TEACHERS' CONCEPTIONS OF HISTORY OF MATHEMATICS

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ABSTRACT

In 1997, history of mathematics was included in the curriculum goals for elementary and lower secondary schools (ages 6-16) in Norway. However, studies suggest that history of mathematics did not get the attention that the curriculum mandated.

To learn more about how this 1997 change in the curriculum may have been approached by mathematics teachers, I did an interview study of four Norwegian secondary and high school teachers. This is a phenomenological study where the goal is to gain more knowledge about teachers' conceptions, to supplement the findings of my earlier studies.

The study shows that the teachers differ in what they consider to be history of mathematics and in what the goal of incorporating it in teaching may be. They include history of mathematics in very different ways and to different degrees, and have different opinions on how that works out.

The knowledge gained in this study may contribute to the discussion of how to successfully integrate history of mathematics in the average mathematics classroom – that is: how to engage "ordinary" teachers in this endeavor.

1 Background – Norway

Curriculum. In the new national curriculum of 1997 (Hagness et al. 1999), history of mathematics got a prominent place. One of the six main goals of mathematics was "for pupils to develop insight into the history of mathematics and into its role in culture and science". In addition, history of mathematics was mentioned in specific goals for several grades.

Textbooks. I did a survey of all the new textbooks in mathematics for primary and secondary schools, about 21,000 pages in all (Smestad 2002). There was not much history of mathematics in the textbooks and the texts were quite closed, and not suited to stimulate pupils' inquiring. The texts also included a number of errors, and mostly consisted of colourless, biographical information (such as year of birth and country of origin) and rarely touched upon the development of concepts, mathematicians' motivations for working on mathematics or how mathematics has been used throughout history.

Teacher education. The TIMSS 2003 study showed that many Norwegian mathematics teachers in primary and lower secondary school had little or no mathematics in their teacher education (Grønmo 2004). History of mathematics must have been only a small part, if at all included. History of mathematics was not included in the in-service courses developed for the new curriculum either (KUF 1997). Therefore, most mathematics teachers should only be expected to know what they have studied on their own.

Teaching. We know little about the actual teaching of history of mathematics in Norway. An evaluation of the new curriculum (Alseth, Brekke, and Breiteig 2003) only mentions history of mathematics in passing.

2 Background – international

No overview of the international situation exists. However, an analysis of the TIMSS 1999 Video Study (Smestad 2004) showed that only 3 % of the 638 mathematics lessons (from seven different countries) included some history of mathematics, and even in these lessons,

the time allotted to history of mathematics was only three minutes on average. The little history of mathematics there was, was mostly lectured (not worked on by pupils) and often consisted of biographical information.

There have been several efforts, both locally and internationally, to help teachers getting insight into the combined fields of history and pedagogy of mathematics, for instance US historical modules (Katz and Michalowicz 2004), an Italian sourcebook (Demattè 2006) and Taiwanese teacher training (Horng 2004), to mention just a few recent examples. There have also been empirical studies to measure the effects of including history of mathematics.

The ICMI Study (Fauvel and Van Maanen 2000) looked at curricula, textbooks and teacher education, in addition to giving many examples, but also pointed to possible problems at the teacher and classroom level (ibid:30-31). Studies of other attempts to change teaching have shown that teacher beliefs have to be taken into account (i.e. Manouchehri and Goodman 1998). Insight into teachers' conceptions may be useful when discussing how to improve the position of the history of mathematics in schools. However, I have found no research on teachers' conceptions of history of mathematics, although the topic has been touched upon in several conferences, for instance by Man-Keung Siu (Siu 2004).

3 The research question

My research question is *"What are mathematics teachers' conceptions of history of mathematics and what are the backgrounds for these?"*

I use the word "conception" in a broad sense. It is an attempt to translate the Swedish word "uppfatning" which can be defined thus: "*What a phenomenon basically means to an individual or the fundamental way in which the individual understands the phenomenon, is called the individual's "uppfattning" of the phenomenon."* (Uljens 1989:10) (my translation). I define it even broader, to include experiences with, and attitudes towards, history of mathematics and also what the teachers' perceive as being history of mathematics.

Thus, in this connection I am not interested in what history of mathematics "is" or what is "actually" happening in Norwegian classrooms – what Ference Marton would call "first order perspective" (ibid). I'm interested in teachers' conceptions, including their conceptions of what history of mathematics is, and of what they do in their classrooms – what Marton would call "second order perspective". And I'm interested in finding a variety of views, not to pinpoint what is "a typical view".

