International Study Group on the Relations Between
the HISTORY and PEDAGOGY of MATHEMATICS
An Affiliate of the International Commission on
Mathematical Instruction

HPM 2008

History and Pedagogy of Mathematics

The HPM Satellite Meeting of ICME 11

14 - 18 July 2008

Centro Cultural del México Contemporáneo, Mexico City, México

http://www.red-cimates.org.mx/HPM2008.htm

Main themes

1. Integrating the History of Mathematics in Mathematics Education.
2. Topics in the History of Mathematics Education.
3. Mathematics and its relation to science, technology and the arts: historical issues and educational implications.
4. Cultures and Mathematics.
5. Historical, philosophical and epistemological issues in Mathematics Education.
6. Mathematics from the Americas.

Invited Speakers

Barbin Evelyne, Université de Nantes, France.

d'Ambrosio Ubiratan, Pontificia Universidade, Catolica de Sao Paulo, Brazil.

Emmer Michele, Università di Roma "La Sapienza", Italy.

Farfán Rosa María, Instituto De Ciencia y Tecnología del DF, & Cinvestav IPN, México.

Parshall Hunger Karen, University of Virginia, USA.

Pengelley David, New Mexico State University, USA.

Van Brummelen Glen, Quest University, Canada.

International Scientific Program Committee

Main Organizers

-Ricardo Cantoral, Departamento de Matemática Educativa, Centro de Investigación y de Estudios Avanzados del IPN, México,
-Florence Fasanelli, American Association for the Advancement of Science, USA
-Alejandro Garciadiego, Departamento de Matemáticas, Facultad de Ciencias, UNAM, México
-Robert Stein, California State University, San Bernardino, USA
-Constantinos Tzanakis, Chair of the HPM Study Group, Department of Education University of Crete, Greece

Members

-Abraham Arcavi, Weizmann Institute of Science, Israel
-Evelyne Barbin, IREM-Centre François Viète, Université de Nantes, France
-Carlos Correia de Sà, Departamento de Matemática Pura da Faculdade de Ciências da Universidade do Porto, Portugal
-Ubiratan d' Ambrosio, Pontificia Universidade, Catolica de São Paulo, Brazil
-Abdellah El Idrissi, Ecole Normal Supérieure, Morocco
-Gail FitzSimons, Monash University, Australia
-Michael Fried, Program for Science and Technology Education, Ben Gurion University of the Negev, Israel
-Fulvia Furinghetti, Department of Mathematics, Universita di Genova, Italy
-Wann-Sheng Horng, National Taiwan Normal University, Taiwan
-Masami Isoda, Graduate School of Comprehensive Human Science, University of Tsukuba, Japan
-Niels Jahnke, Fachbereich Mathematik, Universität Duisburg-Essen, Germany
-Sten Kaijser, Department of Mathematics, University of Uppsala, Sweden
-Victor Katz, Department of Mathematics, University of the District of Columbia, USA
-Manfred Kronfellner, Institut für Algebra & Computermathematik, Technische Universität Wien, Austria
-Ewa Lakoma, Institute of Mathematics, Military University of Technology, Warsaw, Poland
-Joao Bosco Pitombeira de Carvalho, PUC do Rio de Janeiro, Brazil
-Luis Radford, Ecole des Sciences de l' Education, Laurentian University, Canada
-Leo Rogers, Roehampton University, Digby Stuart College, Roehampton University, UK
-Gert Schubring, Institut für Didaktik der Mathematik, Universität Bielefeld, Germany,
-Man-Keung Siu, Department of Mathematics, University of Hong Kong, China
-Bjørn Smestad, Faculty of Education, Oslo University College, Norway
-Jan van Maanen, Freudenthal Institute, University of Utrecht, The Netherlands
-Chris Weeks, Downeycroft, Virginstow Beaworthy, UK
-Greicy Winicki Landman, Department of Mathematics and Statistics, California State Polytechnic University, USA

Local Organizing Committee

- Ricardo Cantoral, co-chair, Departamento de Matemática Educativa, Centro de Investigación y de Estudios Avanzados del IPN, México
- Alejandro Garciadiego, co-chair, Departamento de Matemáticas, Facultad de Ciencias. Universidad Nacional Autónoma de México, México
- Rosa María Farfán, Dirección de Educación, Ciencia y Sociedad del Instituto de Ciencia y Tecnología del DF, México
- Gisela Montiel, Centro de Investigación en Ciencia Aplicada y Tecnología Avanzada del IPN, México
- Gabriela Buendía, Cimate de la Universidad Autónoma de Chiapas, México
- Rodrigo Cambray Nuñez, Universidad Pedagógica Nacional, México
- Edmundo Palacios, Universidad Iberoamericana, México

PROGRAMME & ABSTRACTS

Overall Time Schedule

	Monday 14/7	**Tuesday 15/7**	**Wednesday 16/7**	**Thursday 17/7**	**Friday 18/7**
9:15 - 9:30	**Opening**				
9:30 – 10:30	**Plenary Talk**	**Plenary Talk**	Excursions/ Free Day	**Plenary Talk**	**Plenary Talk**
10:30 - 11:30	**Plenary Talk**	**Plenary Talk**		**Plenary Talk**	**Panel Discussion**
11:30 - 12:00	Break	Break		Break	Break
12:00 – 14:00	**Parallel sessions** (in 4 rooms)	**Parallel sessions** (in 4 rooms)		**Parallel sessions** (in 4 rooms)	**Parallel sessions** (in 4 rooms)
14:00 – 14:30	Lunch on one's own	Lunch on one's own		Lunch on one's own	**Closing**
14:30 – 16:00					
16:00 – 18:00	**Parallel sessions** (in 3 rooms)	**Parallel sessions** (in 3 rooms)		**Parallel sessions** (in 3 rooms)	
	Dinner on one's own	Dinner on one's own		Dinner on one's own	

Detailed programme of HPM 2008

Sunday, 13 July 2008

16:00-20:00

Registration desk is open

Monday, 14 July 2008

8:00-9:15 Registration desk is open (also during the breaks)
Welcome Cocktail

9:15-9:30 OPENING

9:30-10:30 Plenary Lecture: Theme 1 - *Integrating the History of Mathematics in Mathematics Education*

Chair: Alejandro Garciadiego

Patio de los Generales	David Pengelley	Teaching mathematics with primary historical sources: Should it go mainstream? Can it?

10:30-11:30 Plenary Lecture: Theme 5 - *Historical, philosophical and epistemological issues in Mathematics Education*

Chair: Evelyne Barbin

Patio de los Generales	Rosa María Farfán	Matemática educativa. La convergencia de series infinitas

11:30-12:00 Coffee Break

12:00-14:00 Oral presentations

room	Name	Title	Theme
Chair	**C. Tzanakis**		
R1 12.00-12.30	Robert Stein	The Math Wars: A Cultural View	2
R1 12.30-13.00	Walter Meyer	History of The Return of Applications in Undergraduate Mathematics in the United States	2
R1 13.00-13.30	Andreas Christiansen	Bernt Michael Holmboe (1795–1850) and his textbooks in school mathematics	2
R1 13.30-14.00	Osamu Kota	Teaching and Learning of Geometry in Japan In the Late Nineteenth Century	2
Chair	**B. Smestad**		
R2 12.00-12.30	Maria Cristina Araújo de Oliveira, Pietropaolo Ruy	Secondary School Journal: an instrument in the Mathematics Teacher training	2
R2 12.30-13.00	Ildikó Pelczer, Cristian Voica, Fernando Gamboa Rodríguez	A historical overview of analysis exams in Rumania	2

R2 13.00-13.30	Hans-Stefan Siller	Informatics – a subject developing out of Mathematics – a review from 1970 to 2007	2
R2 13.30-14.00	Flávia Soares	Defining math teachers' knowledge: discussion about examinations for elementary and secondary teachers in Brazil in the nineteenth century	2
Chair	**G. Winicki-Landman**		
R3 12.00-12.30	Loreto Cruz-Hernández, Katya Romo-Medrano	El desarrollo de la Matemática en México	2
R3 12.30-13.00	Maria Ângela Miorim	Produciendo Libros de Matemáticas para alumnos y profesores en Brasil en la década de 1930: una mirada sobre la experiencia de la Compañía Editora Nacional	2
R3 13.00-13.30	Alejandro Jiménez León, María Graciela Gutiérrez Vallejo	Aplicando Copyleft y Creative Commons en la difusión y protección de las ciencias, así como su preservación digital en el largo plazo	2
R3 13.30-14.00	Yadira Marcela Mesa, Jhony Alexander Villa Ochoa	Reflexión histórica, epistemológica y didática del concepto de función cuadrática	1
Chair	**U.Th. Jankvist**		
R4 12.00-12.30	Janet L. Beery	Formulating Figurate Numbers	1
R4 12.30-13.00	George Booker	The historical development of early algebra and implications for developing algebraic thinking in the middle years of school	1
R4 13.00-13.30	Kathleen M.Clark, Lydia Dickey	The FSU Cuneiform Tablet Collection: Using Mathematics of the Past to Inform Teaching in the Future	1
R4 13.30-14.00	Albrecht Heeffer	Negative numbers as an epistemic difficult concept. Some lessons from history	1

14:00-16:00 LUNCH BREAK

16:00-18:00 Oral presentations

room	Name	Title	Theme
Chair	**R. Stein**		
R1 16.00-16.30	Funda Gonulates	Prospective Teachers' Views on the Integration of History of Mathematics in Mathematics Courses	1
R1 16.30-17.00	Bjørn Smestad	Teachers' conceptions of history of mathematics	1
R1 17.00-17.30	Robert Peard	Quantitative Literacy for Pre-service Elementary Teachers within Social and Historical Contexts	1
R1 17.30-18.00	Michael Kourkoulos, Constantinos Tzanakis	Contributions from the study of the history of statistics to understand students' difficulties to grasp the concept of variance	1
Chair	**P. Ransom**		
R2 16.00-16.30	Gert Schubring	The debate on a "geometric algebra" and methodological implications	1
R2 16.30-17.00	Wann-Sheng Horng	Conceptualizing PCK in terms of HPM	1
R2 17.00-17.30	Uffe Thomas Jankvist	History of modern mathematics and/or modern applications of mathematics in mathematics education.	1
R2 17.30-18.00	Beverly M.Reed	The Effects of Studying the History of the Concept of Function On Student Understanding of the Concept	1

Chair	G. Buendía		
R3 16.00-16.30	Snezana Lawrence	Teachers thinking dynamically: collaborative teaching practice to introduce the history of mathematics and dynamic geometry software into the teaching of 8 - 14 year olds (*poster & exhibition of material*)	1
R3 16.30-17.00 R3 17.00-17.30	Francisco Gurrola Ramos, Rita Lizbeth Jáuregui Cota	Didáctica del Teorema de Pitágoras: Aplicando Situaciones Didácticas (***Workshop***)	1
R3 17.30-18.00	María Guadalupe Cabañas Sánchez, Ricardo Cantoral	Estudio socioepistemológico del área y la integral (*poster*)	1

Tuesday 15, July 2008

9:30-10:30 Plenary Lecture: Theme 5 - *Historical, philosophical and epistemological issues in Mathematics Education*

Chair: G. Schubring

Patio de los Generales	Evelyne Barbin	Dialogism in mathematical writing: historical, philosophical and pedagogical issues

10:30-11:30 Plenary Lecture: Theme 3 - *Mathematics and its relation to science, technology and the arts: historical issues and educational implications.*

Chair: Robert Stein

Patio de los Generales	Michele Emmer	Mathematics, Art, Architecture and Technology: the Idea of Space

11:30-12:00 Coffee Break

11:30-12:00 Coffee Break

12:00-14:00 Oral presentations

room	Name	Title	Theme
Chair	**E. Barbin**		
R1 12.00-12.30	Edel M. Reilly	Mathematics Apart: Examining the History of Subject Isolation and Its Implications for Mathematics Education	5
R1 12.30-13.00	Mala Saraswathy Nataraj, Michael O.J. Thomas	Using history of mathematics to develop student understanding of number system structure	5
R1 13.00-13.30	Renaud Chorlay, Anne Michel-Pajus	The multiplicity of viewpoints in elementary function theory: historical and didactical perspectives	5
R1 13.30-14.00	Staffan Rodhe	Emanuel Swedenborg's work on differential calculus	5
Chair	**R. Cambray**		
R2 12.00-12.30	Rotaeche Guerrero Rosa Araceli, Gisela Montiel Espinosa	From the history of the angle to its epistemological nature. Contributions to a scholar design (de la historia del ángulo a su naturaleza epistemológica. Aportaciones para un diseño escolar)	5

R2 12.30-13.00	Alonso Apolo Castañeda	Desarrollo del punto de inflexión como objeto escolar; estudio a la obra de L'Hospital	5
R2 13.00-13.30	Antonio Rocío, Gustavo Martínez Sierra	De la epistemología de la fórmula de tercer grado por Bombelli-Cardano a la construcción de una secuencia de actividades para la construcción del significado del número complejo	5
R2 13.30-14.00	Gustavo Martínez Sierra	From the analysis of the articulation of the trigonometric functions to the corpus of Eulerian analysis to the interpretation of the conceptual breaks present in its scholar structure (Del análisis de la articulación de las funciones trigonométricas al corpus del análisis euleriano a la interpretación de las rupturas conceptuales presentes en su construcción escolar)	5
Chair	**R. Cantoral**		
R3 12.00-12.30	Alberto Camacho Ríos, Bertha Ivonne Sánchez Luján	Social practice of the variability notion: an epistemological approach	5
R3 12.30-13.00	Luis Radford	Semiotic Reflections on Medieval and Contemporary Graphic Representations of Motion	5
R3 13.00-13.30	Cecilia Rita Crespo Crespo, Rosa María Farfán, Javier Lezama Andalón	Reflexiones acerca de argumentaciones y matemática en escenarios sin influencia aristotélica y su importancia en el aula de matemática	5
R3 13.30-14.00	Jhony Alexander Villa Ochoa	Elementos para la validación de una generaización matemática. Un analisis epistemológico a la evolución histórica del método inductivo	5
Chair	**C. Tzanakis**		
R4 12.00-12.30	Gabriela Buendía Abalos, Montiel Espinosa Gisela	From history to epistemology: meanings for trigonometric functions (***Workshop***)	5
R4 12.30-13.00			
R4 13.00-13.30	Vasiliki Farmaki, Stelios Negrepontis	Axioms in search of a definition (***Workshop***)	5
R4 13.30-14.00			

14:00-16:00 LUNCH BREAK

16:00-18:00 <u>Oral presentations</u>

room	Name	Title	Theme
Chair	**G. Booker**		
R1 16.00-16.30	George W. Heine	Euler's Contributions to Mathematical Cartography	3
R1 16.30-17.00	Peter Ransom	Yo Ho Ho-ratio: some mathematics of Trafalgar (How Lord Nelson inspired curriculum development in mathematics) (***Workshop***)	3
R1 17.00-17.30			
R1 17.30-18.00			
Chair	**F. Cordero**		
R2 16.00-16.30	Coralie Daniel	The big ideas of mathematics	3

R2 16.30-17.00	Oscar João Abdounur	Mathematics and music in scientific dissemination context: an epistemological/historical approach	3
R2 17.00-17.30	Luis M. Casas García, Ricardo Luengo	Pedagogía, Historia y Matemáticas: el tema de la Medida	3
R2 17.30-18.00	Kay Owens	Diversity in approaches to mathematics education in a cultural context	4
Chair	**G. Montiel**		
R3 16.00-16.30	Julio Omar Palacios Zarco	Un estudio sobre el uso de las gráficas en las obras de Evangelista Torricelli y Daniel Bernoulli	3
R3 16.30-17.00	Leonardo Venegas	El pensamiento matemático de Leonardo da Vinci	3
R3 17.00-17.30	Arlete de Jesus Brito	El libro Geografía General y el Conocimiento Matemático Del siglo XVII	3
R3 17.30-18.00	Eduardo Carlos Briceño Solís, Francisco Cordero Osorio	La construccion del artefacto al instrumento. Un estudio del "uso de las graficas". En busca de una integración tecnológica para el aprendizaje de las matemáticas	3

Wednesday 16, July 2008

Excursions / Free Day

Thursday 17, July 2008

9:30-10:30 Plenary Lecture: Theme 6 - *Mathematics from the Americas*

Chair: Florence Fasanelli

Patio de los Generales	Parshall Karen	The Establishment of a Community of Mathematical Researchers in North America: 1636-1950

10:30-11:30 Plenary Lecture: Theme 6 - *Mathematics from the Americas*

Chair: Ricardo Cantoral

Patio de los Generales	Ubiratan D' Ambrosio	The transmission and acquisition of Mathematics in Colonial and Early Independent Countries in the Americas

