The Alto Douro “wine coopers' mathematics”

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ABSTRACT

In this study, we identify some mathematical procedures used in a traditional job of Alto Douro’s Portuguese Region – wine cooper. We present and discuss the way that the wine coopers use (most of the time, intuitively) well-known mathematical notions to do certain calculations. We propose some tasks to develop with students in mathematical classes in this region, even at the elementary levels, that can be adapted in other regions or countries.

1. Introduction

In this study, we identify mathematical processes used in the Alto Douro's Portuguese Region (located in the interior north-eastern of Portugal) inside of a traditional job – *wine cooper* (in Portuguese: *tanoaria*). We work in a complex Portuguese region with specific social and cultural characteristics that will be introduced in a few words in Section 2 of this paper.

In particular, we recognize some of the mathematical procedures used in this traditional job, we will present and discuss the way that the wine coopers use (most of the time, intuitively) well-known mathematical notions to do certain calculations. We have found several interesting mathematical elements that we would like to share.

Our work began with the interviews of three wine coopers on this region. In our interviews, we realize that the main difficulty of this traditional job is what wine coopers call “*working the round*”. This expression is suggestive of the mathematical concepts involved in building the barrels.

We also present our mathematical interpretation of the processes used by Alto Douro’s wine coopers interviewed. This interpretation is made by comparing the oral description of the empirical processes with the *formal* mathematical knowledge. All these aspects will be described in Section 3.

Along this study we have concluded that mathematical knowledge is present in wine coopers’ activity and at a level that may be used within mathematics classroom. This led us to promote and accomplish experimental activities within the classroom (e.g., Costa *et al.*, 2007a). We think that this is a promising way to emphasize the value of their region *to* and *with* students, teachers, school and community throughout mathematics classroom.

Further more we remembered the questions raised by the ICMI Study (van Maanen, 2007) and we also identify our work with that study:

"The Study should explore the benefit to learners of realising both that they have a local heritage from their direct ancestors (...) but also that every culture in the world has contributed to the knowledge and experience base made available to today's learners."

Therefore, in Section 4, we propose some tasks to apply in mathematical classrooms in this region, even at the elementary levels (e.g., Costa *et al.*, 2007b). We hope that some teachers in our region will adhere to these proposals, and that in other regions or countries, this one could inspire similar works.

2. Alto Douro's Portuguese Region – region of wine production

The Alto Douro's Portuguese Region is located in the interior northeastern of Portugal and near the Douro's river. This region is quite different from the others Portuguese regions due to its social, cultural and geographic characteristics. It has always been dependent on agriculture, namely with the production of wine (a well-known example is the production of "Oporto Wine") and olive oil. Since this region is much closed with the agriculture it promoted the appearance of several traditional jobs related with these fields work. For example, we have wine coopers, basket-makers, tin-makers.

We focus this study in one of these traditional jobs, namely the wine cooper. As in http://en.wikipedia.org/wiki/Cooper_(profession),

"*(...) traditionally, a **cooper** is someone who makes wooden staved vessels of a conical form, of greater length than breadth, bound together with hoops and possessing flat ends or heads. Examples of a cooper's work include but are not limited to casks, barrels, buckets, tubs (...)*"

and

"*(...) the **wine cooper** is craftsman that makes or repairs all kinds of wood (chestnut and oak) barrels mainly to collect, transport and wine making. Nowadays those barrels are only used to keep the special Oporto wine fermentations and now the main activity of wine coopers is handicrafts (...)*".

Figure 1 – Examples of handicrafts made by wine coopers.

This traditional job has almost disappeared, and the same happens with the agricultural jobs of our country and also the youngsters prefer other jobs. As it is mentioned by Monteiro (2001),

"*Esta região devido às mais variadas razões, tem visto a sua população não só diminuir, como também envelhecer. Cada vez nascem menos crianças e são as pessoas de mais idade que se mantêm ligadas à sua terra, vendo partir os jovens em busca de melhor qualidade de vida, longe das actividades rurais.*"[1]

This is also suggested by the Wikipedia text (2007):

[1] "Due to several reasons the population of this region is decreasing and getting older. Nowadays less children are born and the older people are those who still stay in their land, while the young people leave in order to look for a better life, faraway of the rural activities."

"Many people in the region still live in small villages. Traditionally these villages were cut off from the coast due to the lack of good roads, and suffered the effects of poverty and isolation. Immigration was often the only option. Today the situation has improved with better roads, but most villages are still losing population as youngsters there keep moving to bigger towns for better living."