4 Method

I have chosen to do a small case study with qualitative research interviews, within the phenomenological tradition. Due to time restrictions, the number of teachers interviewed is only four. A larger number of teachers would probably have given a broader range of views, but the necessary amount of time was not available.

The earlier studies, of textbooks (Smestad 2002) and lessons (Smestad 2004), do tell us something about teachers' (and textbook writers') conceptions of history of mathematics. This interview study could be seen as part of a bricolage (Denzin and Lincoln 1994), where the new study serves to supplement the findings of the first two.

In such a study, being aware of your own preconceptions in the field is important (Fog 1994:19). My previous studies (Smestad 2002 and 2004) form an important part of my preconceptions. My strong belief that history of mathematics may help improve teaching, is

also important. I will not go into further detail here, but have tried to take my preconceptions into account when analyzing.

4.1 Generalizability, validity, reliability

Generalizability. The simplest form of "generalizability" is the existence proof. For instance, by describing one teacher who claims to have been inspired by a one-day course, I prove that such teachers exist. Such existence proofs may be interesting, but the study makes no attempt to say how many such teachers exist.

Kvale (1997:162) cites Kennedy's discussion on situations where it is up to the receiver of the information to decide whether analytical generalizations can be made, based on a sufficient description of the context. When I describe teachers' conceptions of history of mathematics and the context they're in, it is left to you (the reader) to decide whether the teachers you are concerned with, is in a context sufficiently close to "my" teachers for my findings to be of use.

In any case, my study may be the start of a "register" of possible conceptions of history of mathematics, which can also enrich further discussions (see Donmoyer, cited in Kvale (ibid:163)).

Reliability. One relevant issue concerning the reliability of the findings is the issue of "leading questions". By asking the teachers for their own experiences before going on to ask prepared questions, I reduce the problem of the leading questions. However, the fact that I was there in the first place, asking about history of mathematics, may have influenced the teachers. On the more technical side, I believe the transcription reliability is high, as the sound was never a problem, and I had the opportunity to go back over and over to make sure the transcription was okay.

Validity. I try to secure the validity in several ways. First of all, I tried to ask follow-up questions during the interview whenever I was uncertain about an interpretation. Secondly, I offered a short summary to the participant at the end of the interview including my interpretation, giving the participant the opportunity to comment. Thirdly, I have analyzed the text in several steps, moving through hermeneutical spirals. By presenting preliminary findings at some meetings, I have also been given feedback which I have brought back to the analysis.

Another form of validity testing concerns the teachers' comments on their own conception of history of mathematics. When a teacher claims to be an enthusiast, the claim can be backed up by having lots of concrete examples on how he uses history of mathematics in his teaching. If he had claimed to be an enthusiast, but without being able to give any examples, his claim would have been more in doubt.

4.2 Choice of participants

I wanted my participants to be different, as I wanted to see a variety of conceptions of history of mathematics. Therefore, I wanted to include teachers both from secondary school (kids age 13-16), where history of mathematics was introduced with the new curriculum in 1997, and high school (kids age 16-19), where history of mathematics has been part of the textbooks and curriculum for some years. I also wanted teachers of different ages (because experience may influence the conception) and teachers with different education (both from university and from teacher colleges).

I recruited the participants by contacting schools and asking for teachers who would be willing to participate. Although I stressed that an interest in history of mathematics was not necessary, there was at least one teacher who didn't want to be interviewed because she didn't know any history of mathematics. This may mean that my four teachers are more interested in history of mathematics than the average teacher.

The four interviews were done over a period of several years, which meant that I had the opportunity to analyze previous interviews and improve on my interview guide for the following interviews. For instance, for the first two interviews, I started out by asking for reasons for including history of mathematics in the teaching of mathematics, while for the last two interviews, I started by asking the participant to tell me about the last time he used history of mathematics in his teaching (as mentioned above). I have also confronted participants with opinions given by teachers in earlier interviews, to make the participants discuss further. As this was only done towards the end of the interviews, this has not compromised the analysis of the interviews.

5 The participants

In the following discussion, I will compare the four teachers in key areas, but first I will give an introduction on each participant in the form of short patchworks based on the interviews.