11:30-12:00 Coffee Break

11:30-12:00 Coffee Break

12:00-14:00 Oral presentations

room	Name	Title	Theme

Chair	**Katz Victor**		
R1 12.00-12.30	Fariba Elliee	American women mathematicians in the last quarter of the 19th century	6
R1 12.30-13.00	Harald Gropp	American calendars in the sixteenth century	6
R1 13.00-13.30	Roger Godard	Carl Runge: A Professor of Applied Mathematics at Georg-August Universität, Göttingen	5
R1 13.30-14.00	Adriana, Cesar de Mattos	The role of the referee in the History of Mathematics	5
Chair	**R. Farfán**		
R2 12.00-12.30	Juan Alberto Acosta Hernández, Carlos Rondero Guerrero, Anna Tarasenko	Un enfoque histórico y epistemológico de la noción de linealidad	5
R2 12.30-13.00	Luis Arturo Serna Martínez	Estudio Socioepistemológico de la tangente	5
R2 13.00-13.30	Santiago Ramiro Velázquez Bustamante	Un estudio socioepistemológico del discurso matemático escolar. El caso de la probabilidad elemental	5
R2 13.30-14.00	Rodolfo Godoy Rosas	La Derivada Insensible (*presentation and poster*)	1
Chair	**G. Buendía**		
R3 12.00-12.30	Fabián Valdivia Pérez	Las Matemáticas en la Biblioteca Palafoxiana: un acercamiento a sus libros y a sus temas.	6
R3 12.30-13.00	Mabel Alicia Gay de Niez	Atención a la diversidad: detección del talento en matemática en alumnos de 7º- 8º años de egb 3 de la ciudad de Concordia.	6
R3 13.00-13.30	Everardo Lara González, Natalia Fátima Sgreccia	Nepohualtzitzin, mucho más que un instrumento de cálculo (***Workshop***).	6
R3 13.30-14.00			
Chair	**F. Schweiger**		
R4 12.00-12.30	Jodelle S.W. Magner	Napier's Rods in Today's Classrooms	1
R4 12.30-13.00	Sifis Petrakis	Brouwer's intuitionism as a self-interpreted mathematical theory	5
R4 13.00-13.30	Osamu Takenouchi	Seki-Takakazu's memorial year	4
R4 13.30-14.00			

14:00-16:00 LUNCH BREAK

16:00-18:00 <u>Oral presentations</u>

room	Name	Title	Theme
Chair	**R. Cambray**		
R1 16.00-16.30	José Ismael Arcos Quezada	Rediseño de un primer curso de cálculo con base en la incorporación de algunos elementos del cálculo infinitesimal de los siglos XVII y XVIII	1
R1 16.30-17.00	Mario Dalcín, Mónica Olave	Conjetura y demostración en geometría dinámica a partir del libro de los lemas	1
R1 17.00-17.30	Cristina Ochoviet, Mónica Olave	¿Cómo decidir qué matemática debería saber un futuro docente? Aportes desde la Historia de la Matemática.	1

R1 17.30-18.00	Mónica Olave, Mario Dalcín	Tras los pasos de Arquímedes: Estrategias espontáneas de estudiantes de Bachillerato en el cálculo del área de un segmento de parábola.	1
Chair	**L. Radford**		
R2 16.00-16.30	Winicki-Landman Greisy	Teaching Big Ideas in mathematics through history	1
R2 16.30-17.00	Carlos Rondero Guerrero	Un análisis histórico y epistemológico de la noción de promediación	1
R2 17.00-17.30	Aida Maria Torres Alfonso	La integración de la historia y las nuevas tecnologías en el proceso de formación de matemáticos	1
R2 17.30-18.00			
Chair	**D. Pengelley**		
R3 16.00-16.30	Oleksiy Yevdokimov	*Title missing*	1
R3 16.30-17.00 R3 17.00-17.30	Janet Heine Barnett, Guram Bezhanishvili, Jerry Lodder, David Pengelley, Hing Leung, Desh Ranjan	Learning Discrete Mathematics and Computer Science via Primary Historical Sources: Student projects for the classroom (***Workshop***)	1
R3 17.30-18.00	Ewa Lakoma	On the role of the history of mathematics in mathematics education for the knowledge-based society	1

Friday 18, July 2008

9:30-10:30 Plenary Lecture: Theme 4 - *Cultures and Mathematics*
Chair: Rosa María Farfán

Patio de los Generales	Glen van Brummelen	Crossing Cultures, Oceans, Religions, and the Cosmos: In Search of the Origins of Trigonometry

10:30-11:30 Panel Discussion: Theme 1 - *Integrating the History of Mathematics in Mathematics Education*

Patio de los Generales	Montiel Gisela (coordinator), Cordero Francisco, Camacho Alberto, Castañeda Apolo, Quintero Ricardo	Origen, construcción y difusión del conocimiento matemático: de la historia a la matemática educativa

11:30-12:00 Coffee Break

12:00-14:00 Oral presentations

room	Name	Title	Theme
Chair	**Glen van Brummelen**		
R1 12.00-12.30	Janet Heine Barnett	Mathematics goes Ballistic: Benjamin Robins, Leonhard Euler, and the Mathematical Education of Military Engineers	4
R1 12.30-13.00	Fritz Schweiger	Jacobi's Last Theorem: The history of Jacobi-Perron algorithm	5

R1 13.00-13.30	Cecília Costa	J. Vicente Gonçalves and the "Journal of the Faculty of Sciences of Lisbon University": a contribution to the dissemination of Portuguese mathematical studies	4
R1 13.30-14.00	Cecília Costa, Maria Manuel da Silva Nascimento, Paula Catarino	The Alto Douro "wine coopers' mathematics"	4
Chair	**K. Parshall**		
R2 12.00-12.30	Ada Katsap	Ethnomathematical Problems for School Mathematics: Teachers Posed Math Problems in Context on Their Own Culture	4
R2 12.30-13.00	Meltem Ceylan Alibeyoglu	Mathematics in Zeugma	4
R2 13.00-13.30	Narges Assarzadegan	Dividing and Composing the Squares (*Poster*)	4
R2 13.30-14.00			
Chair	**G. Schubring**		
R3 12.00-12.30	Abdellah El Idrissi	The Al Biruni's Trigonometry	4
R3 12.30-13.00	Kristín Bjarnadóttir	A puzzle rhyme from 1782	4
R3 13.00-13.30	Alejandro Miguel Rosas Mendoza	The infinite series in India in centuries V to the XV	4
R3 13.30-14.00			
Chair	**R. Quintero**		
R4 12.00-12.30	Leonardo Solanilla Chavarro, Ana Celi Tamayo Acevedo	Geometría y Análisis en la Historia Temprana de las Integrales Elípticas	5
R4 12.30-13.00	Parra Fuentes Teresa Guadalupe, Cordero Osorio Francisco	Resignificación de la derivada en la Ingeniería por medio de la concepción Lagrangiana	5
R4 13.00-13.30	María Dolores Gabriela Meza Puesto	Propuesta para integrar la tecnología educativa en la instrucción del Cálculo Integral a nivel de Preparatoria: Herramienta tecnológica PRACTYMATHE	
R4 13.30-14.00	María Magdalena Espinosa Martínez	Comprensión de las Medidas de Dispersión: Caso de la licenciatura en Psicología	

14:00-14:30 CLOSING

TITLES OF PLENARY LECTURES & PANEL DISCUSSION

Barbin Evelyne: *Dialogism in mathematical writing: historical, philosophical and pedagogical issues.*

d' Ambrosio Ubiratan: *The transmission and acquisition of Mathematics in Colonial and Early Independent Countries in the Americas.*

Emmer Michele: *The idea of space: from Euclid to virtual architecture.*

Farfán Rosa María: *Matemática educativa. La convergencia de series infinitas*

Montiel Gisela (coordinator)**, Cordero Francisco, Camacho Alberto, Castañeda Apolo, Quintero Ricardo:** Panel discussion**:** *Origen, construcción y difusión del conocimiento matemático: de la historia a la matemática educativa*

Parshall Hunger Karen: *The Evolution of a Community of Mathematical Researchers in North America: 1636-1950.*

Pengelley David: *Teaching mathematics with primary historical sources: Should it go mainstream? Can it?*

Van Brummelen Glen: *Crossing Cultures, Oceans, Religions, and the Cosmos: In Search of the Origins of Trigonometry.*

TITLES OF PRESENTATIONS & WORKSHOPS IN PARALLEL SESSIONS

Abdounur Oscar João: *Mathematics and music in scientific dissemination context: an epistemological/historical approach.*

Acosta Hernández Juan Alberto, Rondero Guerrero Carlos, Tarasenko Anna: *Un enfoque histórico y epistemológico de la noción de linealidad.*

Araújo de Oliveira Maria Cristina, Ruy Pietropaolo: *Secondary School Journal: An instrument in the Mathematics Teacher training.*

Arcos Quezada José Ismael: *Rediseño de un primer curso de cálculo con base en la incorporación de algunos elementos del cálculo infinitesimal de los siglos XVII y XVIII.*

Assarzadegan Narges: *Dividing and Composing the Squares* (poster presentation).

Barnett Janet Heine: *Mathematics goes Ballistic: Benjamin Robins, Leonhard Euler, and the Mathematical Education of Military Engineers.*

Barnett Janet Heine, Guram Bezhanishvili, Jerry Lodder, Pengelley David, Hing Leung, Desh Ranjan: *Learning Discrete Mathematics and Computer Science via Primary Historical Sources: Student projects for the classroom* (workshop).

Beery Janet L.: *Formulating Figurate Numbers.*

Bjarnadóttir Kristín: *A puzzle rhyme from 1782.*

Booker George: *The historical development of early algebra and implications for developing algebraic thinking in the middle years of school.*

Briceño Solís Eduardo Carlos, Cordero Osorio Francisco: *La construccion del artefacto al instrumento. Un estudio del "uso de las graficas". En busca de una integración tecnológica para el aprendizaje de las matemáticas.*

Buendía Abalos Gabriela, Montiel Espinosa Gisela: *From history to epistemology: meanings for trigonometric functions* (Workshop).

Cabañas Sánchez María Guadalupe, Cantoral Ricardo: *Estudio socioepistemológico del área y la integral* (poster presentation).

Camacho Ríos Alberto, Sánchez Luján Bertha Ivonne: *Social practice of the variability notion: an epistemological approach.*

Casas García Luis M., Luengo Ricardo: *Pedagogía, Historia y Matemáticas: el tema de la Medida.*

Castañeda Alonso Apolo: *Desarrollo del punto de inflexión como objeto escolar; estudio a la obra de L'Hospital.*

Cesar de Mattos Adriana: *The role of the referee in the History of Mathematics.*

Ceylan Alibeyoglu Meltem: *Mathematics in Zeugma.*

Chorlay Renaud, Michel-Pajus Anne: *The multiplicity of viewpoints in elementary function theory: historical and didactical perspectives.*

Christiansen Andreas: *Bernt Michael Holmboe (1795–1850) and his textbooks in school Mathematics.*

Clark Kathleen M., Dickey Lydia: *The FSU Cuneiform Tablet Collection: Using Mathematics of the Past to Inform Teaching in the Future.*

Costa Cecília: *J. Vicente Gonçalves and the "Journal of the Faculty of Sciences of Lisbon University": a contribution to the dissemination of Portuguese mathematical studies.*

Costa Cecília, da Silva Nascimento Maria Manuel, Catarino Paula: *The Alto Douro "wine coopers' mathematics".*

Crespo Crespo Cecilia Rita, Farfán Rosa María, Lezama Andalón Javier: *Reflexiones acerca de argumentaciones y matemática en escenarios sin influencia aristotélica y su importancia en el aula de matemática.*

Cruz-Hernández Loreto, Romo-Medrano Katya: *El desarrollo de la Matemática en México.*

Dalcín Mario, Olave Mónica: *Conjetura y demostración en Geometría Dinámica a partir del libro de los lemas.*

Daniel Coralie: *The big ideas of mathematics.*

de Jesus Brito Arlete: *El libro Geografía General y el Conocimiento Matemático del siglo XVII.*

El Idrissi Abdellah: *The Al Biruni's Trigonometry.*

Elliee Fariba: *American women mathematicians in the last quarter of the 19th century.*

Espinosa Martínez María Magdalena: *Comprensión de las Medidas de Dispersión: Caso de la licenciatura en Psicología.*

Farmaki Vasiliki, Negrepontis Stelios: A*xioms in search of a definition* (Workshop & oral presentation)

Gay de Niez Mabel Alicia: *Atención a la diversidad: detección del talento en Matemática en alumnos de 7º- 8º años de EGB 3 de la ciudad de Concordia.*

Godard Roger: *Carl Runge: A Professor of Applied Mathematics at Georg-August Universität, Göttingen.*

Godoy Rosas Rodolfo: *La Derivada Insensible* (Oral and Poster presentation).

Gonulates Funda: *Prospective Teachers' Views on the Integration of History of Mathematics in Mathematics Courses.*

Gropp Harald: *American calendars in the sixteenth century.*

Gurrola Ramos Francisco, Jáuregui Cota Rita Lizbeth: *Didáctica del Teorema de Pitágoras: Aplicando Situaciones Didácticas* (workshop)

Heeffer Albrecht: *Negative numbers as an epistemic difficult concept: Some lessons from history.*

Heine George W.: *Euler's Contributions to Mathematical Cartography.*

Horng Wann-Sheng: *Conceptualizing PCK in terms of HPM.*

Jankvist Uffe Thomas: *History of modern mathematics and/or modern applications of mathematics in mathematics education.*

Jiménez León Alejandro, Gutiérrez Vallejo María Graciela: *Aplicando Copyleft y Creative Commons en la difusión y protección de las ciencias, así como su preservación digital en el largo plazo.*

Katsap Ada: *Ethnomathematical Problems for School Mathematics: Teachers Posed Math Problems in Context on Their Own Culture.*

Kota Osamu: *Teaching and Learning of Geometry in Japan in the Late Nineteenth Century.*

Kourkoulos Michael, Tzanakis Constantinos: *Contributions from the study of the history of statistics to understand students' difficulties to grasp the concept of variance.*

Lakoma Ewa, *On the role of the history of mathematics in mathematics education for the knowledge-based society*

Lara González Everardo, Sgreccia Natalia Fátima: *Nepohualtzitzin, mucho más que un instrumento de cálculo* (workshop).

Lawrence Snezana: *Teachers thinking dynamically collaborative teaching practice to introduce the history of mathematics and dynamic geometry software into the teaching of 8 - 14 year olds.*

Magner Jodelle S.W.: *Napier's Rods in Today's Classroom.*

Martínez Sierra Gustavo: *From the analysis of the articulation of the trigonometric functions to the corpus of eulerian analysis to the interpretation of the conceptual breaks present in its scholar structure (Del análisis de la articulación de las funciones trigonométricas al corpus del análisis euleriano a la interpretación de las rupturas conceptuales presentes en su construcción escolar).*

Mesa Yadira Marcela, Villa Ochoa Jhony Alexander: *Reflexión histórica, epistemológica y didática del concepto de función cuadrática.*

Meyer Walter*: History of the return of applications in Undergraduate Mathematics in the United States.*

Meza Puesto María Dolores Gabriela: *Propuesta para integrar la tecnología educativa en la instrucción del Cálculo Integral a nivel de Preparatoria: Herramienta tecnológica PRACTYMATHE.*

Miorim Maria Ângela: *Produciendo Libros de Matemáticas para alumnos y profesores en Brasil en la década de 1930: una mirada sobre la experiencia de la Compañía Editora Nacional.*

Nataraj Mala Saraswathy, Thomas Michael O.J.: *Using history of Mathematics to develop student understanding of number system structure.*

Ochoviet Cristina, Olave Mónica: *Cómo decidir qué matemática debería saber un futuro docente? Aportes desde la Historia de la Matemática.*

Olave Mónica, Dalcín Mario: *Tras los pasos de Arquímedes: Estrategias espontáneas de estudiantes de Bachillerato en el cálculo del área de un segmento de parábola.*

Owens Kay: *Diversity in approaches to mathematics education in a cultural context.*

Palacios Zarco Julio Omar: *Un estudio sobre el uso de las gráficas en las obras de Evangelista Torricelli y Daniel Bernoulli.*

Parra Fuentes Teresa Guadalupe, Cordero Osorio Francisco: *Resignificación de la derivada en la Ingeniería por medio de la concepción Lagrangiana.*

Peard Robert: *Quantitative literacy for pre-service elementary teachers within social and historical contexts.*

Pelczer Ildikó, Voica Cristian, Gamboa Rodríguez Fernando: *A historical overview of analysis exams in Rumania.*

Petrakis Sifis: *Brouwer's intuitionism as a self-interpreted mathematical theory.*

Radford Luis: *Semiotic Reflections on Medieval and Contemporary Graphic Representations of Motion.*

Ransom Peter: *Yo Ho Ho-ratio: some mathematics of Trafalgar (How Lord Nelson inspired curriculum development in mathematics)* (workshop).

Reed Beverly M.: *The effects of studying the history of the concept of function on student understanding of the concept.*

Reilly Edel M.: *Mathematics apart: examining the history of subject isolation and its implications for mathematics education.*

Rocío Antonio, Martínez Sierra Gustavo: *De la epistemología de la fórmula para las soluciones de una ecuación de tercer grado de Bombelli-Cardano a la construcción de una secuencia de actividades para la construcción del significado del número complejo.*

Rodhe Staffan: *Emanuel Swedenborg's work on differential calculus.*

Rondero Guerrero Carlos: *Un análisis histórico y epistemológico de la noción de promediación.*

Rosas Mendoza Alejandro Miguel: *The infinite series in India in centuries V to the XV.*

Rotaeche Guerrero Rosa Araceli, Montiel Espinosa Gisela: *From the history of the angle to its epistemological nature. Contributions to a scholar design (De la historia del ángulo a su naturaleza epistemológica. Aportaciones para un diseño escolar).*

Schubring Gert: *The debate on a "geometric algebra" and methodological implications.*

Schweiger Fritz: *Jacobi's Last Theorem: The history of Jacobi-Perron algorithm.*

Serna Martínez Luis Arturo: *Estudio Socioepistemológico de la tangente.*

Siller Hans-Stefan: *Informatics – a subject developing out of Mathematics – a review from 1970 to 2007.*

Smestad Bjørn: *Teachers' conceptions of history of mathematics.*

Soares Flávia: *Defining math teachers' knowledge: discussion about examinations for elementary and secondary teachers in Brazil in the nineteenth century.*

Solanilla Chavarro Leonardo, Tamayo Acevedo Ana Celi: *Geometría y Análisis en la Historia Temprana de las Integrales Elípticas.*

Stein Robert : *The Math Wars: A Cultural View*

Takenouchi Osamu: *Seki-Takakazu's memorial year.*

Torres Alfonso Aida Maria : *La integración de la historia y las nuevas tecnologías en el proceso de formación de matemáticos .*

Valdivia Pérez Fabián: *Las Matemáticas en la Biblioteca Palafoxiana: un acercamiento a sus libros y a sus temas.*

Velázquez Bustamante Santiago Ramiro, Lozano René Santos: *Un estudio socioepistemológico del discurso Matemático escolar. El caso de la probabilidad elemental.*

Venegas Leonardo : *El pensamiento Matemático De Leonardo Da Vinci .*

Villa Ochoa Jhony Alexander: *Elementos para la validación de una generalización matemática. Un analisis epistemológico a la evolución histórica del método inductivo.*

Winicki-Landman Greisy: *Teaching Big Ideas in mathematics through history .*

Yevdokimov Oleksiy : ***Title to be specified***

PLENARY LECTURES OF INVITED SPEAKERS AND PANEL DISCUSSION: ABSTRACTS

***Theme* 5**

Dialogism in mathematical writing: historical, philosophical and pedagogical issues.