Our actual experience close to some Portuguese schools, allowed us to realize that, beyond the difficulties in learning Mathematics identical to the ones recognised at a national level, in this region other difficulties exist resulting upon these geographical and socio-cultural characteristics. As quoted by Goodwin and Stumper (2007), we also agree that

"[c]ulture provides a means for students who usually do not fully participate in the mathematics class, to make connections to mathematical content. When students feel that the mathematics in the class does not relate to them, or their culture, they may feel unconnected or even invisible."

We have identified certain common elements in these students, namely the active participation, intense and (most of the time) hard of these youngsters in the domestic tasks as well as in the rural work. This reality has impelled us to create and adapt specific strategies, which took into consideration these students, and furthermore aim to contribute to the young population settlement in their own region.

3. "Working the round"

As we have already said, during our study we had the opportunity to get in touch with several wine coopers of this region, and we select three of them to constitute our cases study. We observed them in activity in their work's place and made one semi-structured interview to each one.

In our interviews, we realize that the main difficulty of this traditional job is what wine coopers call "*working the round*". This expression is suggestive of the mathematical concepts involved in building the barrels. They usually have a round form, joining the cylindrical and truncated conical forms, as can be seen in Figure 2.

"*Working the round*" is present in everything that they need to perform (including the dimensions) all pieces to make the barrels as well as in the tools used. In the following subsections we illustrate this study with some examples of the Alto Douro "wine coopers' mathematics".

3.1 Volume computation of the barrel

In this subsection we present how the wine coopers do volume computations of their barrels. A barrel has three key dimensions: the head, the length and the bilge as illustrated in Figure 2.

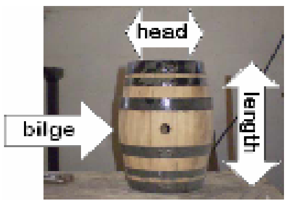

Figure 2 – Key dimensions of a barrel: the head, the length and the bilge.

From the interviews, we noticed that wine coopers intuitively consider the barrel as an approximation of a cylinder. Accordingly, the diameter corresponds to the head of the barrel and the high corresponds to the length. They do not give any importance to the bilge of the barrel. In their own words (translated from [Costa *et al*., 2006]):

Interviewer (I): The head is 20. How about the bilge?

Wine cooper (C1): We don't care with the bilge ... because if we give more fire [during barrel making] *it will be more straighten (…)*

One of the wine coopers gave us the data that he empirically uses to build a barrel with a certain volume. We organized them in Table 1.

Head (d) (centimeters, cm)	Length (h) (centimeters)	Empirical Volume (V) (liters, l)
75	150	500
63	95	250
…	…	…
20	40	12
20	38	10
16	25	5

Table 1 – Data organized from an interview with a wine cooper.

The wine coopers gave the following explanation about the dimensions that a barrel must have in order to have a certain volume, where the empirical factor *605* emerged from those interviews (translated from Costa *et al*., 2006):

(C2): After that, we have to multiply it by 605 (…)

We present our mathematical interpretation of the empirical factor *605*. We compared the oral description of the empirical process used by wine coopers with the mathematical form of computing the volume of a cylinder.

Let us consider the general case of a barrel with the following dimensions (all in the same unit): *head* (diameter of the cylinder) d and *length* (high of the cylinder) h.

From the interviews and following the wine coopers' instructions, we have for the barrel volume the value V given by:

$$V = (d/2)^2 \times (h/2) \times 605.$$

Recording the formula for the volume V' of a cylinder

$$V' = \pi \times r^2 \times h, \quad (1)$$

where r is the radius of the base of the cylinder, we obtain that

$$V = r^2 \times h \times \frac{605}{2} = r^2 \times h \times 302{,}5 = r^2 \times h \times 3{,}025 \times 10^2 \approx V' \times 10^2, \qquad (2)$$

which give us the formula (1), up to the factor 10^2, with a new approximation for the value of number π.

Based on these considerations, we present our mathematical interpretation of the empirical factor *605* in (Costa *et al.,* 2007, d) and we obtained another and, as far as we know, an unknown approximation for the number π:

$\pi \approx 3{,}025$ – *"the wine coopers' pi".*

3.2 Compass of contraction

Now we present a tool – a compass of contraction (see Figure 3), build and used by one wine cooper, that hides a lot of mathematics.

Figure 3 – Compass of contraction

This tool is used to determine the dimensions of the pieces of wood that joint together form the barrel – the wood staves. These pieces in the middle are larger than in the tops. The compass of contraction indicates the relation between these two dimensions, which is previously fixed by the wine cooper.