Teacher 1 (T1)

Teacher 1 is in his 40s, and has taught in secondary school for about 10 years. On the introduction of history of mathematics in 1997, he says: *"Well, personally, I think we are struggling with this, we are focused on the mathematics, the four basic operations, on geometry, fractions... We tend to push aside the history of mathematics and not spend enough time on it."* and *"I haven't made much effort to get acquainted with the history, I guess that is something I will have to do eventually"*. He also says that it's difficult to work on history of mathematics, because his pupils are not motivated for mathematics. *"It's not easy to motivate students in this age to… (hesitates) I think the students have to be a bit older to understand that [history of mathematics] is important."* Moreover, *"We are working with the exam in mind, and as far as I've seen for these ten years, history of mathematics is not part of these exams."*

On his own relationship with history of mathematics: *"in teacher education, we had a little history of mathematics, especially connected to numeral systems, but then I worked a lot with small children after becoming a teacher, and it's obvious that you don't work with history of mathematics with them."*

On what would be needed to include more history of mathematics, he mentions that as long as the textbooks do not include more of the history of mathematics, it's tempting to skip it.

Teacher 2 (T2)

Teacher 2 is also in his 40s, and has been teaching in high school for 20 years.

"When the new curriculum[1] came, a professor came and lectured for a whole day on the history of mathematics. It was an extremely good lecture, and very interesting, and […] there was little of it that I really had any knowledge of in advance. But I've always been interested,

[1] This refers to the new curriculum for high schools in 1994, not the one for primary and secondary schools in 1997 mentioned earlier.

since I was a kid, in Leonardo da Vinci, stories from antiquity, in architecture, science and so on." Apart from this lecture, he has also picked up information from the textbooks.

"I tend to start every new topic with history of mathematics, as an anecdote or as something to spice up the subject. For some, it's just a short entertainment, for others it is more fundamentally formative." "I may use only five minutes. To focus the pupils. They come in, lazy. I make a story out of this. If these five minutes make the pupils more motivated or concentrated, it's worth it."

"I would like to know more. Someone could write a concise history of mathematics, popularized. Or I could have had another course."

Teacher 3 (T3)

Teacher 3 is in his 60s, and has been teaching in high school for 40 years.

"I've always been interested in history of mathematics. My first source was an incredibly inspiring mathematics teacher in high school." "I got Mathematics for millions as a gift from my parents while I was still in high school. And when I'm on holidays, I always visit book stores and buy some book that I enjoy reading. And during my studies, I had many inspiring lecturers. They often used historical bits, especially lives of mathematicians, for instance Abel and Galois."

"I try to spice up the teaching with a little history now and then, when it is natural, but I think most mathematics teachers don't do that." "When we have finished going through the curriculum and have about a month left before the exam, we start repeating it. But then they get the chance to try other methods of presenting the subjects, by means of lectures or small projects. Then I encourage them to some historical connection." "The main point is to motivate the pupils."

"It is enough with one enthusiastic teacher in a school for it to spread to the other teachers, if conditions are favourable."

Teacher 4 (T4)

Teacher 4 is in his 50s, and has been teaching in secondary school for almost ten years.

"What I know about history of mathematics is what I have read in books. I saw a book once, was very inspired by it. I thought this sounded very exciting, because I also had an idea that all mathematics came from Greek mathematics. It does not."

"I have used history of mathematics to motivate the pupils. My school has many pupils from other parts of the world, and I want to remove the misconception that all mathematics is from the western world. I ask a little about mathematics they use in their home countries, and we talk about counting methods and calendars." "We also work on house-building, and discuss the pyramid, African huts that are circular and cylindrical, the eskimos' igloos, Indian tents, sami tents… Then we can calculate areas. After a while, my pupils realize that the circular form is better than the western which is quadratic or rectangular. So the African pupils, they get improved self-esteem or status by knowing that "we build, in our culture, smarter houses".

He thinks the pupils like the variation and to discuss their own culture. At the same time, prejudices are broken down. Both "strong" and "less strong" students think it is exciting.

He thinks history of mathematics can be used lower in the school system as well. Himself, he works on history of mathematics 1-2 times a month, maybe 2-3 lessons each time.

His impression is that history of mathematics is not used very much. If it is to be used more, courses are needed. In high school, lots of history of mathematics is put into the textbooks, but there is so much mathematics that has to be taught as well.

6 Discussion

After analyzing the interviews, it has become apparent that the teachers differ on many key questions. I will now discuss some of these differences.