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The notion of dialogism was introduced by the Russian semiotician Mikhail Bakhtin. For him, every sentence or every discourse must be understood as a rejoinder in a dialogue: it is an answer to other sentences, or discourses and it is intended to be received by somebody. Our purpose in this lecture is to explore the meaning and the implications of this notion for mathematical texts. The consequences for historical works are clear, in the sense that one should pay attention to the nature of texts (letters, papers, books), to the texts known to the authors, and so on. In a philosophical perspective, the notion of dialogism leads to a reflection on mathematical proof. At the classroom level, dialogism is an interesting notion for the reading of ancient texts by pupils and thinking about the persons for whom the pupils write.
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***Theme* 6**

The transmission and acquisition of Mathematics in Colonial and Early Independent Countries in the Americas

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I analyze the History of Mathematics in the Americas according to the following periods: 1. Pré-Columbian ; 2. Conquest and early colonial times [16th and 17th centuries]; 3. The established colonies [18th century]; 4. The struggle for Independence [late 18th and early 19th centuries]; 5. Independent nations [mid 19th century on].
In this talk I will discuss the periods 2. 3. and 4, that is, from mid-16th, to mid 19th centuries, with particular emphasis given Middle and South America. These periods are considered in light of the interests and conveniences of bringing European mathematics to the New World. More then "what mathematics", we investigate the reasons and the methods used for the transmission and acquisition of mathematics in the colonies and early independent countries.
The main question is "What was the role of mathematics in the colonial projects of European Empires and in the project of building up new nations after Independence?".
The theoretical framework for this study are adapted from the proposal of models of development, developed in the sixties mainly by CEPAL, in Santiago de Chile, and from the historians of science concerned with the inumerous factors involved in the social construction of knowledge. I focus the moments of social changes (or attempted change) in history and reflect upon the perception of how mathematics might help to achieve the goals, specially interests, of all those individual and groups involved in the process. It is important to point out how mathematics helps to achieve these goals. If we are to understand why and how certain knowledge-making practices become prevalent and impact our lives we may progress.
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***Theme* 3**

The idea of space: from Euclid to virtual architecture

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The discovery (or invention) of non-Euclidean geometry and of the higher dimensions (from the fourth on), the new idea of space to summarize, is one of the most interesting examples of the profound repercussions that mathematical ideas will have on humanistic culture and on architecture. The paper will discuss the elements necessary to give sense to the word *Space.*

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***Theme* 5**

Matemática educativa. La convergencia de series infinitas

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Presentaremos el estudio epistemológico de una investigación, sobre convergencia de series reportada ampliamente en (Farfán, 1997, 2003) y cuyo resultado se puede sintetizar como: *determinar el estado estacionario del sistema conduce, necesariamente, a un estudio de la convergencia de una serie trigonométrica infinita.* Asimismo ofrecemos una panorámica de los diversos estudios que se derivan, en especial el proyecto de un lenguaje gráfico y la reproducibilidad de situaciones didácticas desde una perspectiva socioepistemológica. Con ello pretendemos ilustrar el desarrollo de una línea de investigación en matemática educativa.

El marco de referencia lo constituye el trabajo de Fourier y alrededor de él una problemática crucial para el desarrollo del cálculo pues se reestructuras, los conceptos fundamentales del análisis matemático del siglo XVIII, como son: el de función, el papel del álgebra, el continuo real, así como la interpretación física de las soluciones, y se inicia el estudio de la convergencia de series infinitas, pilar fundamental del Análisis Matemático moderno. Salta a la vista la importancia singular de la obra de Fourier, tanto para la ingeniería como para el análisis matemático mismo. Como resultado de la revisión de la obra de Fourier, formulamos nuestra hipótesis central de investigación la cual radica en que, ***para la construcción de la noción de convergencia de series infinitas, se precisa de un ambiente fenomenológico estrechamente relacionado con la estabilidad de sistemas fluidos.*** De suerte tal, que determinar el estado estacionario del sistema conduce, necesariamente, a un estudio de la convergencia de una serie trigonométrica infinita.

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***Theme* 1**

***Panel Discussion*: Origen, construcción y difusión del conocimiento matemático: de la historia a la matemática educativa**

Montiel Gisela (coordinator) (CICATA-IPN[†]),
Cordero Francisco (Cinvestav-IPN[‡]),
Camacho Alberto (ITC II[§]),
Castañeda Apolo (CICATA-IPN),
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En la matemática educativa se reconoce a la *matemática escolar* como un cuerpo autónomo de conocimientos que toma a la *matemática* como su saber de referencia, pero se distingue de ella, no solamente por su explícita pretensión didáctica, sino también por el profundo cambio de su epistemología (Cantoral, 1995).

Para reconocer la base de *significados naturales* subyacentes a los procesos y conceptos matemáticos, la *investigación en matemática educativa* analiza las ideas que señaladas en la historia y en la epistemología establecen explicaciones sobre la construcción de un cierto conocimiento matemático, desde su gestación hasta los procesos de institucionalización a los que se ve sometido como saber matemático erudito y como saber matemático escolar.

En este sentido valoramos a la historia de las matemáticas más allá de su aportación motivacional, como fuente de problemas o como elemento esencial de la cultura que transmite la escuela en la formación del ciudadano. Ello implica ver en la historia al hombre haciendo y usando matemáticas en un contexto social específico y no sólo la producción matemática final que logra.

Con base en algunos ejemplos, producto de nuestras investigaciones, el Panel pretende mostrar a la comunidad cómo desde la Matemática Educativa hacemos uso de la historia de las matemáticas para la construcción de explicaciones del fenómeno didáctico y para extraer elementos de naturaleza epistemológica que fundamenten el rediseño del *discurso matemático escolar*.

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***Theme* 6**

The Evolution of a Community of Mathematical Researchers in North America: 1636-1950.

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This paper explores the various factors which effected the emergence of communities of mathematical researchers in North America beginning in the last quarter of the nineteenth century and their development in the twentieth. The analysis for the United States will hinge on a periodization defined largely by broader political and social influences; contemporaneous developments in Canada will be highlighted.

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***Theme* 1**

Teaching mathematics with primary historical sources: Should it go mainstream? Can it?

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Many are now teaching mathematics directly with primary historical sources, in a varietyof courses and levels. How far should this be taken? Should we adapt or redesign standard courses to a completely historical approach, chiefly from primary sources? If so, what are the obstacles to achieving this? Materials? Instructor attitudes? What should and can we do about such things?

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***Theme* 4**

Crossing Cultures, Oceans, Religions, and the Cosmos: In Search of the Origins of Trigonometry.

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It is a strange paradox that calculus and linear algebra recently have been reacquainted with inspirations in science and industry; yet trigonometry — which owes its very existence to outside needs — remains virtually untouched. It was born in Hipparchus's fusion of astronomical models with Babylonian calculation, forming the first truly exact science. It was rejuvenated with Muslim requirements to determine the direction of Mecca; it flourished with the beginning of European ocean-going navigation. Although trigonometry has a colourful history crossing the boundaries of ancient Greece, medieval India and Islam, and the West, students usually remain unaware both of its cultural richness and of the reasons that these cultures cared. We shall explore examples of these little-known stories accessible to students learning the subject, providing interesting historical motives for some of the more peculiar twists and turns encountered in the classroom.

PRESENTATIONS & WORKSHOPS IN PARALLEL SESSIONS: ABSTRACTS

***Theme* 3**

Mathematics and music in scientific dissemination context: an epistemological/historical approach

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The aim of this presentation is to show the use of an exhibition in museum "Estação Ciência" of the University of São Paulo to approach historical and epistemological aspects of the relationship between mathematics and music. The exhibition attempts, amongst other goals, to create an environment for professionals involved with teaching/learning processes; to experience activities that will enhance their ordinary knowledge and activities especially in the scientific area and its connection between mathematics and music; make the meaning of acoustical and musical concepts accessible to the laymen; promote to the visitor the opportunity to experience situations historically contextualized involving simultaneously mathematical, musical and physical concepts through direct or analogical reproduction of experiments that can trigger the interest and curiosity of the visitors by the study and development such concepts further; etc. In establishing a context for teachers to experience activities of culture and extension to their curricular activities, one values the history of mathematics particularly concerning its relationships with music, making accessible the historical context in which such relationships emerged.
The methodology involved in the project suggests that the exhibition is composed by 8 modules distributed in rooms that should convey historically the central ideas of the relationship between mathematics and music: 1) Motivational and relevant ideas for the grasp of the Harmonic Series concept; 2) The experiment of the monochord: ratio vs. musical intervals and the mathematical systematisation of the scale; 3) The Renaissance: the birth of music as an experimental science; 4) Scales and temperament; 5) Harmonic Series/Fourier Series; 6) Consonance and dissonance: from the arithmetic symbolism to a physical conception; 7) The Music of the Spheres; 8) From the speculative mathematics to the empirical mathematics: a scientific revolution in music. Each module contains mechanical and/or multimedia interactive devices, and make use of multiple representation interconnected through texts and/or hypertexts with a historical and epistemological approach which aim the development and strengthening of the mathematical and musical concepts associated to each room by the visitors.
The project leads to the production of a catalogue of the exhibition that should be available to the public at the institution which ensures the access of researchers and the community interested; strengthen the museum "Estação Ciência" by means of making available to its collection the material designed and produced for this project ensuring the continuation of the scientific diffusion of the relationship between mathematics and music as well as enabling the institution to use and/or produce material that can be applied in schools, especially in the area of physics, mathematics and music.

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***Theme* 5**

Un enfoque histórico y epistemológico de la noción de linealidad

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Se presenta un estudio histórico y epistemológico acerca de la noción de linealidad, donde se muestran aspectos relevantes sobre su génesis y desarrollo, haciendo un rescate de la misma, hasta mostrar su aparición en el Álgebra Lineal.

Se han identificado cuatro escenarios históricos, el primero se remonta a las culturas ancestrales egipcia, china y babilónica, donde el cobro de impuestos se calculaba por medio de la proporción directa y la progresión aritmética. En el segundo, se toma como referencia principal a autores clásicos de la cultura griega, quienes emplearon a la noción de proporcionalidad como elemento epistemológico de construcción de muchos de sus resultados.
El tercer escenario, se caracteriza por ser una época en donde se presenta un enlace conceptual entre la noción de proporcionalidad y la noción de linealidad, donde Fermat y Descartes, dieron pleno sentido a los trabajos de Apolonio sobre lugares geométricos.
El último escenario, parte del origen del Álgebra Lineal, cuyo sustento epistemológico está dado por la noción de linealidad, la cual genera conceptos tales como: dependencia e independencia lineal, rango, entre otros. Se plantea que es a través de la articulación de saberes como se pueden explicitar las relaciones conceptuales de la linealidad, para su incorporación en la Didáctica de la Matemática. En este trabajo se considera a la linealidad como una noción que ha evolucionado en la historia, a partir de necesidades socioculturales de las épocas referidas.
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***Theme* 2**

Secondary School Journal: An instrument in the Mathematics Teacher training

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This study presents an analysis of the content of articles on Mathematics and Technical Drawing[**], published in the Secondary School Journal, between 1957 and 1963, in order to indicate a panorama of theoretical and practical recommendations, emphasised and defended in these texts, regarding to teaching and learning processes of notions, concepts and procedures related to geometry. This journal have been structured as an instance to train teachers, as it has emerged in a context in which the great majority of Brazilian teachers of secondary school was self-taught because, in 1957, only 16% of in-service teachers were graduated from Philosophy university courses. For this analysis, we assume that pedagogical press, being the result of editorial strategies addressed to teachers, plays a meaningful role in spreading ideas and knowledge taken as necessary and fundamental to docent act. In this paper, we identify recommendations about teaching and learning process of geometry contents that are highlighted in the docent professionalizing process according to the *appropriation* perspective (Certeau 1994, Chartier 1991 e Carvalho 2006) of the educational legislation of the 50's decade, by different authors of analysed articles.
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***Theme* 1**

Rediseño de un primer curso de cálculo con base en la incorporación de algunos elementos del cálculo infinitesimal de los siglos XVII y XVIII.

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Los cursos de cálculo que actualmente se ofrecen en las aulas de escuelas y universidades, difieren del cálculo infinitesimal de fines del siglo XVII en varios aspectos, de entre los cuales uno muy importante es que en el cálculo escolar actual no se aceptan las cantidades infinitamente pequeñas. Sin embargo, cuando se analizan los libros de texto de otras ciencias básicas y de la ingeniería, se puede constatar que el cálculo que ahí se utiliza es más bien próximo al de fines del siglo XVII, con algunos elementos de los tres siglos posteriores.
Así pues, al menos si se trata de un curso de bachillerato o en una escuela de ingeniería, resulta

plausible hacer una presentación en la que se acepten las cantidades infinitesimales y en la que el énfasis en los fundamentos lógicos se traslade a un interés por abordar y resolver determinados problemas en los ámbitos de la geometría o las ciencias. Con base en ello se puede recuperar el indudable valor didáctico de las concepciones infinitesimalistas de los siglos XVII y XVIII.
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***Theme* 1&4**

Dividing and Composing the Squares

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Geometrical/ artful patterns are valuable in visualization and understanding geometrical/ mathematical concepts. In this paper, I explain some patterns and methods of dividing and composing squares, based on Abu'l- Wafa Buzdjani's book (4^{th} Hejira/ AD 10^{th} century). In addition, I applied activities as a workplace format in high school geometry classroom. Participants were 14 students grade 11 at a High school in Isfahan. They cut and assembled squares by scissor and painted them. Then they found some applied patterns in geometrical based tiling in Islamic constructions in Isfahan. Finally, they presented their works in power point format to others. Notation to such patterns and arts helped them to better understanding geometrical concepts such as rotation, symmetry, reflection, transformation, mapping, translation and dilation.
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***Theme* 4**

Mathematics Goes Ballistic: Benjamin Robins, Leonhard Euler, and the Mathematical Education of Military Engineers

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Efforts to understand the trajectory of cannonballs are an interesting example of the tensions between practical and theoretical knowledge. Although Galileo's 1638 parabolic trajectory was an important theoretical step forward, field gunnery practice was guided by the Tartaglia's 1537 "mixed motion" model through the eighteenth century. In 1742, Benjamin Robins published New Principles of Gunnery, and revolutionized the study of ballistics by suggesting the projectile's initial velocity – not its range – was the appropriate parameter to consider in accounting for air resistance. In 1745, Leonard Euler produced a German translation of New Principles, adding his own extensive commentary. Euler's annotated translation quickly became a standard text – Napoléon Bonaparte studied ballistics from the French version – thereby influencing the education of artillery officers and, eventually, of all engineers. This paper surveys the contributions of Robins and Euler to mathematical ballistics theory, examines the influence of these developments on the education of eighteenth century military engineers, and considers the extent to which the history of ballistics theory supports the thesis that the drive to reconcile practical knowledge with theoretical knowledge can be a critical element in shaping mathematical theory. We close with comments concerning the use of this history in today's classroom.

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***Theme* 1**

Learning Discrete Mathematics and Computer Science via Primary Historical Sources: Student projects for the classroom

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We discuss and present excerpts from classroom project modules based on primary historical sources, being developed by an interdisciplinary faculty team for courses in discrete mathematics, graph theory, combinatorics, logic, and computer science. The goal is to provide motivation, direction, and context for these subjects through student projects based directly on the writings of the pioneers who first developed crucial ideas and worked on seminal problems. Each module is built around primary source material close to or representing the discovery of a key concept. Through guided reading and activities, students explore the mathematics of the original discovery and develop their own understanding of the subject. We describe a dozen projects already available, and give substantial selections from two of them, on Pascal's elucidation of mathematical induction in his treatise on the arithemetical triangle, and Euler's seminal paper in graph theory on the Königsberg Bridge Problem. We also discuss the details of classroom implementation of teaching with historical projects. Preliminary evaluation shows a statistically significant benefit to students' performance in subsequent courses from a course with a historical project. Further evaluation and project development is underway, and two web sites provide expanded materials and information. Ongoing support is provided by the US National Science Foundation.