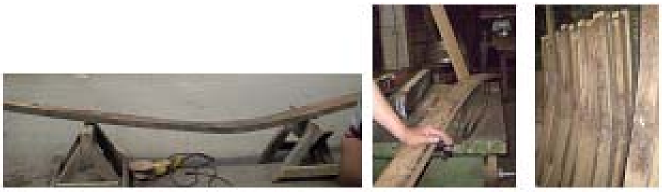

Figure 4 – Example of wood staves

From our interviews we notice that the wine cooper who builds this tool consider the proportion of 22% to be the one that produces a more graceful barrel.

Now we describe this compass of contraction. Given two small iron (or wooden) bars with same length, we joint them together in a point X in order to obtain two isosceles triangles, [AXB] and [CXD], as shown in Figure 5.

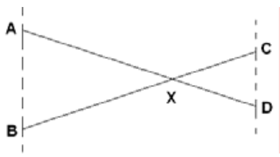

Figure 5 – Schematic representation of a compass of contraction.

Triangles [ABX] and [CDX] are congruent, thus the lengths of the corresponding sides are proportional. These sides are built with the convenient length in order to obtain the desired proportion. Using the proportion fixed by this wine cooper – 22% – for any length $\overline{\mathrm{CD}}$ of the line segment CD, the length $\overline{\mathrm{AB}}$ of the line segment AB is 22% greater than the first.

Note that this tool could also be named as "compass of expansion" for similar reasons.

3.3 Hoops of the barrel

As barrels are made of wood staves and bound with iron hoops, so those hoops are important elements in their structure.

Figure 6 – Examples of iron hoops

In this case the wine cooper needs to estimate the total length of strip of iron to make the hoops for a barrel.

In this Portuguese region the strip of iron is sold by using its weight and normally the strip is rounded. Being so, in order to know how many metres has a strip (coiled) the wine cooper measures the external diameter and the internal diameter, computes their average and multiplies it by the number of laps.

4. Some proposals of classroom mathematical tasks

In this section we present the design of some proposals that could be adapt and adopt within the classroom, inspired in the mathematical processes used by Alto Douro's wine coopers. We have already worked in some proposals that were made in a chapter of a book (Costa *et al.*, 2007c). The development and application of each proposal depends on the working context of the classroom, school, etc. Our goal is to present open proposals and to promote the use of this kind of material within the classroom. Nowadays, with the call to promote thinking in the real world in schools, a challenge to enhance the teaching and learning of mathematics through means that move away from the traditional approach is presented.

4.1 "Build your own barrel"

In group, students will build a (small) barrel with a fixed capacity. First of all, they have to do some research about the mathematical processes used by wine coopers. In a second phase, they will build the barrel with the adequate size. Afterwards, they will verify if the capacity of the barrel is the desired one. Finally, they could present the final product and report the project phases to community. Teachers should encouraged students to invite parents, grand parents and even a wine cooper to appreciate their project work.

Figure 7 – "Build your own (small) barrel"

4.2 "Build your own compass of contraction"

a) In small groups, students will build (in wood or in a strong card) a compass of contraction using a fixed proportion (e.g. 24%). In a composition and using all the needed schemes, they should explain the mathematical processes used.

b) In a small composition students must explain why a compass of contraction could also be named as a compass of expansion.

Figure 8 – "Build your own compass of contraction"

4.3 "Hoops!"

In pairs, students will estimate the metres of a strip of coiled iron presented in the Figure 9. They must write a technical report explaining mathematical procedures about what they thought and how they will estimate the length of the strip.

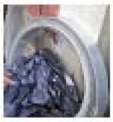

Figure 9 – Strip of coiled iron

5. Final Remarks

Throughout this work, we realize that the study of traditional activities has many mathematical concepts that could and should be detailed and used to promote classroom and school activities. We have seen and concluded that mathematical knowledge is present in the wine coopers' activity and at a level that may be used within mathematics classroom, even at the elementary levels.

Bearing in mind the characteristics of this Portuguese region, we believe in the viability of promoting the application of new methodologies and accomplishing experimental activities within the classroom. For instance, a school project may be drawn in the same way as we have suggested: choosing an activity; preparing interviews and interviewing artisans of the chosen activity; working in class the collected material; and presenting the project results to the community.

With our present work we do not forget the words of Gerdes (2003):

"Relative to research, it is my opinion that among the most urgent tasks is the study of mathematical elements in cultural spheres that are under siege (...). Humanity will loose an enormous source of knowledge, an enormous potential for broadening the reflection on mathematical thinking if these elements are not studied today and in the near future."

So, we also hope to give a contribution to preserve some of the cultural elements in Alto Douro's Portuguese Region.

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