6.1 What is history of mathematics?

While I have not asked how they will define history of mathematics, different conceptions nonetheless are apparent in the material. One main difference is between those who think history of mathematics is about the *development* of mathematics and those who think history of mathematics is about the *use* of mathematics (throughout history).

Some illustrative examples:

T2: *"Then we talk about how fertile this area [the Nile delta] is, and that it is very important to keep your land, and that they have used trigonometry and surveying to sort everything out after the flooding, when the borders are washed out."*

T2: *"We in Europe has been a bit behind when it comes to mathematics and that there are certain things that we think we have discovered, but which have been discovered earlier in both Egypt and China."*

T3: *"Geometry is, of course, connected to ancient mathematics. [… We also work on] probability theory, when and how did that come in, and a bit about these erroneous deductions, d'Alembert's error and so on. […] And a bit later the function concept, which came late when it comes to precision."*

T4: *"I often ask pedagogical questions about mathematics, what is mathematics, where does mathematics come from and […] what kind of mathematics do they use in their home countries, so for instance we touch upon counting methods, what kinds of techniques are used there?"* T4 also mentions building houses and creating a right angle, but also mentions who developed "our" numeral system.

T4 seems to be more on the *use* side, particularly in the way he lets pupils look at how mathematics is used in their cultures today. T3 is more on the *development* side, while T2 has both perspectives evenly included. T1 says too little about this to be analysed. My impression is that the textbooks of high school (which T2 and T3 have used) have been on the development side, while the new curriculum for primary and lower secondary school (relevant for T1 and T4) was more on the use side. Whether there may be a connection here could be a topic for further study.

6.2 What kind of background on history of mathematics do the teachers have?

T1 mentions his teacher education as his main source, and mentions Egyptian numeral systems as an example of what he learned. T2 points to a one-day course by a professor in 1994, which *"was an extremely good lecture, and very interesting, and […] there was little of it that I really had any knowledge of in advance"*. He also mentions that the textbook he uses have historical introductions to the chapters, and that these have been useful. T3 had a very inspiring teacher in high school, and has also read lots of books (one of the first being

"Mathematics for millions") and had good lecturers at university. T4 have read articles and books, including Paulus Gerdes and Ubiratan D'Ambroiso.

This confirms my earlier point: even though history of mathematics has been included in the curriculum, teachers do not automatically get any help in updating their knowledge. Two of the teachers (T3 and T4) managed fine on their own, while T2 got a one-day course which kindled an interest. T1 never got any input after teacher education, according to his own story.

6.3 Are the teachers interested in history of mathematics?

Of these four teachers, three claim to be very interested in history of mathematics, while T1 is not. His lack of interest is apparent all through the interview: *"Well, personally I haven't done much with history of mathematics", "I guess that is something that I will have to do eventually"*. The other three teachers show their enthusiasm both in words and in their way of speaking. T2: *"I think [history of mathematics] is so great…"*, T3: *"I've been very interested in [history of mathematics] for many years."* T4: *"I got very excited and thought [history of mathematics] sounded very thrilling."* The interest of T2, T3 and T4 are also apparent in that they can give many examples of history of mathematics relevant for teaching.

6.4 How do the teachers include history of mathematics in their teaching?

T1 mentions how he talks about Niels Henrik Abel when the Abel competition comes up every year, but does not have many more examples of inclusion of history of mathematics. (He mentions last having worked on history of mathematics "last year".) T2 says that he starts every new chapter with talking a bit about history of mathematics – "maybe just five minutes", he calls it "anecdotes", "stories" and "spice".

T3 also includes history of mathematics in the introduction to a topic, but mostly in projects, topic days and work towards the oral exam – ways of working in which the students are more than passive listeners. He has from time to time cooperated with other subject teachers. T4 includes the history of mathematics in small "projects", 1–2 a month, each lasting 2–3 hours, often using concretes.

The ICMI Study (Fauvel and Van Maanen 2000:213-) mentions lots of different ways of including history of mathematics in the teaching of mathematics. The textbooks tend to tell stories from history of mathematics without giving ideas for further work (Smestad 2002). This may lead teachers to fall into the pattern of the textbooks, while it may take more interest and enthusiasm to include history of mathematics in a richer variety of ways.