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***Theme* 1**

Formulating figurate numbers

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The multiplicative formula for figurate numbers (or binomial coefficients) we use today appeared in Western Europe in verbal form in the late 1500s and in symbolic form in the early 1600s. In this presentation, we first recount the early history of figurate numbers and especially of multiplicative means for computing them. We then focus on the development of multiplicative formulas for figurate numbers in the late sixteenth and early seventeenth centuries by Cardano, Faulhaber, Briggs, and Harriot. Throughout the presentation, we explore what it means to "have a formula for" a mathematical relationship. Indeed, the story of figurate number formulas has implications both for how we teach the history of mathematics and for how we teach mathematics. For example, students (and instructors) may be surprised to learn what historians mean when they report that Cardano had the first multiplicative formula for figurate numbers. While students may imagine a symbolic formula, in fact Cardano's description was verbal and employed a numerical example. In all of our mathematics courses, we should keep in mind, if not discuss outright, how conceptions of what constitutes a mathematical formula have varied over time and also how our and our students' views may differ. Students (and instructors) might argue from their own experience that a few good examples, or even just one well-

chosen example, can convey a general formula at least as well as a symbolic formula. On the other hand, symbolic notation seems to aid greatly in further development of ideas. The history of multiplicative formulas for figurate numbers is interesting in its own right and provides rich fodder for a broader discussion of mathematical formulas.
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***Theme* 4**

A puzzle rhyme from 1782

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A puzzle of thirty birds, known in many ancient cultures, is found in an Icelandic spelling book for children from 1782, transposed into Icelandic context so well that it seemed genuinely domestic. Its form is a three-verse rhyme governed by complex Old-Germanic rules of prosody. The birds are sold for units typical for Icelandic middle-age trade commodities, used up to recent times. The composition is completely adjusted to the Icelandic culture and may therefore be considered as Icelandic ethnomathematics of early modern times.
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***Theme* 1**

The historical development of early algebra and implications for developing algebraic thinking in the middle years of school

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Throughout history, algebra has been a cornerstone of mathematics, and we can trace the roots of algebraic thinking deep into the bedrock of mathematics.

Navigating through Algebra NCTM 2001

Algebraic thinking is essential for coping with the problems that continually arise in a rapidly changing world. While algebra has largely been viewed as a formal system that is mastered in high school, recent trends in curriculum thinking have focussed instead on the ways of thinking that underpin these formal ways of operating, recognising that such thinking needs to develop from the earliest days of school (NCTM 2001). However, there has been little attempt to use a knowledge of the historical development of algebraic thinking to suggest paths, problems and opportunities for the teaching and learning of algebraic ideas in the early years of schooling. This paper will examine the historical roots of algebra and the uses they might suggest for programs in the middle years of school as well as any obstacles or limitations that might be anticipated both in developing students' thinking and in the tasks that might be used.

Building algebraic thinking involves recognising general mathematical relationships and expressing these in increasingly sophisticated ways, at first seeing patterns, then describing these with words or diagrams before leading to the use of symbols that can express generalities concisely. Determining patterns among numbers, objects and geometric shapes have clear links to the development of similar thinking in earlier times while the forms of description invoked parallel the 'syncopated' algebra that arose around 1000 years ago, involving words and rudimentary symbols, before a fully symbolic system was proposed and accepted. Exploring a variety of routes to algebra (Mason 1985) can enable algebra to become more than 'generalised arithmetic' and lead to representations in terms of tables and graphs as well as the symbols that more obviously grow out of number work. In particular, an understanding of why and how the concepts of patterning and algebra have emerged in mathematics can then provide a richer background to algebraic thinking to teacher and student alike. In this way, as Avital (1995) pointed out, 'the history of mathematics can supply a structure of understanding relating reasons with results'.

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***Theme* 3**

La construccion del artefacto al instrumento. Un estudio del "uso de las graficas": En busca de una integración tecnológica para el aprendizaje de las matemáticas

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La tecnología son consideradas como recursos didácticos, lo que conlleva nuevas formas de abordar la enseñanza y aprendizaje de la matemática, pero estas no han sido suficiente para que tal aprendizaje se integre al humano. Esto ha llevado a la creación de un marco llamado génesis instrumental que estudia la construcción del artefacto al instrumento hecha por el estudiante el cual ha sido capaz de integrarlo a su actividad matemática. Con base a lo anterior la socioepistemología está haciendo estudio del uso del conocimiento en situaciones especificas con la perspectiva del "uso de las gráficas" en donde precisamente en una situación aprendizaje con el uso de tecnología se pretende evidenciar que es el "uso de las graficas" la que norma tal integración entre artefacto y estudiante de tal forma que favorece la construcción del instrumento que le permite construir conocimiento matemático.

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***Theme* 5**

From history to epistemology: meanings for trigonometric functions.

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Euler introduced for the first time the trigonometric functions into the calculus. This is a historical fact, but can this have some –positive and rich- implications in trigonometric functions teaching? A historical review brings forward illustrative elements for mathematics teaching, but the epistemological analyses of that historical fact are the ones which give a meaningful basis for it.

Before Euler, the geometric aspects of the sine and cosine were the object of study and not its analytical characteristics. Katz (1987) has mentioned that trigonometric functions may have been avoided because no one saw any *reasonable use* for them as yet. What we want to emphasize here, is that this reasonable use is totally socio-cultural. This use appeared and was developed within the scientific task of *physics mathematization*; this task plays the role of a *reference practice* witch favored the development of activities like modeling, measuring or calculating. Specifically, the necessary uses of trigonometric function emerge via differential equation while Euler was dealing with the motion of a harmonic oscillator. We believe that in this interest to describe analytically a periodic movement there is an implicit *prediction practice*. Montiel (2005) has pointed out this historical moment as the second one in the social construction of the trigonometric function: *predicting is the practice that regulates the physics mathematization*.

A didactical implication of this epistemological moment refers to the periodicity of trigonometric functions. In the didactic system these functions are the more familiar examples of periodic functions: in some cases they are thought as the only periodic ones. An epistemological analysis shows (Buendía, 2004; Buendía & Cordero, 2005), in contrast, that periodicity has its own "historical life" as a property that describes a certain repetitive movement and Euler (XVIII century) as the first one to discuss this property with the trigonometric functions. So, if our didactic system has reduced this property only to

describe a certain function, what can give significant meanings to the periodicity of trigonometric functions? We believe that predicting: that practice that was also in Euler's work.
In this workshop, we propose a review of several extracts from Euler's *Introduction to Infinitesimal Analysis* to point out elements that can conform practices epistemologies for trigonometric functions. We will work also a way to bring these social practices epistemologies into the didactic system: how does a practice like prediction has to be intentionally introduce by means of a situation that favors the knowledge re-signification.

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***Theme* 1**

Estudio socioepistemológico del área y la integral

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En esta contribución se presenta parte de una investigación más amplia en la que estudiamos al fenómeno didáctico de reproducibilidad, vinculado a la explicación escolar de la integral definida como área bajo la curva (Cabañas y Cantoral 2006). Investigación que realizamos desde la aproximación socioepistemológica a la investigación en Matemática Educativa, aproximación teórica que involucra las componentes cognitiva, didáctica, epistemológica y social. La aproximación socioepistemológica plantea como tarea fundamental el examen del conocimiento situado, aquel que atiende a las circunstancias y escenarios socioculturales particulares, caracterizando al conocimiento como el fruto entre la epistemología y factores sociales (Cantoral y Farfán, 2003). Así, desde las componentes epistemológica y social nos preguntamos: ¿Cuáles son los usos y contextos de la noción de área previos a su definición a la manera de A. Cauchy? y ¿Cuáles son los contextos y procedimientos en que se presenta la integral definida a partir de la obra de Cauchy? Entendiendo por *usos* a las formas en que es empleada o adoptada una determinada noción (Cabañas, 2006). Por *contextos* a los entornos situacionales en los que se considera un hecho y los *procedimientos* a las formas de organización (Cordero, 2005).
Se caracterizaron tres tipos de usos previo a la definición de la integral a la manera de Cauchy: El área es susceptible de comparar, conservar y medir. Los contextos se caracterizaron como estático y dinámico. En el primero ubicamos al método exhaustivo –utilizado por los griegos de la época clásica–; al método de los indivisibles –utilizado por Cavalieri–; y al método basado en una propiedad de las progresiones geométricas –usado por Fermat–; el método de las transformaciones –utilizado por Leibniz–. En el contexto dinámico se ubican a las cantidades que varían unas respecto de otras y, eventualmente, todas varían respecto de la variable universal: el tiempo.
Los contextos y procedimientos en que se presenta la integral definida a partir de la obra de Cauchy se caracterizaron como sigue: En el contexto de la concepción de función y de continuidad y los procedimientos en las concepciones de función primitiva y la distribución de puntos en el intervalo de integración donde la función es continua.

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***Theme* 5**

Social practice of the variability notion: an epistemological approach

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In the navigation practice and the geography of the middle XVII century, the variable notion was recognized like a "mistake" that made the needle of the compass when deflect it about the magnetic north, in regard to true north [Juleu, 1723]. The deviation of this angle called "magnetic declination". During that time the geometers gave to the variable for giving a mean to the magnetic declination, taking it like a "mistake", was fundamental for the transition which followed the "variability" concept in the definition the a "theory of mistakes", for one hand, and of the derivative concept, for the teaching, for the another hand. This paper establishes a brief historical development of the definition of variability, considering its genesis like a social practice such as suggest the socio-epistemology.

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***Theme* 3**

Pedagogía, Historia y Matemáticas: El tema de la Medida

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En este trabajo presentamos el proyecto de innovación educativa desarrollado en la región de Extremadura, en España, pero que puede servir a profesores de cualquier parte del mundo para llevar a cabo experiencias similares adaptadas a sus ámbitos geográficos y culturales.
El objetivo de la actividad ha sido conocer y valorar la utilidad de las Matemáticas en una de las actividades más cotidianas: medir. Se ha investigado el uso que a través de la Historia hicieron nuestros antepasados de las unidades e instrumentos de medida, desde los antiguos sistemas hasta el sistema métrico decimal.
A partir de este centro de interés, como recurso pedagógico, se han planteado actividades para alumnos y profesores, de tipo etnográfico (recogida de información entre personas de edad, recuperación de instrumentos en desuso), de tipo histórico (revisión de textos antiguos), o de mayor contenido matemático (cálculo de medidas, construcción de aparatos o resolución de problemas relacionados).
El trabajo se completa con un CD interactivo donde se recoge toda la información y actividades del proyecto.
Se ha conseguido con estas actividades integrar varias áreas del currículo, trabajar de forma colaborativa entre profesores y hacer participar de las tareas escolares a personas del entorno próximo de los alumnos.

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***Theme* 5**

Desarrollo del punto de inflexión como objeto escolar; estudio a la obra de L'Hospital

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Durante el siglo XVII una nueva perspectiva de la difusión de la ciencia alentó en varias ciudades el surgimiento de Academias Científicas, con lo que se fortaleció la comunicación entre los académicos y promovió la instauración de publicaciones científicas vinculadas con las academias. En 1696, cuando la naturaleza del cálculo era un tema de discusión en los círculos académicos, L'Hospital publicó su obra *Analyse des infiniments petits* un trabajo que mostraba, desde una perspectiva diferente a la de Leibniz, el *nuevo* cálculo.
L'Hospital, (1696) organizó el saber *a manera de un curso* atendiendo a la presentación gradual de las ideas, agregó explicaciones a los conceptos del cálculo y problemas. La conformación de este *discurso* orientado a la difusión (no expertas), obligó la caracterización y definición de los objetos matemáticos como *curva, ordenada, diferencia, máximo, mínimo, punto de inflexión.* A partir de la publicación de este trabajo el punto de inflexión adquirió un nuevo significado, se configuró como objeto vinculado a la segundas diferencias y evolucionó como objeto escolar hasta nuestros días. (Cantoral, 1998). Esta investigación muestra el tratamiento del punto de inflexión en la obra de L'Hospital, describe su tratamiento, su caracterización y muestra cómo se configuró como objeto escolar.
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***Theme* 5**

The role of the referee in the History of Mathematics

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The aim of the Royal Society of London, like in every scientific society, is to "organise" the Science, within the certain, in time and place, context. In order to do that, the "society" uses the judgment procedures and structures for every single theme it is submitted. Within the Royal Society of London the referees decide about the works which they are going to publish. The decisions of the referees are submitted to the authority of the council for the final approval. In fact, the decisions of the referees determine the "co-optation"[††] or the exclusion of proposes in the sciences field.
In particular, my study examines the judgment structures, e.g. the opinions of the referees, for the Cayley's paper: *An introductory Memoir upon quantics* published in 1854 in the *Philosophical Transactions of Royal Society.*
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***Theme* 4**

Mathematics in Zeugma

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State of the Problem

The time period we are living in now is the century, where reflections of the pluralist public comprehension is also seen in the education. The changing demographic composition of the society in the last years have carried the education to a multi-cultural point. At this point, the mathematics have also defined a new fields for itself. The Ethno-mathematics is one of them. The fact that the culture is a part of the mathematics lesson, it helps the students to increase their academic successes, (Banks, 1989), it helps with the creation of more equitable learning environment, (NCTM, 2000) and it

correlates the mathematics with the other disciplines, (Zaslavsky, 1998; Moses-Snipes, 2005). But, in many times, mathematics becomes a field that the students fear and they could not communicate with.

Aim of the Work

The multi-cultural mathematical activities should not be thought to be separate from the mathematics field, it should become a field which is debated within the mathematics education. It has been aimed to help the 6th class students to create the mathematical concepts by using the Zeugma mosaics, to realise the relationship between the mathematics and the culture and to understand the value of the Zeugma as an inherited culture with this work. Zeugma (Greek: Ζεύγμα) is an ancient city of Commagene; currently located in the Gaziantep Province of Turkey in the South East of Turkey. Belkis/Zeugma with its historic, archaeological, strategic importance, is a unique and priceless asset, which had been lying buried in the depths of history and only recently saw the light of day. The writers of ancient times, Pliny and Strabon describe the city of 'Zeugma' in its various aspects and the information gained from their Works forms an important part of today's knowledge of the city. There are a number of exceptional assets that make Zeugma important and valuable in terms of its historical chronology, archaeology and art.

The Method

The sample group of the work is the 6th class students who are studying in the Private Darussafaka Elementary School and the Private Istek Acıbadem Elementary School. The sample groups have been chosen amongst the volunteers. The 6 student of the group of the 18 students are girls and the rest 12 students are boys. The work has been carried out during the education year of 2006-2007 after the school hours. The work which has been applied, has been prepared in line with the new educational programmes and has been related with the Social Studies, Computing, Art and Turkish teachings.

Six Zeugma mosaics have been chosen in line with the 6th class mathematic syllabus applications, nine work sheets have been prepared in relation to these and they have been applied. Some of the achievements from the mosaics are to solve and construct the problems in relation to the time measurement units and to determine the relationship between the angles, the sides and the diagonals of the square and the rectangle.

The students have created the two mosaics again, which have been chosen, on the computer medium by using a special computer programme with the help of the computing lessons teachers of the school. The students have enlarged the Demeter mosaic by certain ratio with the help of their art and mathematics teachers and created a Demeter mosaic by using different colour stone pieces. Two excursions have been organised to support the work. As the last point, the "Zeugma" documentary which has been prepared by the TRT, have been shown to the students in order that they shall recognise the Zeugma Antique City more closely. A research home work in relation to the "Zeugma" have been given to the students after they watched the film. The research homework has been shared with the group and a discussion has been carried out.

Limitations

Since the work has been carried out outside of the school period, it became necessary to go out of the planned timing due to the fact that the students had different works and additional time had been required for.

The coordination with teachers of different disciplines could not have been achieved in the level that it has been required.

Result

It has been observed that the contribution of the students had been high during the work.

Two questions have been asked to the students who have participated at the end of the work:

a. Did you have pleasure from this work?
b. What do you think that you have learned/realised with this work?

99 % of the students have answered "yes" to the first question. The following results have been observed from the answers of the students to the questions and from the observations of the administrator as the results of the meetings with the students:

- ☐ They learn the concepts, which they learn in the mathematics lessons, easily,
- ☐ They observe the richness of the mathematical ideas of the traditional public,
- ☐ They realise the relationship of mathematics with the art,
- ☐ They comprehend the applications of the subjects that they learn in the mathematics in the daily life.

- ☐ The requirement of the understanding of the value of the Zeugma have been stated by the students.

Three questions have been asked to the teachers who have participated in the work.

a. Do you think that the students have taken any advantage from this work?
b. Do you think that you, as the teachers, have taken any advantages from this work?
c. Does this work had any additions in your field of work? How?

The following results have appeared after these questions and after the non-constructed meetings with the teachers:

When the work has been studied as the whole, benefits have been obtained not only for the students but for the teachers of the different branches. The teachers who have participated in the work, have found chances to examine and to relate the subjects that they are in their fields in more depth.

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***Theme* 5**

The multiplicity of viewpoints in elementary function theory: historical and didactical perspectives

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The so-called rigorization of Analysis in the 19th century is a standard topic in the history of mathematics, and has indeed provided material for didactical research works centred either on notions (e.g. limit, continuity) or on shifts in levels of abstraction (Advanced Mathematical Thinking). For four years, members of the "history of mathematics" group of the French IREM network endeavoured to establish new connections between historical and didactical questions. For the Paris group, the starting point was the identification of four viewpoints on functions : point-wise, infinitesimal, local and global. We gathered historical material – sometimes standard, sometimes less well-known – showing typical interactions between these viewpoints at different stages of the rigorization process. We also tried to identify the contexts in which these viewpoints first emerged, then were explicitly differentiated one from the other. We eventually devised two epistemological models – the "world of quantity" and the "world of sets" – in order to describe two distinct forms of "functional thinking". These high-level descriptive tools helped us gain new insights into didactical questions relevant for the teaching of Analysis at elementary or advanced level. After a short case-study, we will present the main features of the epistemological models. We shall eventually consider more general teaching perspectives.

