell, a swan for 2 ells and a bunting for 1/10 of an ell, gives:

1 swan and 2 ducks make 3 birds for 3 ells and
9 swans and 10 buntings make 19 birds for 19 ells,

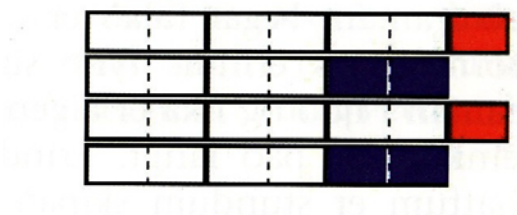
which demands two positive whole numbers to multiply 3 and 19 to add up to 30.

The number 19 takes great space and leaves no room for a solution, so doubling is tried and the solution 1·3 + 3·19 = 60 so

1 swan and 2 ducks make 3 birds for 3 ducks.
3·9 swans and 3·10 buntings make 57 birds for 57 ells.
A total of 28 swans, 2 ducks and 30 buntings would cost 60 ells so the solution is that14 swans, 1 duck and 15 buntings are worth 30 ells.

The solution is also stated in a verse, where the end-rhymes come up in pairs, two and two, a more conventional form. A prosaic translation is attached as well as a schematic picture of its pattern.

Álptir fjórtán eru hér til,	Fourteen swans exist,
og einum titling fleira,	and one more bunting,
á einni gjöri eg önd þér skil,	I answer for one duck,
ekki færðu meira.	you will not have more.



(Einarsson and Björgvinsson, 2001)

6. Publications of the Icelandic Puzzle

This puzzle was included in a comprehensive collection of Icelandic puzzles, nursery rhymes and various games (Árnason and Davíðsson, 1887, I, 129–130), which demonstrates that the puzzle belonged to pastime activities of the general public in the 19th century.

Björn Gunnlaugsson (1788–1876) discussed this puzzle in a manuscript of the second volume of his mathematics book, *Tölvísi* (1865), a volume that was never published. He tells the reader that the puzzle was printed in a spelling booklet (Pálsson, 1782), used by his dear mother to teach him when he was, as he said, a spelling child. The mother's effort bore excellent fruit, as Gunnlaugsson became the first Icelandic learned mathematician. The author of the spelling book, the reverend Pálsson, was a learned man and a poet. He studied in Copenhagen in the early 1740s (Ólason, 1949, 205–206). He may have become acquainted with the international puzzle there and composed the Icelandic verses, but nothing definite is known about that.

In his manuscript (Lbs. 2397), mathematician Gunnlaugsson used the puzzle as a demonstration of how to solve Diophantine equations by a method of chain fractions. Gunnlaugsson's equation is 4x + 19y = 270 where x is the number of ducks and y is the number of swans. A quick inspection offers the solution (x y) = (1, 14). Gunnlaugsson's method is very elaborate but he comes up with four solutions, of which three, (20, 10), (39, 6) and (58, 2), produce 0 or negative numbers of buntings. As a farmer's son Gunnlaugsson was a practical man and interpreted even the negative solutions sensibly. However, it is not likely that more than a couple of Icelanders would have understood his solution of the 30-bird puzzle. It would hardly have promoted interest in the puzzle among the readers to study his solution, even if it had ever been published. Gunnlaugsson was not, of course, focussing on the puzzle, but on the solution technique.

The first 400-page volume of his book *Tölvísi* was published by the Icelandic Literary Society, the same as later published the puzzle collection. The society took pride in introducing all kinds of knowledge to the people of Iceland, who were seeking independence from Denmark and a better life in the second half of the 19th century. However, the book was called "one of these books that everyone praises but no-one reads" (P. + B., 1883).

The next person to bring up the 30-bird verses was mathematician Dr. Ó. Daníelsson (1877–1957) in his *Arithmetic* (1938, 149) for lower secondary level schools. Daníelsson, who only printed the last two verses as an exercise, worked out the first half of its solution, which was unusual for him. He noted that the number of buntings is 30 – x – y, where x and y are the numbers of ducks and swans. The price of the lot is then $\frac{1}{2}x + 2y + \frac{30 - x - y}{10} = 30$ to be reduced to the equation 4x + 19y = 270. At that point Daníelsson remarked that the solution should be easily solved, considering that x + y must be less than 30, thus trying to meet his pupils half-way. Daníelsson was using the puzzle, which may have been known to some of the young people, to introduce the technique of translating a general text into a mathematical equation. However, by that very act their minds were led away from the more primitive and possibly more natural way to search for a solution, towards the synthetic technique of setting up and solving an equation, even if they were allowed to use guessing at the end.

7 Summary and Conclusions

This ancient puzzle, transposed once upon a time into Icelandic context so well that it seemed genuinely domestic, gives rise to some comments. It was printed in a spelling textbook in the late 18th century and included in a collection of Icelandic puzzles in the late 19th century. These facts point to that puzzles of this kind were widely known, although there was no formal instruction in arithmetic. The domestic context of the puzzle   the verses, the birds and the trade units   facilitated its understanding.

Instead of formal arithmetic teaching, first prescribed in 1880, the Icelanders trained their mathematical intuition by the rhythm of verses and the complex rules governing them: length of lines, length of syllable sets, and the placement of the alliterative pattern, in addition to the regular rhyme. More advanced training was gained by coping with arithmetic puzzles as a pastime on long dark winter nights, or when watching over sheep herds on long, light summer nights.

Iceland's first two mathematicians, Gunnlaugsson and Daníelsson, seized the opportunity to use the puzzle to demonstrate advanced mathematical technique for its solution. Both these men were rooted in the old society of farming, growing up as farmers' sons in the environment of livestock and birdlife. Their intention was probably to facilitate people's mathematical understanding of their novel technique. But, by that very act, attention to the puzzle was transferred from the act of problem-solving to the advanced technique of setting up equations. Solving a puzzle by setting up an equation came to be viewed as the correct and sophisticated way, which reached only the privileged few, and that contributed to the alienation of the common people from grappling with puzzles. It disrupted the verbal tradition.

In addition, society was rapidly changing. The medieval trade units disappeared completely in the twentieth century, and the rural environment grew alien as the population moved to towns. The puzzle verses have not been seen in print in the last half a century. However, the art of composing four-line stanzas is still widely pursued, often serving as a comic mirror of the events of the day.

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