6.5 What goals do the teachers have when including history of mathematics?

One major difference may be 'history as a tool' vs. 'history as a goal' (Jankvist 2007). History can be a tool for understanding the mathematics (i.e. by being inspired by history in working on misconceptions) or as a tool for motivation. It can also be a goal in itself. Here are some relevant quotes:

T1: *"It's not easy to motivate students in this age to… (hesitates) I think the students have to be a bit older to understand that [history of mathematics] is important."*

T2: *"So for some, it is just a little diversion, and a little refreshment, but for others, it can be more fundamentally formative, a further suggestion for further studies or as a peg to hang it on, to link to other things they have learned."* He also stresses that history of mathematics may explain the importance of mathematics and place "the subject in our culture and other cultures".

T3: *"I have […] tried to spice up the teaching, if I can call it that, with some historical passages now and then." "The main examples I can give have to do with motivating the pupils to work on the topics that may seem dry and boring."* But he also mentions that he has used examples to motivate for deeper understanding, for instance he mentions d'Alembert's misconceptions.

T4: *"I have often used it as a motivation in my teaching, to include the historical mathematics."* He stresses that he both aims to give pupils cultural understanding through the history of mathematics, and to motivate them by recognition of their own culture.

It seems that T1 only sees the "history as a goal" part, while the other three see history both as a goal and as a tool. Only T3 gives concrete examples of how history of mathematics may benefit the learning of mathematics directly.

The 1997 curriculum was unclear about what was the point of including history of mathemathics. When the new 2006 curriculum was prepared, however, the committee decided that history of mathematics was to be regarded as a tool, not as a goal. And as the new curriculum was supposed to leave the choice of "tools" to the teachers, history of mathematics was not mentioned.[2]

6.6 What do the teachers think of their pupils' reaction?

Unsurprisingly, their opinion on how the pupils react, also varies. While T1 says that "They try to follow", but that "it's a bit dry" for them, T2 says that "They find it fascinating." T4 says that they find it "exciting to work on their own cultures and learn about others".

It is tempting to interpret this as a result of vicious/virtuous cycles: enthusiasm leads to enthusiastic teaching, which then leads to interested pupils, a result which strengthen the teacher's enthusiasm.

6.7 What kind of resources do the teachers feel they need?

T1 says that *"This is about the interests of the teachers. If I had more interest in this area, I might have had some books on the shelf and could have used them. But a day has only 24 hours…"* The other three teachers want more courses for teachers, and T2 also mentions that there should have been a "concise history of mathematics, popularized", which could have been sent to teachers.

It should be noted that there exists no such "history of mathematics for teachers" in Norwegian.

6.8 What do these teachers think of other mathematics teachers' conceptions of history of mathematics?

T1 thinks he is representative of teachers at his school at least. T2 says he doesn't know much about what the other teachers does in their classrooms. T3 and T4 feels that other teachers are less interested than themselves. T3: *"I know many teacher who can be characterized as enthusiastic mathematics teachers, with good knowledge of history of mathematics, […] but I'm afraid they are not among the youngest ones."* T4: *"No, [history of mathematics] is not used much. That is the impression I have."*

So, at least on this point, they mostly seem to agree: the average teacher is more interested than T1 but less than T3 and T4.

[2] Based on personal communication.

6.9 Should history of mathematics be in the curriculum?

Bodil Kleve studied mathematics teachers' interpretation of the 1997 curriculum, and showed how three teachers enacted the curriculum in three different ways (Kleve 2007). With that in mind, it is worth discussing whether curriculum changes matter at all. The four teachers in my study disagree on this. T2 doesn't explicitly address this question, but refers to the curriculum document when talking about the point of including history of mathematics. T1 notes that history of mathematics has not been included in exams, and uses that as an argument for not stressing that particular part of the curriculum. T3 argues against putting it in the curriculum: *"Then it becomes compulsory, but is that a good idea? Well, possibly it is easier to get it to the pupils in that way, but… then there is a job to do with all the mathematics teachers in Norway, and I'm not sure they have so much knowledge in advance [on history of mathematics]."* T4 argues strongly for having it in the curriculum: *"I say that if it is not there, I'm afraid that it will not be included at all. If it's in the curriculum, it can remind a teacher that there is something. And obviously, we teachers are very different, and if there is something that doesn't appeal to me, I wouldn't teach it either, because then I would teach in a way that's not very positive."*

Have the curriculum documents influenced these teachers? From these quotes and from what they say elsewhere, my impression is that T1 is not much influenced by the curriculum. T2 is not as much influenced by the curriculum document as of the course and textbooks which may be a result of the curriculum. T3 and T4 would have taught history of mathematics anyway, but T4 feels that other teachers would be influenced.