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***Theme* 2**

Bernt Michael Holmboe (1795–1850) and his textbooks in school Mathematics

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Bernt Michael Holmboe (1795-1850) was the teacher at Christiania Kathedralskole that discovered Niels Henrik Abel's (1802-1829) unique skils in mathematics, and who gave him guidance and private tuition. Holmboe was at the Kathedralskole from 1818 till 1826, and after that he was professor at the University of Christiania until his death in 1850. He published the first edition of Abel's complete

works in 1839. Holmboe wrote several textbooks, two of them in basic mathematics and three in more advanced mathematics, and he was probably one of the most influential persons in the development of school mathematics in the first half of the 19th century in Norway. His presentation of geometry in the books was, however, not without opposition. In 1835, the applied mathematician Christopher Hansteen (1784-1873) wrote a textbook in geometry where he challenged the traditional Euclidean geometry, and he introduced the subject mater in a very un-Euclidian way. The controversy that followed in the newspapers has later been called the «dispute about parallelism». The core of it was whether one in mathematics education should -as in the case of Hansteen- let utilitarian considerations overrule logical deduction and theoretical thinking.

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***Theme* 1**

The FSU Cuneiform Tablet Collection: Using Mathematics of the Past to Inform Teaching in the Future

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This presentation describes the rediscovery of a collection of 25 cuneiform tablets sold to Florida State University in 1922 by Edgar J. Banks, the work to translate and publish the text of the tablets (with Eleanor Robson), and the development of instructional materials based upon two tablets, whose text provides accounts of agricultural labor.
The current work on the project is being conducted with a secondary mathematics education graduate student. Goals of the project include creating experiences (for prospective and in-service teachers and K – 16 students) to engage with artifacts that represent arithmetical processes used by people for daily life and that will strengthen the view that mathematics is a human construction. The development of instructional materials based upon the wage and labor information documented in FSU 22 and FSU 23 is aimed at creating examples of how to incorporate humanistic perspectives when teachers, secondary and post-secondary students examine artifacts from ancient civilizations that contain evidence of arithmetical or mathematical processes. The presentation intends to focus on the progress and process of developing instructional materials based upon FSU 22 and FSU 23. Descriptions of a graduate student's participation in key project activities are also included.

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***Theme* 4**

J. Vicente Gonçalves and the "Journal of the Faculty of Sciences of Lisbon University": a contribution to the dissemination of Portuguese mathematical studies

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In this study we bring to light an initiative of J. Vicente Gonçalves – the creation of a scientific journal in 1950 – that promote the exchange of mathematical knowledge between Portuguese mathematicians and foreign mathematicians. We describe this journal. We present a list of the mathematicians who that published in this journal. We report a process developed by J. Vicente Gonçalves to make the exchange of institutional journal with one scientific publication. We present examples of scientific mathematical discussions motivated by the existence of this journal.

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***Theme* 4**

The Alto Douro "wine coopers' mathematics"

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In this study, we identify some mathematical procedures used in a traditional job of Alto Douro's Portuguese Region – wine cooper. We present and discuss the way that the wine coopers use (most of the time, intuitively) well-known mathematical notions to do certain calculations. We propose some tasks to develop with students in mathematical classes in this region, even at the elementary levels, that can be adapted in other regions or countries.
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***Theme* 5**

Reflexiones acerca de argumentaciones y matemática en escenarios sin influencia aristotélica y su importancia en el aula de matemática

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Este trabajo presenta una caracterización de algunos escenarios culturales que se dieron a lo largo de la historia que no tuvieron influencia aristotélica y en los que las argumentaciones utilizadas difirieron de las que se originaron en Grecia. En estas culturas, se abordaron y trabajaron algunos conceptos matemáticos, como por ejemplo el cero y el infinito, cuya aceptación, abordaje y tratamiento científico tardó varios siglos en la cultura occidental. Esto pone de manifiesto el carácter de construcción cultural de las formas de argumentación y la posibilidad de construir conceptos matemáticos sobre la base de otras formas de pensamiento. Por otra parte, en el aula de matemática, se detecta la presencia de algunas formas de argumentación no correctas para la lógica aristotélica y que no han sido aprendidas ni trabajadas en escenarios escolares. Ante la no aplicación de los métodos que serían considerados válidos, se obtuvieron, en esta investigación, declaraciones de los estudiantes sobre su posición frente a formas de argumentación clásicas, que denotan en algunos casos, la no aceptación de las formas clásicas y el logro de mayor convicción de ellas para las que los estudiantes utilizan.
Nuestra cultura, con base aristotélica, ha construido formas de argumentación basadas en la lógica clásica. Sin embargo, las situaciones que evidencian el carácter de construcción social de la argumentación matemática, consideramos tienen que ser tenidas en cuenta en el discurso matemático escolar. El enfoque socioepistemológico utilizado en esta investigación, permite plantear la necesidad de fijar la atención en las formas de argumentación no clásicas que se presentan en el aula y analizar de qué manera podrían contribuir a la construcción de conceptos matemáticos escolares.
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***Theme* 2**

El desarrollo de la Matemática en México

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De acuerdo al libro de Bell, hay tres etapas de desarrollo en la matemática: *Etapa prenewtoniana* que concluye con el descubrimiento del *calculus* y la ley de la gravitación universal por Newton, en el siglo XVII. En la segunda, *etapa del formalismo*, siglo XVIII, se solucionan muchos problemas; en la tercera,

etapa del rigor, siglo XIX, se redefinen los conceptos clásicos: Números, función, límite, derivada, integral, etc. En el siglo XX, la aplicación de la matemática, la ciencia y la tecnología genera un crecimiento increíble de la computación y de la exploración espacial. En México hubo trabajos formales de matemática y ciencia, de manera paralela al mundo, desde la antigüedad. En los inicios del siglo XX, en el Antiguo Colegio de San Ildefonso de la Ciudad de México, sede de la Universidad Nacional, y después de la ENP de la UNAM, un grupo de profesores de matemáticas de bachillerato cimentó nuestros actuales avances científicos y tecnológicos. Es importante conocer a quienes promovieron a la Facultad de Ciencias, la Sociedad Matemática Mexicana, la Sociedad Mexicana de Física, los institutos de investigación, los modernos observatorios astronómicos y la visión de nuestra matemática y ciencia en el contexto internacional.

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***Theme* 1**

Conjetura y demostración en Geometría Dinámica a partir del libro de los lemas

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Reportamos una investigación realizada en torno a la enseñanza de la demostración con profesores de matemática de enseñanza media. La misma se llevó adelante en un mini-curso para profesores de enseñanza media (4 instancias de 2 horas cada una) en las que se trabajó en un ambiente dinámico (con *The Geometer's Sketchpad*) y que tenía por objetivos involucrar a los profesores en actividades de formulación de conjeturas y elaboración de pruebas así como enseñar el manejo de las herramientas básicas del software. A partir de algunas proposiciones que figuran en el *Libro de los Lemas* de Arquímedes (obra compuesta por quince proposiciones referidas a cuestiones de geometría elemental) se diseñaron actividades que involucraban el trabajo en un ambiente dinámico. Reportamos aquí las conjeturas y pruebas elaboradas por parejas de profesores sobre una actividad elaborada en torno a la Proposición 4 donde interviene el *arbelos*.

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***Theme* 3**

The big ideas of mathematics

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In my case studies research project involving a cohort of very able New Zealand mathematics students, various aspects of the experiences and the characteristics of such students were highlighted in new ways. In this paper, I consider two of those aspects. One is the basic human need to socialise. The other is the indication of differences among these students. The students had in common the fact that the mathematics they enjoyed for study, competitions or their own private pleasure stymied most of their social peers. Yet, it was not enough to describe them simply as mathematicians. Differences among them had several dimensions. Each individual's aptitude wove itself among her/his preferences and skills in subject areas. It is conjectured that the patterns of those differences give clues for identifying ways in which understanding differences (in terms of aptitudes as well as in terms of discipline-based preferences and skills) could help make the big ideas of mathematics more commonly part of everyday knowledge than is presently so; and, through that, help able students establish points of contact for interactions with their social peers.

In this paper, annotated drawings and diagrams are used with written text, for when I set out to describe the complexities of the dimensions of differences among the case studies students and the varieties in their social relationships, I found the key to my conveying layers of meaning lay in focusing on number

and spatial relationships for, first, drawing models and, then, writing words. I was prompted to recall examples in history when the outputs of individual people, cultures and periods of time had intrinsically linked the skills of participating in the arts, philosophising, and developing ideas in mathematics.
Such a period characterises our own times. Changes in technology have made the arts of design and visual representation as important to modern students as those of any other art or area of skill. Changes in patterns of employment have increased workplace and leisure time focus on design and visual representation in computer graphics, fashion, architectural and interior design, and publicity projects as well as in the methods of expressing the skills associated with traditional trades and professions. The big ideas of mathematics, as well as the utilising of its procedures, can be important tools in all of these activities of this new technological age. Understanding differences among the students of my case studies and realising they are likely to be found among other students, too (for similar differences have been corroborated by other researchers), endorsed that which is illustrated by history: different ways of knowing are influenced by time, by place, and by the outlooks and knowledge of the individuals who happen upon each idea. Historians of mathematics have significant material to offer educational programmes seeking to move from seeing classroom mathematics as tasks in calculation to seeing it as the exposition of the big ideas of mathematics. A bonus for students and teachers is the extent to which the utilisation of the events of its history can enable the exposition of the big ideas of mathematics to transcend the cultural and geographic boundaries often implicit in mathematics curricula.
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Theme 3

El libro Geografía General y el Conocimiento Matemático del siglo XVII.

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Las matemáticas, en los inicios de la Edad Moderna, se clasificaban en “puras”, como por ejemplo, la geometría, la aritmética y trigonometría y en “mixtas”, que también abordaban aspectos empíricos y, por eso, se consideraban físico-matemáticas. Entre esas últimas estaban la música, la mecánica, la óptica y la geografía, cuyos textos nos muestran conocimientos matemáticos “puros” utilizados y difundidos, en aquella época. Vamos a analizar las matemáticas contenidas en un libro de ese contexto, el Geografía General (1650) de Bernhard Varenius, publicado, por la primera vez en Holanda. Ese libro, considerado el primero de geografía científica, tuvo entre los años de 1650 y 1750 veinte y cuatro ediciones. La primera de ellas la he hecho Newton, en 1672. En ese libro, las matemáticas, como las comprendemos hoy, tienen destaque especial y su análisis nos muestra como, en la época, se representaban los números decimales no enteros, como se elaboraban algunos conceptos trigonométricos y como ocurrían algunas aplicaciones de la geometría proyectiva.
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Theme 4

The Al Biruni's Trigonometry

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Al-Biruni was a mathematician of the eleventh century. He was interested in several sciences and produced many mathematical and astronomical works and results. In this short workshop, we would like to present a history of trigonometry during the Arab-Islamic Civilization, revolved around the works of Al-Biruni. Of course, we will not claim to be aware of dozens of al Biruni trigonometry's works. We refer especially to two treaties:

1-The Canon Masudicus (Al-Qanun Al Masa'oudi) especially the third chapter. This chapter describes al Biruni's method for calculating the sine table (chords). To calculate some sines, He makes use of various artifices and exhibits an ingenious method on quadratic interpolation. One of Al-Biruni's specificity is that he often located his own works in relation to these of his ancestors (Greeks and Hindus), and contemporaries.
2 - The calculation of circle's chord (Istikhraj al-aoutar fi ad'daira): This treaty is undoubtedly characterized by the fact that it is based on an interesting and relatively forgotten geometric property: the broken chord theorem for witch Al-Biruni gives twenty proofs and some corollaries. He made use of that property to solve problems relating to trigonometry, geometry, astronomy and algebra!
In this presentation, we will focus on:

- History of trigonometry during the Arab-Islamic period;
- Trigonometric formulas and properties used by Al-Biruni;
- The broken chord theorem, its proofs and its corollaries;
- Various applications of the broken chord theorem;
- Calculating of sin40° (three methods) and sin 1 °.

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***Theme* 6**

American women mathematicians in the last quarter of the 19th century

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By the last decades of the 19th Century, the North-American scientific community had begun to not only supplement the traditional undergraduate colleges by graduate schools in general, but also by promoting research and the training of future researchers in particular. The introduction of the doctorate degree signified the achievement of original research work and implied the capability for further production and also became a foremost requirement for entrance into the professoriate. Regarding science in general and mathematics in particular, this development within higher education produced useful results.
This timeliness proved to be even more crucial for the entering of women into the mathematical community. It was no mere coincidence that women contributed at all levels of the American mathematical community in the last quarter of the 19th Century. The time was indeed ripe, since three efforts converged in this period to create productive conditions for women in mathematics:
1. More than hundred years of advocacy and advancement in women's education culminated in a general opening of graduate schools and subsequent awarding of doctorates to women.
2. Women desirous of active participation in the mathematical community not only possessed the intellect but also the stamina to withstand the hardship of being the first ones.
3. Women operated in an arena with many influential mathematicians and others who not only sympathised but also supported their active participation.

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Theme

Comprensión de las Medidas de Dispersión: Caso de la licenciatura en Psicología

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La presente investigación gira en torno a caracterizar la comprensión de los estudiantes de la licenciatura en Psicología de las medidas de dispersión. Para ello se identificaron procesos, estrategias, cálculos, tipos de expresión, explicaciones e interpretaciones que pusieron en juego cuando acometieron cuestiones que implican esas medidas, luego de su enseñanza. Por sus aplicaciones y su contribución

formativa, el interés se centró en el rango, la desviación media absoluta, la desviación estándar y la varianza.
Se consideró la aportación de Pollatsek, Lima y Well (1981) se tomó en cuenta el planteamiento sobre tipos de conocimiento analógico, de cálculo y funcional derivados de su reflexión sobre la comprensión instrumental o relacional de la media; mientras que de Steinbring (1991), nos referimos a su propuesta de procesos de desarrollo del concepto matemático y cómo se construye su significado a través de la interacción social en el aula.
Para el desarrollo de la investigación, de orden cualitativo (Eisner, 1998), se tomaron en cuenta los siguientes espacios de estudio con sus respectivos instrumentos de investigación: para la propuesta institucional —compuesta por Plan, programa de estudio, lecciones de los libros de texto— se empleó guía de análisis; para la enseñanza en el aula se analizaron las sesiones de estudio de medidas de tendencia central y de dispersión a partir de un guión de observación; para el estudio del desempeño de los estudiantes, se aplicó un cuestionario antes y después de la enseñanza además se realizó una entrevista a seis de acuerdo a su desempeño en el cuestionario. En el estudio participaron el docente y su grupo de 1er semestre (30 estudiantes) de la licenciatura en Psicología de una Universidad privada.
Para caracterizar la enseñanza en el aula se observaron cinco sesiones de estudio y se video grabaron para su análisis posterior; se apeló a los tres tipos de conocimiento en las sesiones sobre las medidas de tendencia central; imperó en la enseñanza el uso instrucciones o series de pasos a seguir hasta completar procedimientos de obtención de resultados numéricos, el conocimiento analógico a partir de gráficas dio cuenta sólo de los desvíos respecto a la media y no de alguna otra medida estadística; entonces su conocimiento se reduce al de expresiones simbólicas, definiciones en lengua natural y valores numéricos de la medida.
La estrategia de enseñanza de dictar y escribir en el pizarrón, así como no encomendar investigaciones o trabajos extraescolares, condujo a una relación de enseñanza unidireccional, donde al estudiante se le impuso el papel de receptor pasivo, que en el mejor de los casos reproducirá lo estipulado desde la enseñanza.
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***Theme* 5**

Axioms in search of a definition

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The fundamental difference between the modern axiomatic method, enunciated by Hilbert, and the ancient, as practiced in Euclid's *Elements*, lies with the role of the basic definitions. In Hilbert's *Grundlagen* the set of axioms constitutes an implicit definition of the, otherwise undefined and philosophically neutral, basic concepts; in the *Elements*, the postulates, as we argue on the basis of the anthyphairetic interpretation of the Platonic Beings, have only an empirical and heuristic role, and are in need and in search of the suitable definitions, whose origin and cause is in the upper realm of Platonic ideas, capable of generating these postulates below. The restored role of the ancient postulates reveals them, not as the finished products presently conceived and widely criticized as antipedagogical, but, on the contrary, as partial empirical constructs, ideal for turning the students into small researchers in search of a definition.
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***Theme* 6**

Atención a la diversidad: detección del talento en Matemática en alumnos de 7º- 8º años de EGB 3 de la ciudad de Concordia

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Se presenta la investigación realizada en la ciudad de Concordia, Entre Ríos, Argentina durante los años 2004-2005 para detectar alumnos de 7º- 8º años de EGB 3 con talento en matemática. Se selecciona una muestra aleatoria estratificada de 726 alumnos provenientes de 13 escuelas de gestión pública y privada, del núcleo urbano y rural de la ciudad. En la fase de screening se aplican: test de Matrices Progresivas de Raven (escala general), escala de Renzulli-Hartman sobre los aspectos de creatividad, aprendizaje, liderazgo y motivación; nominación por pares y nominación del docente de matemática. De esta fase se seleccionan, con criterio inclusivo, 183 alumnos que son evaluados en la fase confirmatoria con una prueba de nivel superior. Se aplica Análisis Factorial de Correspondencias entre las variables estudiadas y se confirman a 42 alumnos como alumnos con talento en matemática, (5, 78% de la muestra).
Se observa en los docentes una concepción incompleta del alumno con talento específico, no así, en la nominación por pares.
Los alumnos detectados participan exitosamente en distintos certámenes propuestos por la Olimpíada Matemática Argentina, durante el año 2005.
La investigación sirve como disparador de inquietudes para la Dirección de Educación Especial de la Provincia de Entre Ríos.
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***Theme* 5**

Carl Runge: A Professor of Applied Mathematics at Georg-August Universität, Göttingen

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Carl Runge (1856-1927) published major works on numerical methods during the years of 1895, 1901, 1903, 1905, 1908, respectively, on numerical solutions of ordinary differential equations, on the Lagrangian interpolation, on the Fast Fourier Transform, the trigonometric interpolation, and lastly on the numerical solution of partial differential equations by finite differencing. All these achievements had a definite major impact on the European mathematical community. Runge obtained a position at Hanover University in 1886, where he remained for 18 years. Then, in 1904, Felix Klein offered him the first chair of applied mathematics at the prestigious Göttingen University. Felix Klein's objective was to diversify his department. Therefore, in this present paper, we examine the impact of Runge's work on applied mathematics, his intuitionist and constructive philosophy.
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***Theme* 1**

La Derivada Insensible

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Objetivo: Mostrar un desarrollo armónico de múltiples aplicaciones en problemas resueltos usando el concepto de derivada. Mediante una breve Historia presentatoría de la derivada.
Contenido:

1. Introducción.
2. Problemas geométricos.
3. Problemas Algebraicos.
4. Problemas de dinámica.
5. Observaciones Históricas
6. Conclusiones.