7 Conclusion

Putting history of mathematics into the curriculum in Norway did not work well. One important reason for this is that teachers' conceptions of history of mathematics were not taken into account. There seems to have been a superstition that once it was included in the curriculum, teachers would know what to do. My studies show clearly why such an idea is wrong.

The four teachers in this study seem to have different opinions on what history of mathematics is and what the point of including history of mathematics in the curriculum may be. They also include history of mathematics in different ways and to different degrees. Their levels of enthusiasm are different, they feel differently about what their students think of history of mathematics, and have different opinions on what they need to include history of mathematics more. As such differences were revealed in a study of only four participants, one may speculate on which breadth of conceptions may be present in the teacher population as a whole.

These different factors should be taken into account when efforts are made to include history of mathematics in mathematics teaching on a national basis.

REFERENCES

– Alseth, Bjørnar, Gard Brekke, and Trygve Breiteig (2003), *Endringer og utvikling ved R97 som bakgrunn for videre planlegging og justering: matematikkfaget som kasus, Rapport / Telemarksforsking. Notodden 02/2003*. Notodden: Telemarksforsking.
– Demattè, Adriano (2006), *Fare matematica con i documenti storici - una raccolta per la scuola secondaria di primo e secondo grado*: Editore Provincia Autonoma di Trento - IPRASE del Trentino.

- Denzin, Norman K., and Yvonna S. Lincoln (1994), *Handbook of qualitative research*. Thousand Oaks, Calif.: Sage.
- Fauvel, John, and Jan Van Maanen (2000), *History in mathematics education, An ICMI study*. Dordrecht: Kluwer Academic Publishers.
- Fog, Jette (1994), *Med samtalen som udgangspunkt: det kvalitative forskningsinterview*. København: Akademisk Forlag.
- Grønmo, Liv Sissel (2004), *Hva i all verden har skjedd i realfagene?: norske elevers prestasjoner i matematikk og naturfag i TIMSS 2003*. Oslo: Institutt for lærerutdanning og skoleutvikling, Universitetet i Oslo.
- Hagness, Randi, Jorunn Veiteberg, Nasjonalt læremiddelsenter, and Kirke- utdannings- og forskningsdepartementet (1999), *The Curriculum for the 10-year compulsory school in Norway*. [Oslo]: National Centre for Educational Resources.
- Horng, Wann-Sheng (2004), "Teacher's professional development in terms of the HPM: A story of Yu", in, *HPM 2004*, Uppsala, Sweden: Uppsala Universitet.
- Jankvist, Uffe Thomas (2007), "Empirical research in the field of using history in mathematics education", *Nordic Studies in Mathematics Education* 12 (3):83–105.
- Katz, Victor J., and Karen Dee Michalowicz, eds. (2004), *Historical Modules for the Teaching and Learning of Mathematics*: Mathematical Association of America.
- Kleve, Bodil (2007), *Mathematics teachers' interpretation of the curriculum reform, L97, in Norway*, [Kristiansand]: Agder University College, Faculty of Mathematics and Sciences.
- KUF (2004), *Plan for etterutdanning i matematikk*. Kirke-, utdannings- og forskningsdepartementet, 20.03.1997 1997 [cited 14.06. 2004]. Available from http://odin.dep.no/odinarkiv/norsk/dep/kuf/1997/publ/014005-990176/dok-bu.html.
- Kvale, Steinar (1997), *Det kvalitative forskningsintervju*. Oslo: Ad notam Gyldendal.
- Manouchehri, Azita, and Terry Goodman (1998), "Mathematics curriculum reform and teachers: Understanding the connections", *Journal of Educational Research* 92 (1):27.
- Siu, Man-Keung (2004), ""No, I do not use history of mathematics in my class. Why?"", in, *HPM 2004*, Uppsala, Sweden: Uppsala Universitet.
- Smestad, Bjørn (2002), *Matematikkhistorie i grunnskolens lærebøker: en kritisk vurdering*, *HIF-rapport 2002:1*. [Alta]: Høgskolen i Finnmark Avdeling for nærings- og sosialfag.
- Smestad, Bjørn (2004), "History of mathematics in the TIMSS 1999 Video Study", Paper read at History and Pedagogy of Mathematics, at Uppsala, Sweden.
- Uljens, Michael (1989), *Fenomenografi: forskning om uppfattningar*. Lund: Studentlitteratur.