Material:
Lap-top

Cañón
Software: Cabri.
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***Theme* 1**

Prospective Teachers' Views on the Integration of History of Mathematics in Mathematics Courses

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The purpose of this study was to investigate prospective mathematics teachers' attitudes about integrating History of Mathematics in mathematics teaching, their suggestions about the means and methods for integrating History of Mathematics and their expectations about the motivational and conceptual benefits of such an integration. The study also investigated how prospective mathematics teachers changed in terms of their content knowledge and attitudes about integrating History of Mathematics following the instruction that exemplified the use of history of mathematics. The pretest-treatment-posttest quasi experimental design was used. The intervention took place during a "Teaching Methods in Mathematics" course. It contained a number of tasks that required students to participate in activities designed to improve competencies regarding the integration of history of mathematics in mathematics courses. Results indicated an increase in students' attitudes and content knowledge about integrating history of mathematics in mathematics teaching. The increase in students' attitudes was not found to be significant but the increase in the number of strategies students stated for the possible uses of history of mathematics in mathematics classrooms found to be significant. Moreover, the quality of the examples stated for the possible uses of history of mathematics in the classroom found to be significant.
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***Theme* 6**

American calendars in the sixteenth century

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Calendars and their history relate different fields of science and culture such as mathematics and astronomy on the one side but also politics, religion, archaeology, and ethnology on the other side. In the same sense their investigation does not only lead to new insights in the history of mathematics and astronomy, but also to a further understanding of foreign cultures in general. Nearly 500 years ago the Spanish conquest of Mesoamerica changed this part of the world considerably.
This talk will discuss several calendars of the sixteenth century in America, mainly in Mesoamerica, i.e. pre-Columbian calendars like the Maya calendars and the Aztec calendars as well as the Spanish calendars, the Julian and the Gregorian one. Moreover, the pre-Columbian calendars are discussed from our modern point of view as well as from the point of view of the Spanish conquistadores in the sixteenth century.
Before the Spanish conquest of Mesoamerica in the beginning of the sixteenth century among many peoples two calendar systems had been in use, a 365 day calendar (in Maya culture called Haab) and a 260 day calendar (in Maya culture called Tzolkin). Together they form a calendar round of circa 52 years. Moreover, the Maya used the long count calendar, the counting of days from a zero date in the far past. Since these calendars are already discussed in the author's paper of the HPM Uppsala Proceedings the focus here will be on how the Spanish conquistadores of the sixteenth century describe and discuss these calendars. It turns out that these reports are quite poor since the Spanish understanding was more misleading than explaining. The very foreign culture of Mesoamerican peoples (from the Spanish point

of view) in general and the unusual construction of the calendars in particular as well as a special Christian perspective led to an unsufficient description throughout the whole century.
The more the Spanish conquered the country and the more they pushed the Mesoamerican culture aside the more the European Christian (the Julian) calendar came into use in Spanish America. On the one hand, this calendar replaced the indigenous calendars step by step. On the other hand, this solar calendar had been shifted by ten days during the centuries. Calendar reform plans finally led to a replacement of the Julian calendar by the Gregorian calendar in 1582 in Catholic countries. The consequences for Spanish America will be discussed in further detail here.
At the end an outlook towards the centuries which followed will conclude my paper concerning the knowledge and use of pre-Columbian calendars as well as the propagation of the European calendar. Altogether, this calendar history clearly shows how the encounter of two different worlds happened and how the victorious culture destroyed most but not all of the defeated culture.
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***Theme* 1**

Didáctica del Teorema de Pitágoras: Aplicando Situaciones Didácticas.

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Este material pretende ser un material para la reflexión didáctica a partir de la obra matemática denominada "Teorema de Pitágoras", consiste de una secuencia didáctica propuesta como taller con una semana de duración utilizando diferentes técnicas demostrativas e interactivas, con actividades donde se reconstruyen algunas de las demostraciones mas famosas de este teorema que consideramos el mas conocido y didáctico de los teoremas matemáticos. Permite la elaboración de situaciones didácticas que tienen una recuperación histórica, las cuales serán objeto de un proceso de análisis utilizando ingeniería didáctica.
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***Theme* 1**

Negative numbers as an epistemic difficult concept: Some lessons from history

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Historical studies on the development of mathematical concepts will serve mathematics teachers to relate their students' difficulties in understanding to conceptual problems in the history of mathematics. We argue that one popular tool for teaching about numbers, the number line, may not be fit for early teaching of operations involving negative numbers. Our arguments are drawn from the many discussions on negative numbers during the seventeenth and eighteenth centuries from philosophers and mathematicians as Arnauld, Leibniz, Wallis, Euler and d'Alembert. Not only the division by negative numbers poses problems for the number line, but also the very idea of quantities smaller than nothing has been challenged. Drawing lessons from the history of mathematics we argue for the introduction of negative numbers in education within the context of symbolic operations.

> Mon enthousiasme pour les mathématiques avaient peut-être eu pour base principale mon horreur pour l'hypocrisie; l'hypocrisie à mes yeux, c'était ma tante Séraphie, Mme Vignon, et leurs prêtres. Suivant moi, l'hypocrisie était impossible en mathématiques, et, dans ma simplicité juvénile, je pensais qu'il en était ainsi dans toutes les sciences où j'avais ouï dire qu'elles s'appliquaient. Que devins-je quand je m'aperçus que personne ne pouvait m'expliquer comment il se faisait que: moins par moins donne plus?
> (From *The Life of Henry Brulard* by Stendhal, 1890)

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Theme 3

Euler's Contributions to Mathematical Cartography

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Euler's appointment as a full professor to the Academy was not the most exciting news in St. Petersburg in 1730. Possibly that news was carried by Vitus Bering, returning from five years exploring the Siberian far east. It fell to the Academy geographer Nicolas Deslisle and his young colleague, Leonhard Euler, to organize the mass of data that Bering brought back.
More than 40 years laters, Euler published a series of three articles about mathematical cartography: We examine Euler's interest in cartography in the context of the developing science of cartography, the developing Russian nation-state, and the internal politics of the St. Petersburg Academy.
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Theme 1

Conceptualizing PCK in terms of HPM

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In this presentation, I will try to conceptualize some aspects of PCK in terms of HPM, namely, genetic reflection on concepts / theories, historical enlightening, cultural cognition / re-orientation, methodological re-invention, vertical mathematisation, and reflection at advanced standpoint. Empirical data will be drawn upon to make sense of the above indicators concerning quality assurance of high school mathematics teacher professional development, which is part of the outcome of a three-year project funded by National Science Council, Taiwan in 2005-2008.
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Theme 1

History of modern mathematics and/or modern applications of mathematics in mathematics education

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The idea of this paper is to discuss the integration of history of modern mathematics and/or history of modern applications of mathematics in mathematics education as well as the possible teaching and learning benefits of introducing a newer history of mathematics over an old(er) one – something which seems to be done most often when integrating history. Three cases of the history of modern mathematics or modern applications of mathematics are presented and later discussed in terms of their possible contributions to the use of 'history as a goal' and 'history as a tool'. As a means for further illustration of this, empirical data from concrete implementations of two of the cases are also presented and discussed.
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Theme 2

Aplicando Copyleft y Creative Commons en la difusión y protección de las ciencias, así como su preservación digital en el largo plazo

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La libre difusión de las ciencias exactas ha permitido el desarrollo de la humanidad; no puede concebirse ha esta, sin la presencia del conocimiento y conforme él ha evolucionado, el hombre lo ha hecho. Tanto conocimiento como libertad están ligados: el hombre aprende, enseña y hace uso de sus conocimientos en la medida que tiene libertad para hacerlo. Desafortunadamente la difusión del conocimiento se enfrenta a dos corrientes donde se debate el acceso y uso de la información; la primera busca limitar su difusión a través del endurecimiento de leyes (copyright) y el uso de las TIC's; en contra parte, esta la corriente que promueve la libre difusión a través del uso de las licencias de conocimiento libre como Copyleft y Creative Commans. Por otra parte es necesario cuidar la preservación de la información digital, la cual tiene una vida útil en promedio de 6 a 7 años debido a la obsolescencia del hardware o software usado para almacenar o procesar la información. No hacerlo implicará la perdida de un volumen considerable de recursos digitales. De nada servirá proteger a la información bajo licencias de conocimiento libre, si esta se pierde por cuestiones de obsolescencia tecnológica en el mediano plazo.

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***Theme* 4**

Ethnomathematical Problems for School Mathematics: Teachers Posed Math Problems in Context on Their Own Culture

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In our era the existing system of education faces a dilemma concerning the place of the individual and his customs, traditions and knowledge. And what of the teacher, should he be obligated to adjust his teaching, lesson contents and tasks to the socio-cultural background of his pupils? Indeed, the teacher must understand how the culture his pupils come from shapes their perceptions and expectations. If we will accept this thesis as a correct one regarding math teachers, the orientation in math teachers' education should be developing their multi-cultural perception in mathematical education.

Besides, similar dilemma rises from math itself. Many different fields of mathematics spawned from the study of problems borrowed from man's need to find urgent solutions to them. History provides numerous examples. Raind's papyrus deals with issues of daily life and agriculture and teaches how to perform mathematical operations involving fractions. The Bible explains how to observe the commandments in their exact form and fosters the knowledge of Euclidean geometry while the Quran teaches its readers how to make justice in division of inheritance, and on the way teaches a lesson in percentages. The appearance of Mathematics as a discipline created a need in study books and inaugurated a tsunami of mathematical problems, known to us as 'word problems', composed for study purposes only. It seems, this in its turn triggered the "math problem haters'" era. The roots of this 'word problem syndrome' lay in the absence of pupils' ability to cope with information-heaped text of the problem, perceived by them as tasteless information. The question is therefore this: how can we transform the tasteless text into a narrative that shall assist the pupil to make sense of the math problem?

The ethnomathematical problem is one of the answers to that question. Ethnomathematical problem is a mathematical problem in which the verbal text uses a narrative to describe mathematical practises present in the customs, traditions and daily experiences of different socio-cultural groups. The numerical value of the problem solution must be examined in social context**.**

The idea to integrate a program of ethnomathematics (D'Ambrosio, 1985, 2006) in "History of Mathematics" educational college course stemmed from the desire of helping teachers to get acquainted with mathematical practices in Jewish and Bedouin cultures, who are the two sectors comprising the course's population. As a part of the course's duties the teachers had to conduct a field investigation

involving screening and identification of the components embedded in mathematical practise found in texts and tales of their own society and culture. Following this investigation the teachers posed ethnomathematical problems in areas of geometric transformations, graphs, combinatorics, etc. The paper contains several problems and cases of teaching in which teachers used ethnomathematical tools in school math teaching. The discussion would focus on the unique nature of the contents and text styles of these problems. It also examines how math and culture can meet in the math classroom in aim to facilitate solvers' coping with mathematics word problems and also to become familiar with practical mathematical heritage of their own people.

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***Theme* 2**

Teaching and Learning of Geometry in Japan nn the late Nineteenth Century

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Until early 1870s, Japanese people learned traditional Japanese mathematics,. Mathematics was regarded as a practical science, and they learned mathematics through problem-solving. After the Meiji Restoration (1867 - 1868), Japanese Government intended to modernize Japan by introducing Western civilization into Japan. Since 1872, mathematics has been taught in Japan mainly in Western style. Among the various branches of Western mathematics, the most difficult one for Japanese in early 1870s was geometry, as Euclidean geometry is of quite different nature from geometry. At first, Euclidean geometry was considered relating with mensuration. The value of Euclidean geometry, however, was recognized soon. Geometry was regarded as a subject for mental discipline. Teaching of geometry was gradually improved. In this way, teaching of geometry had got the right track by the end of the nineteenth century.

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***Theme* 1**

Contributions from the study of the history of statistics to understand students' difficulties to grasp the concept of variance.

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In the present paper a didactically oriented study of the history of statistics underlies the importance of some students' difficulties concerning the initial understanding of the variance, which are very often neglected or misunderstood by the usual teaching of statistics. The study of the history also helps in understanding their depth and fundamental character:

(a) The study of history of statistics points out the importance of the context of the treated situations in the emergence and the evolution of understanding of the sums of squared distances from a central point or a central line (i.e. variance, method of least squares,…). In this evolution an issue that is pointed out to be particularly important is the **complexity** of the involved situations: The concept of variance for uni-dimensional distributions is initially conceived and developed in the context of simple probabilistic situations (i.e. binomial distribution, normal distribution (De Moivre 1733), …, central limit theorem (Laplace 1810)). The Method of Least Squares (MLS) for bi-(and multi-)dimensional distributions was initially conceived by Legendre at 1805 in the context of data treatment in problems of astronomy and

geodesy. During the18th century, probabilistic thinking and the treatment of data in astronomy and geodesy followed distinct paths. These paths gradually converged, and their synthesis culminated with the works of Gauss and Laplace (that concerned precisely the MLS), in 1809 and 1812 respectively.
However, both, the aforementioned probabilistic situations and problems of geodesy and astronomy, were by far simpler than the the problems referring to social phenomena. Transferring the important statistical methods and aggregates (e.g MLS) developed for treating data in astronomy and geodesy to the social science was not easy. A laborious evolution for almost a century and overcoming important conceptual barriers was necessary for giving a coherent meaning to these methods and aggregates in the context of problems of social sciences, and for creating adapted tools that permitted efficient statistical treatments of problems in this domain. A main reason for these difficulties was the great inherent complexity of social phenomena (i.e. they are influenced by a very large number of often interrelated factors).
(b) History of statistics point out that there is an intimate relation between statistics and Physics. This relation is not limited in the fruitful developments realised in the context of problems of Geodesy and Astronomy, during the 18th and the early 19th century. Later developments both in statistics and physics, continuing until the 20th century, have gradually permitted to understand that basic statistical concepts (such as the sum of squares distances from a central point or from a regression line, the binomial and the normal distribution) have a deep physical meaning and were involved in the modelisation (and thus in the understanding) of fundamental Physical phenomena (such as the absolute temperature of ideal gases and solid bodies, the Brownian motion,…).
In the usual introductory courses of statistics very often they are used exclusively (or almost) situations examples that are related to social phenomena[‡‡], whereas meaningful examples from other domains, such as Physics or Geometry, are absents.
Our analysis of students' behavior points out that (i) at the introductory level, is difficult to elaborate a coherent meaning for the variance (and more generally for the sum of squared distances from a center (central point, central line,…)) in the context of situations related to social phenomena, (ii) when introductory teaching of statistics is restricted to use exclusively (or almost) such situations examples, this restriction can activate important epistemological obstacles against students understanding of the variance.
The study of history of statistics helps in a deeper understanding of these students' difficulties. Moreover the history of statistics inspires different ways for introducing the variance that may facilitate students' initial understanding of the variance. Further didactical analysis point out that one of the most promising such way may be based on the use of adequate simple Physical models.
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***Theme* 1**

On the role of the history of mathematics in mathematics education for the knowledge-based society

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The main aim of an education for the knowledge-based society – apart from making an opportunity for learners of having an access to the knowledge and its operative using – is to shape the competence of lifelong learning.
Current dynamic transformations in the field of science, engineering, economy and also rapid social changes, which occur under the influence of information and communication technologies, anticipate a permanent development that will surely accompany future generations in all areas of life.
All these changes have implications for the field of education. The principle of Jacques Delors: "Learning to learn" shall become the central idea for all current and future educational affairs. The mathematical knowledge shall take especially important position in general education, since the world uses so intensively information and communication technologies, that every domain becomes more and more formalized and mathematized in order to be more and more explored and developed by mathematical models and methods supported by new technologies. Thus shaping the ability to learn

[‡‡] many of which are labeled also as "every day life" phenomena.

mathematics not only in a frame of school or academic system but also independently of them, is extremely important for future adult life in the knowledge-based society.
Contrary to this indispensable idea, in recent years we are witnessing a strong tendency to prepare more and more specialized curricula at university level: more specific professional subjects and less hours devoted to learning basic subjects – among them to mathematics. This trend of reducing an amount of hours for mathematics teaching is also possible to observe at the secondary school level. Mathematics educators have still more and more difficult problem to solve – how to prepare students to learn, to understand and to study mathematics individually. The history of mathematics can play very important role in solving this essential problem. In this presentation I will show on some examples how the knowledge on the history of mathematics can help in developing fundamental mathematical concepts like the concept of theorem and its proof and also the concept of probability. In this approach which is based on my research studies, the knowledge on historical development of mathematical concepts serves as a tool to give to students an opportunity to learn according to their cognitive development. This approach can be supported by using new technologies, according to students' current natural environment.

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***Theme* 6**

Nepohualtzitzin, mucho más que un instrumento de cálculo.

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Consideramos que en el Nepohualtzitzin se resume la concepción metafórica de la matemática ancestral indígena. El nombre de este instrumento de cálculo es de origen náhuatl y su estructura numérica corresponde a la maya, pues utiliza los valores de uno y de cinco como son los puntos y las rayas. En náhuatl significa: Ne, sufijo de persona; Pohual, cuenta; Tzitzin, lo trascendente. Puede traducirse como "El que cuenta para trascender", idea consustancial a su diseño.
Resulta fascinante descubrir que los números poseen espíritu a través de su metáfora y que nuestro ser contiene cifras; que somos un microuniverso con un orden matemático, el cual, al coincidir con la cuenta cósmica del macrouniverso, se convierte en una asociación numérica sublime que nos conduce a una conducta de vida ascendente hacia la plenitud.
Al incluir en las clases el aspecto histórico de la construcción del conocimiento matemático, no se lo ubica al alumno desde una posición pasiva de receptor de verdades acabadas; sino que trabaja los contenidos desde una posición más cercana a su construcción, desde sus evoluciones y limitaciones, lo cual le otorga un carácter más humano y más acorde a una concepción de aprendizaje científico como proceso permanente de construcción.
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***Theme* 1**

Teachers thinking dynamically collaborative teaching practice to introduce the history of mathematics and dynamic geometry software into the teaching of 8 - 14 year olds.

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This talk will summarise the findings and present the tools used in a collaborative teaching project which run over two academic years in secondary and primary school in Kent, UK. The project focused on developing collaborative teaching practice between teachers from different phases/key stages in order to promote greater continuity of teaching and learning. The focus of the project was the teaching of geometry in historical context, using dynamic geometry software. A range of topics taught, the pedagogy developed, and the path of discovering and using the historical context by the mathematics teachers with no previous historical training will be traced.
Historical context allowed teachers to overcome the difficulties of developing an organic programme which could be applied in primary as well as in secondary setting. Some difficulties, originating from the fact that the primary school teachers did not have formal mathematical training and were uncomfortable with the teaching of geometry were overcome by the collaborative teaching practice and by the focus on the history of mathematics as a starting for all investigations. This approach allowed them not only to explore the opportunities available to them when teaching mathematics, but also gave them an opportunity of learning in more depth about the topics of interest to them. Historical context helped all teachers involved to 'scaffold' their own understanding and detailed knowledge of geometry topics and this in turn enabled them to transfer knowledge easier to children and to involve them in the process of discovery. The historical element also offered a focus and method of research.
As geometry was at the centre of this project, the different lessons were related to inter-disciplinary applications of geometry such as acoustics and architecture/engineering. A particularly interesting topic was the development of different systems of spatial representation - using two-dimensional geometry to represent three-dimensional space, which will be examined here in more detail. A set of lesson plans and worksheets, as well as teacher/student observations on the use of dynamic geometry software to explore this topic will be made available.
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***Theme* 1**

Napier's Rods in Today's Classroom

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Often students, of all ages, do not consider mathematics to have a history connected to individuals. This session will present some historical mathematics of Napier and connections, in particular his famous "Rods" or "Bones," with the current use of a lattice for multiplication. Participants will be introduced to a set of Napier Rods to do some simple calculations and then apply this same idea to that of a lattice to see the connections. If time allows, other historical connections for lattice multiplication will also be presented. Participants will take away their own set of Napier rods to be used with their students.
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***Theme* 5**

From the analysis of the articulation of the trigonometric functions to the corpus of eulerian analysis to the interpretation of the conceptual breaks present in its scholar structure (Del análisis de la articulación de las funciones trigonométricas al corpus del análisis euleriano a la interpretación de las rupturas conceptuales presentes en su construcción escolar)

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This article presents the results of an investigation on the construction of knowledge from the *socioepistemological approach.* We are particularly interested in the study of the processes present in the articulation of conceptual mathematics systems to what we have called *processes of mathematical*

convention and articulation (Martínez-Sierra, 2003, 2005). More specifically, the aim here is to present our advances in the quest to identify the present processes of mathematics convention of the articulation of the trigonometric functions (TF) to the corpus of Eulerian analysis. We will also present the interpretations that said analysis has allowed us to make in order to become aware of the conceptual breaks in the scholastic construction of the trigonometric functions.

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***Theme* 1**

Reflexión histórica, epistemológica y didática del concepto de función cuadrática.

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El concepto de función ha sido considerado como un elemento fundamental para la construcción de pensamiento matemático, en gran parte por las múltiples aplicaciones en la modelización de situaciones de variación relativas a contextos cotidianos y a las demás ciencias. En este artículo, se presentan los avances de un proyecto de investigación en el que pretende diseñar una propuesta didáctica mediante la cual se pueda construir el concepto de función cuadrática vía la modelización de fenómenos de variación. Se presentan particularmente avances del proyecto en cuanto al rastreo histórico y algunos elementos que pueden ser tenidos en cuenta en el momento de construir una didáctica del concepto de función cuadrática.

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***Theme* 2**

History of the return of applications in Undergraduate Mathematics in the United States

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Changing university education in the U. S. is like political change in democratic countries: many people need to be persuaded and there is much argument. The advice of prestigious leaders and committees is often ignored as each department makes its own decisions. Money mostly plays the role of helping in the attractive presentation of alternatives. One of the significant changes of the last century or so is the change in emphasis given to teaching applications of mathematics to students specializing in mathematics. We outline how recent increases in this emphasis have come about and we discuss social factors that probably influenced the increase in the teaching of applications in the second half of the twentieth century.

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***Theme* ??**

Propuesta para integrar la tecnología educativa en la instrucción del Cálculo Integral a nivel de Preparatoria: Herramienta tecnológica PRACTYMATHE

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Este trabajo presentará un estudio donde se podrá observar si el manejo de la herramienta tecnológica PRACTYMATHE, que se diseñará específicamente para este proyecto y se usará de manera complementaria a las clases que los alumnos toman en el aula, promueve que se facilite el aprendizaje del Cálculo Integral. Será posible evaluar si los alumnos adquieren el mismo nivel académico que el programa de estudios vigente exige, y se podrá observar si en los alumnos se desarrollan las competencias del aprendizaje colaborativo para el dominio del Cálculo Integral.

Al término de esta investigación el producto será un sistema que a través de una página web permita aplicar a los alumnos exámenes de opción múltiple con reactivos seleccionados al azar a partir de una base definida por los autores, y que registre un historial que pueda ser utilizado para fines de evaluación por los profesores y para fines de investigación por los autores.
El profesor podrá ver la evaluación de todos los exámenes reportados por los alumnos, podrá ver el detalle de un examen específico reportado por un alumno: cuáles fueron las preguntas que se le hicieron, cuáles fueron sus respuestas, cuáles eran las respuestas correctas, esto realizado sobre un tiempo límite para resolver el examen.
El sistema tendrá la versatilidad en las preguntas y respuestas porque será posible capturarse en formato de texto, HTML o imagen (JPG, GIF, PNG).
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***Theme* 2**

Produciendo Libros de Matemáticas para alumnos y profesores en Brasil en la década de 1930: una mirada sobre la experiencia de la Compañía Editora Nacional

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Este trabajo presenta resultados parciales de un proyecto de investigación más amplio intitulado "La Companhia Editora Nacional y la Producción de Libros Didácticos de Matemática". Considerando el impreso como objeto cultural, presenta un estudio histórico de las publicaciones relacionadas a la enseñanza de la matemática que componen las Colecciones *Atualidades Pedagógicas* y *Livros Didáticos* de la *Biblioteca Pedagógica Brasileira* de la Companhia Editora Nacional, producidas en la década de 1930. Además de una presentación contextualizada de los libros y de sus autores, buscamos caracterizar aspectos materiales y textuales de las obras, así como acompañar el movimiento de sus ediciones. Para lograrlo, utilizamos distintas fuentes primarias. Tal como sostiene Chartier (1990), cuando afirma que "no existe texto fuera del soporte que lo da a leer, que no hay comprensión de un escrito, cualquier que sea, que no dependa de las formas a través de las cuales llega al lector" (p.127), los libros han sido nuestras principales fuentes, complementadas por análisis de catálogos y fichas de movimiento de ediciones de la Editora Nacional.
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***Theme* 5**

Using history of Mathematics to develop student understanding of number system structure

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The use of the historical development of mathematical concepts to inform current teaching and learning is a debatable process. In order to investigate the value of this approach in this study we considered the use of a combination of historical development of number systems and modeling with concrete materials as a way of enhancing students' knowledge and understanding of place value. Additionally, we also looked at place value in different number bases and linked multiple representations in an attempt to strengthen understanding of the structure of the number system. The results suggest that this historical and concrete approach helped students to understand positional notation to the extent of generalizing it to other bases, and this may have further implications for the learning of algebra.
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***Theme* 1**

Cómo decidir qué matemática debería saber un futuro docente? Aportes desde la Historia de la Matemática

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Presentamos el análisis de una situación de clase del nivel secundario en la que se están enseñando los números complejos. Esta situación da cuenta de cómo algunos conocimientos de Historia de la Matemática pueden ser de gran utilidad a los docentes en su desempeño profesional y permiten además obtener infornación acerca de la matemática que podría incluirse en un plan de estudios para formar profesores de matemática.

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***Theme* 1**

Tras los pasos de Arquímedes: Estrategias espontáneas de estudiantes de Bachillerato en el cálculo del área de un segmento de parábola

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Presentamos el análisis del trabajo de estudiantes de Bachillerato (16 – 18 años) de Uruguay que, sin haber recibido instrucción específica sobre cálculo integral, se enfrentan al cálculo del área bajo una curva. Detectamos que las intuiciones y argumentaciones de un número considerable de los estudiantes se refieren a la idea de agotar el área bajo la curva inscribiendo más y más polígonos usando argumentos que se acercan al método de exhaución utilizado por Arquímedes (siglo III a.C.) en su *Cuadratura de la parábola*.
Los hallazgos antes detallados se dieron en el marco de una investigación que pretendía detectar las estrategias que utilizan los estudiantes al enfrentarse al cálculo del área bajo una curva. Comparamos el trabajo de nuestros estudiantes con el realizado por Arquímedes y creemos que lo hallado nos brinda elementos para realizar el diseño de secuencias apropiadas para introducir la integral definida teniendo en cuenta los aportes de la Historia de la Matemática y tomando como punto de partida las estrategias espontáneas de los estudiantes, apoyándonos en diseños que tengan en cuenta los procesos cognitivos de quienes aprenden.

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***Theme* 4**

Diversity in approaches to mathematics education in a cultural context

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The purpose of the study was to explore the ways in which different schools met cultural contexts. Cultural context and language are important aspects of children's learning in school. The study involved observations and interviews in four countries. The themes that arose from the analysis were aspects of cultural context, meeting language differences in different ways, maintaining culture in different ways, teaching in a cultural context, teaching mathematics in a cultural context, having an emphasis on national values, using national language appropriately, and developing context-specific strategies for diversity. Each theme is illustrated by descriptions from the different contexts and discussed in terms of their impact on the learning in that cultural context. The differences were often unexpected but significant for our understanding about how school systems and teachers mediate context.

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***Theme* 3**

Un estudio sobre el uso de las gráficas en las obras de Evangelista Torricelli y Daniel Bernoulli

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En la actualidad, las matemáticas son percibidas por los estudiantes como una ciencia que siempre se ha desarrollado de forma lineal, con un orden lógico insoslayable que hace referencia a las lecciones de los libros de texto y que está hecha por genios únicos e irrepetibles encerrados en algún lugar del mundo solo pensando en la solución de un problema específico (en el mejor de los casos los estudiantes solo conocen algunos acontecimientos aislados o particulares en la vida de algún matemático importante sin saber realmente cual fue el proceso que los llevo a ese descubrimiento). Es por tal motivo, que al integrar elementos históricos en la enseñanza de las ciencias y en particular en la matemática permitirá dar una idea de la naturaleza social del conocimiento además que, para la didáctica de las matemáticas, los estudios epistemológicos son una fuente importante que brinda elementos que no se encuentran presentes en el discurso matemático escolar y que pueden incidir positivamente en el contexto educativo. Dado a los elementos anteriormente expuestos, la presente investigación acepta el hecho que la construcción del conocimiento es de una naturaleza eminentemente social y que los estudios epistemológicos brindan elementos que no se encuentran presentes en el discurso matemático escolar. Por lo tanto creemos conveniente usar una perspectiva de investigación en la didáctica de las matemáticas que involucre a la dimensión epistemológica que nos permita ver los elementos que hacen posible la construcción de cierto conocimiento, la dimensión cognitiva para entender los procesos mentales que realizan los individuos, la dimensión didáctica para su implementación en el contexto educativo y la componente social que se encarga de normar todas las dimensiones anteriores a través de las prácticas sociales que tienen los grupos humanos (Cordero, 2006). Para la socioepistemología como visión teórica, la importancia de lo social juega un papel predominante ya que señala que el conocimiento matemático se resignifica a través de las prácticas institucionales que tienen los individuos es decir, la matemática tiene sentido y significado propios en contextos socioculturales específicos. Por tal motivo, esta investigación centra su atención en el uso de las graficas que existen en la disciplina de mecánica de fluidos para encontrar elementos que no se encuentren en el contexto matemático que permitan una resignificación de las gráficas en el discurso matemático escolar. Como elementos de esta investigación definimos dos epistemologías, una relacionada con el discurso de la mecánica de fluidos actual y la otra relacionada con el discurso de la mecánica de fluidos en la época que el cálculo surgió y se institucionalizo como parte de las matemáticas en el siglo XVIII, la obra que tomamos como referencia es el trabajo de Daniel Bernoulli ya que es un parte aguas en el desarrollo teórico de la mecánica de fluidos y como referencia insoslayable de los trabajos sobre hidrodinámica en el siglo XVIII el trabajo de Evangelista Torricelli en el siglo XVII. Esto dará elementos que permitirán una reorganización del discurso matemático escolar y una resignifiación del conocimiento matemático más acorde con las necesidades que tienen los grupos humanos para que de esta forma, la enseñanza logre hacer de la matemática un conocimiento funcional.

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***Theme* 5**

Resignificación de la derivada en la Ingeniería por medio de la concepción Lagrangiana

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En este escrito presentamos un marco de referencia para resignificar la derivada en un dominio diferente a la matemática misma, como la Ingeniería. Tratando de dar cuenta de la relación entre ambos dominios ya que coincidimos con Cantoral y Farfán (2003) cuando mencionan que la matemática, en

especial la del nivel superior, está al servicio de otros dominios científicos y de otras prácticas de referencia en donde adquiere sentido y significación. Para lograr tal fin seleccionamos el tema "Conservación de la masa" de Mecánica de Fluidos en donde está presente la derivada, y para favorecer tal resignificación usamos la idea de Lagrange sobre ésta. Este trabajo está fundamentado en la Aproximación Socioepistemológica que incorpora de forma sistémica cuatro componentes para la construcción social del conocimiento: su naturaleza epistemológica, su dimensión sociocultural, los planos de lo cognitivo y los modos de transmisión vía la enseñanza (Cantoral y Farfán, 2003).
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***Theme* 1**

Quantitative literacy for pre-service elementary teachers within social and historical contexts

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Concern with elementary teachers' subject knowledge in mathematics and science has been extensively documented in the literature over the past two decades in both mathematics and science education. In addition, there is evidence that many students begin teacher education displaying misconceptions in both mathematics and science. There is general agreement that these students' misconceptions are acquired during their school experiences and that negative attitudes contribute to poor classroom teaching which in turn contributes to poor pupil attitudes, beliefs and performance outcomes. If these pupils go on to become teachers, a cycle of negativity may be created and that if change is to occur, it must come through suitable intervention at the tertiary level. It is therefore important to make the most efficient use of the limited time available to improve the general mathematical and scientific competencies of pre-service primary teachers. This paper will outline an attempt at appropriate intervention by the implementation of an integrated unit, Quantitative Literacy, which has been developed by the author. The unit consists of an integration of topics from mathematics and science in which mathematical and scientific thinking, beliefs, and problem solving are examined entirely within social and historical contexts. This paper will present some of the integrated historical topics of the unit.
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***Theme* 2**

A historical overview of analysis exams in Rumania

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In the present paper we give a historical review of the mathematical analysis problems given at admission exams to the Faculty of Mathematics of Bucharest, Romania. The University of Bucharest was founded in 1864 and since 1866 till 1962 the mathematics career was part of the Faculty of Physical-Mathematical sciences. Admission exams were introduced in 1947 and till 1976 consisted of a written and an oral examination; since then admission is based on three written exams: algebra, geometry and mathematical analysis. The Universities had the authority to elaborate the admission subjects, however between 1980 and 1990 these were established centrally, by the former Ministry of Education. We present a qualitative evaluation of the analysis problems given at the admission exam between 1947 and 1995. Our analysis is made along three aspects: coverage of the analysis curricula by exam problems, problem types and problem difficulty. The study reveals five problem types, usually present in an analysis exam. These problems gave a fair coverage of the curriculum. Their difficulty varied over time, but after 1980 problems became more algorithmic. One of the most interesting conclusions is that, in some periods, exams seemed to reflect more political purposes than educational ones.
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***Theme* 5**

Brouwer's intuitionism as a self-interpreted mathematical theory

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We introduce the concept of a self-interpreted mathematical theory, construing Brouwer's intuitionistic analysis as an important example of such a theory. Brouwer's aim was to show evidence of all the mathematical properties of the continuum by unfolding its intuitionistic meaning, without using any axioms. Our criticism of Kleene's formalization of intuitionistic analysis derives from this point of view. We give a reconstruction of Brouwer's analysis from the self-interpreted theory point of view, based only on definitions of the fundamental concepts. We argue though, that Brouwer's proof of fan theorem is not intuitionistically acceptable, therefore a different approach on the proof of fan theorem is needed, in order to secure the self-interpreted character of intuitionistic analysis. Finally, we discuss the benefits of incorporating elements of intuitionistic analysis into teaching mathematical analysis, based on some crucial points of our reconstruction of Brouwer's intuitionism.

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***Theme* 5**

Semiotic Reflections on Medieval and Contemporary Graphic Representations of Motion

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In the first part of this presentation, an ongoing semiotic investigation of some 14th century attempts at representing motion in geometrical terms is presented. In particular, some questions related to the generality of mathematical discourse and some epistemic issues embedded in the problem of representing space, time and velocity in geometric terms are addressed. A comparison between the mathematical signs in the discursive treatment of motion and the corresponding geometric one in the work of Oresme and some of his followers, suggests two different, although interrelated, forms of "narratives" to express and to think about motion in mathematical terms. The structures of signification of these narratives rest on the mutual constitution of a numeric-analytic and a geometric-synthetic set of meanings. In the second part of the presentation, the aforementioned set of meanings is further investigated through a classroom episode in which contemporary students deal with the geometric-analytic representation of motion in a Cartesian plan. Some educational implications are discussed.
En la primera parte de esta presentación se expone una investigación en curso sobre algunos intentos hechos en el siglo XIV para representar el movimiento en términos geométricos. En particular, se discuten algunas preguntas relacionadas con la generalidad del discurso matemático y ciertas presuposiciones epistémicas subyacentes al problema de la representación geométrica del espacio, el tiempo y la velocidad. Una comparación entre los signos matemáticos presentes en el tratamiento discursivo del movimiento y los signos correspondientes al tratamiento geométrico en el trabajo de Oresme y algunos de sus continuadores, sugiere la existencia de dos narrativas diferentes, aunque relacionadas, para expresar y pensar matemáticamente el movimiento. Las estructuras de significación de esas narrativas reposan en una constitución mutua de dos conjuntos de significados, uno numérico-analítico y otro geométrico-sintético. En la segunda parte de la presentación, estos conjuntos de significados son investigados a través de un episodio de sala de clase en el que estudiantes contemporáneos discuten en torno a la representación del movimiento en un plano cartesiano. Al final, se consideran algunas implicaciones educativas.

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***Theme* 3**

Yo Ho Ho-ratio: some mathematics of Trafalgar
(How Lord Nelson inspired curriculum development in mathematics)

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This workshop is based on mathematics masterclasses with 13/14 year old learners for 2.5 hours and a series of 10 one-hour lessons with lower secondary pupils in my mathematics class at school. It covers the history of the battle of Trafalgar (21 October 1805), and how Lord Nelson may have used mathematics in a variety of situations. The session will be done in the period costume of a sailor of 1805, explaining the social conditions and mathematics of the time.
The mathematics includes

1) mental geometry, visualising ships from different positions
2) links between algebra and geometry in the piling of cannonballs
3) using a mathematical text of the time to calculate the number of cannonballs in a pile
4) making a pair of parallel rulers
5) comparing two data sets using statistics
6) working with probability on the crown and anchor game
7) equations of motion to calculate the cannonball's speed and damage done

The workshop will give people a chance to work through some of the materials and see how historical incidents can be used to motivate learners. A CD-ROM with all the worksheets, pictures and notes will be given free to all those attending the session.
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***Theme* 1**

The effects of studying the history of the concept of function on student understanding of the concept

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This study examines the mathematical learning that occurred when students studied the history of the concept of function. Students experienced an in-depth study of the history of functions during a 5-week unit in the junior-senior level History of Mathematics course. They completed a series of worksheets, readings, and problems.
The research methodology was a teaching experiment and the framework for analysis of data was *APOS* (Action, Process, Object, Schema) Theory. All 17 students enrolled in the course completed an extensive initial questionnaire and 6 were selected to participate in an in-depth interview to reveal their understanding of the function concept. During the unit, each student wrote a series of reflections about his or her understanding. After the unit, students completed a second questionnaire and participated in another in-depth interview to discern the changes in their thinking about the concept.
The findings support the notion that studying the history of a mathematical concept enables a deep reflection of ideas. Four of the six participants notably strengthened their function conceptions. Two moved an entire APOS level. Five of the six exhibited an increased ability to recognize a function in a given scenario. Growth was most profound in the area of graphical representations.
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***Theme* 5**

Mathematics apart: examining the history of subject isolation and its implications for mathematics education.

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This article offers a historical examination of the roots and consequences of subject isolation. The article first discusses calls to make mathematics a more integrated part of students' overall learning. It then explores factors which led over time to subject isolation and, in particular, the heavy separation of mathematics from other academic disciplines. Next the article discusses the negative consequences of this isolation. Finally, the article moves to a discussion of how using writing to teach mathematics can help overcome many of the problems caused by mathematics' subject isolation.

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***Theme* 5**

De la epistemología de la fórmula para las soluciones de una ecuación de tercer grado de Bombelli-Cardano a la construcción de una secuencia de actividades para la construcción del significado del número complejo

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En el presente artículo se ofrecen resultados de una investigación sobre construcción del conocimiento desde la *aproximación socioepistemológica.* En particular estamos interesados en el estudio de los procesos presentes en la articulación de los sistemas conceptuales matemáticos a los que hemos llamado *procesos de convención y articulación matemática* (Martínez-Sierra, 2005). De manera específica este trabajo indaga sobre qué alternativas pueden ser factibles para la construcción escolar del significado de los números complejos, bajo la hipótesis de que su significado puede ser construido a través del proceso de convención matemática. El análisis de la producción de los estudiantes, al trabajar una secuencia de actividades diseñada por nosotros en base a la hipótesis anterior, da evidencia de que a pesar que los estudiantes insistían en que "las raíces cuadradas de números negativos no existen**",** nuestra secuencia los indujo a operar con ellos y de esta manera construir un significado en el plano operativo.

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***Theme* 5**

Emanuel Swedenborg's work on differential calculus.

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Emanuel Swedenborg (1688-1772) is well-known as a reformator of Christianity. Very little is written about his mathematical efforts in the 1710s. He is possibly the first Swede to have studied the Leibnizian calculus. In an earlier thesis I have found that Swedenborg's source for his writing was parts of Charles Reyneau's *Analyse Demontrée* (published in 1708). Now I have been lucky to find Swedenborg's own copy of the French book in the Library of the Royal Swedish Academy of Sciences. There is a note in it that says that Swedenborg bought the book in Paris on 1st of Septembre 1713. Furthermore there are lots of mathematical notes in it that gives more information on Swedenborg's thoughts on infinitesimal calculus. A comparison between his own manuscripts, his notes and Reyneau's book will set the light on a young scholar's road in apprenticing Leibnizian calculus. In my talk I will tell you a little of the background to Swedenborg's interest in the up-to-date mathematics and give you some interesting examples of his interpretation of *Analyse Demontrée*.

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***Theme* 1**

Un análisis histórico y epistemológico de la noción de promediación.

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En este trabajo es analizada desde la perspectiva histórica y epistemológica, la noción de promediación, considerada como una idea germinal de donde se desprenden los diferentes conceptos de promedio, como la media aritmética, armónica y geométrica entre otros. Es por ello que es importante realizar su rescate para la didáctica de las matemáticas, para lo cual es necesario considerar las filiaciones y rupturas epistemológicas, lo que permite dar evidencias de diferentes momentos históricos donde la noción evoluciona.

De inicio, el rescate epistemológico se realizó desde la perspectiva conceptual de Arquímedes expresada en su obra "El Método", en donde el estudio del equilibrio, esta sustentado precisamente en la noción de la equilibración física, y cuya matematización se estructura por medio la noción de promediación. Un constructo epistemológico fundamental en donde se expresa la promediación es *"el exceso y el defecto"*, lo que le permite a Arquímedes relacionar lo mecánico con lo matemático (Torrija, 1999). Ello se manifiesta en su estudio sobre el equilibrio de los planos, el cálculo del centro de gravedad y la cuadratura de la parábola, entre otros aspectos (Dijksterhuis, 1987).

Como muestra de lo anterior, se tiene lo que podría llamarse su cuarto "lema", de los diez que giran en torno al concepto de centro de gravedad.*"El centro de gravedad de cualquier recta (segmento de recta), es el punto que divide a la recta en dos partes iguales"*.

Otra muestra de los procedimientos de Arquímedes están dados a través de la relación con las sumas de potencias tratando de establecer cuadraturas, resultados equivalentes a las integrales definidas (Edwards, 1979),

$$1+2+3+\cdots+n=\frac{n}{2}(n+1) \qquad \rightarrow \qquad \int_0^a x\,dx = \frac{a}{2}$$

$$1^2+2^2+3^2+\cdots+n^2=\frac{n}{6}(n+1)(2n+1) \qquad \rightarrow \qquad \int_0^a x^2\,dx = \frac{a^3}{2}$$

Un resultado central de la edad media, está dado por la Escuela de Merton, y es el que se refiere al llamado teorema de la velocidad media, (Edwards, 1979), es decir, la distancia que recorre un cuerpo uniformemente acelerado está dada por,

$$s=\frac{1}{2}(v_0+v_f)t\,,$$

Donde la velocidad uniforme, es el promedio o la media aritmética de la velocidad inicial v_0 y la velocidad final v_f, en el intervalo de tiempo t. Por su parte, Oresme, probó la Regla de Merton, con una verificación geométrica asociada al área de un trapecio. Obsérvese que el argumento central de este teorema está dado por el concepto de promedio.

Otra forma de filiación epistemológica de la premediación es dada por Wallis, quien realizó cálculos con los indivisibles aritméticos, lo que se muestra en la Arithmetica infinitorum, (Edwards, 1979), los que a su vez conceptualiza como una forma de promedio. De manera tal que el área bajo la curva, $y = x^3$, la determinó empíricamente cuando después de calcular el término n-ésimo del cociente de sumas de la misma potencia 3,

$$\frac{0^3+1^3+2^3+\cdots+n^3}{n^3+n^3+n^3+\cdots+n^3}=\frac{1}{4}+\frac{1}{4n},$$

Obtiene que para valores de n suficientemente grandes, o sea, $n \rightarrow \infty$,

$$\int_0^1 x^3 dx = \frac{1}{4}$$

Finalmente, es posible dar cuenta que la noción de promediación evoluciona de tal forma que aparece en la didáctica actual en diferentes contextos, en el Cálculo a través de los teoremas de los medios, para derivadas e integrales, y posteriormente en la Probabilidad y Estadística, en donde el promedio o media, está dado por el valor esperado de una variable aleatoria X, discreta y continua respectivamente,

$$E(X) = \sum_x x \; p(x)$$

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

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***Theme* 4**

The infinite series in India in centuries V to the XV

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In most of history books the main reference to the appearance of concepts like sequence and series is the work by western mathematicians.

Many recent historical studies have been made in all around the world and evidences of the appearance of sequences and series concepts have been found in India, Egypt, China, etc.

This oral presentation will show some awesome advances in the series expansion achieved by the ancient Indian mathematicians. We will talk about *Madhava* who calculated some series like

$$r\,q = \frac{r \cdot r\sin q}{1 \cdot r\cos q} - \frac{r \cdot (r\sin q)^3}{3 \cdot (r\cos q)^3} + \frac{r \cdot (r\sin q)^5}{5 \cdot (r\cos q)^5} - \frac{r \cdot (r\sin q)^7}{7 \cdot (r\cos q)^7} + \cdots$$

$$\arctan q = q - \frac{q^3}{3} + \frac{q^5}{5} - \frac{q^7}{7} + \cdots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

About 300 years before Newton and Leibniz.

This historical approach to series could lead us to design new school activities in a way that students get motivated in the study of mathematics.

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***Theme* 5**

From the history of the angle to its epistemological nature. Contributions to a scholar design (de la historia del ángulo a su naturaleza epistemológica. Aportaciones para un diseño escolar)

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The scholar notion of angle has played an ambiguous role in school. The traditional assumption in schools is that when the concept is defined, characterized, manipulated and its typology exhibited in the mathematics classroom, its use, application and interpretation in other subjects should pose no problem for the students. To the contrary of that assumption, it is in other subjects where the most common conflicts in the handling of this notion are found.
The nature of the concept of angle has been the topic of debate for over 2,000 years and the discussion is not over yet (Matos, 1990). Matos made an historical account of the concept of angle with the aim of understanding how angle was conceived, the properties that were attributed to it, the problems which were resolved and even those that were not, using the concept. This historical review provides important factors to consider when we think of the conflicts the student presents in the classroom when dealing with the concept.
Historically, we find that the angle was used, applied and defined as a quality, a quantity and/or a relation. Considering the definitions and historic uses along with the most common obstacles and conceptions in the student, we have designed a sequence of activities that seek to favor the epistemological nature of the concept.
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***Theme* 1**

The debate on a "geometric algebra" and methodological implications

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A crucial methodological question confronting various approaches regarding the use of history of mathematics for teaching is whether and how original texts can be presented as teaching texts. Given that texts, which are older than, say the nineteenth century, use to be not directly readable and understandable – for several essential reasons (conceptualization, notation, language, epistemology, etc.). For use in teaching, one will try, hence, to "modernize" somewhat the original.
Inevitably, modernization will result in some "distortion" and the question is which degree and kind of distortion can claim to be legitimate or tolerable for the aim of teaching. Particularly sensitive in this regard is the relation between geometry and algebra, viz. the transformability of earlier, largely geometrical texts into - for moderns - readable, algebraized texts. A seminal case study for the legitimacy of distortion will be presented by the debate on the existence of a "geometric algebra" in Greek mathematics, provoked in 1975 by Sabetai Unguru and having famous mathematicians (van der Waerden, Freudenhal, Weil) as reactors. The methodological questions for historiography of mathematics, as implied in this debate, will be shown in their relevance for use of historical texts in teaching.

***Theme* 1**

Jacobi's Last Theorem: The history of Jacobi-Perron algorithm

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J. L. Lagrange proved that the regular continued fraction expansion of a quadratic irrational number becomes periodic. In a paper published after his death C. G. Jacobi proposed a generalization of the regular continued fraction algorithm. Most probably he hoped that for a pair of cubic numbers this algorithm would become periodic. Paul Bachmann stated that according to results of Hermite and Charve on ternary quadratic forms this conjecture should be true. O. Perron developed a profound theory of Jacobi's algorithm and its generalization to higher dimensions. It was here that Perron stated a theorem on positive matrices which later on became a good tool under the names of Perron and Frobenius. L. Bernstein collected a large number of examples for special families of irrational numbers

for which he could prove periodicity. Among others this story shows again that number theory is a rich source of problems which are easy to state but hard to solve.
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***Theme* 5**

Estudio Socioepistemológico de la tangente

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El presente trabajo de investigación nos habla acerca de la dificultad que tienen los estudiantes de Cálculo en poder establecer un vinculo entre la noción de recta tangente vista en cursos anteriores al de Cálculo y una noción de recta tangente dinámica como se requiere en Cálculo, por tal motivo se lleva a cabo un análisis documental tomando como marco de referencia a la socioepistemología, la cual es una aproximación teórica que toma en cuenta cuatro componentes que son: la didáctica, la cognitiva, la epistemológica y la social. En nuestro trabajo se emplearon dos componentes que son la epistemológica y la social. Se tomaron en cuenta por lo tanto los escenarios socioculturales propios de los diferentes escenarios científicos de cada época. Con motivo de llevar a cabo el análisis se divido el trabajo en tres momentos de acuerdo a el uso que se le daba a la recta tangente los cuales son: tangente geométrica, tangente variacional y tangente implícita. Con el presente trabajo se pueden recatar elementos que se han perdido en la historia y que pueden contribuir a resignicar el Discurso Matemático escolar.
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***Theme* 2**

Informatics – a subject developing out of Mathematics – a review from 1970 to 2007

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The use of computers and calculators in mathematical education since the late seventies has initiated a small revolution in Mathematics. Suddenly some techniques in calculation like using a slide rule or working with spreadsheets for logarithms were dispensable. Nevertheless data-processing was an important part of mathematical education at this time. Therefore some motivated teachers in Austria worked within an experiment to establish computer application in mathematics education. But the importance of computer applications was growing and so an optional course was started in some schools. Most of the teachers of these courses were mathematicians. That's the reason why a lot of mathematical topics were discussed in this earlier informatical education.
By and by programming languages and mathematical application programs were developed. Some of them, especially the graphical representation with the help of structograms and the languages BASIC or PASCAL, were implemented in mathematical education. The level of education rose through this use and the increasing importance of computer science in education was activating the awareness. So after a while, in the year 1985, a curriculum was set up for Informatics. Since then it is a school subject on its own in Austria, where special topics are taught. But the tight binding to mathematics was never lost. So it was obvious that students had to work with mathematical aspects like greatest common divisor, number systems or logical topics where you can find a close context to Informatics. Since 1989/90 another structural change occurred. Informatics became universal and all pupils at the age of 15 had the chance to take this subject for another 3 years as an optional subject. In education the programming of applications to different mathematical aspects was common for the first time. But when Computer Algebra Systems were introduced in education, they were also part of instruction because it was easy to create little imperative or functional programs.
In the last few years programming became less important. Application software like spreadsheets or picture editing software is taught more often. Hence this is not seen as a balanced informatical

education K.J. Fuchs and H.-St. Siller are trying to enforce the implementation of programming paradigms with the help of hand-held calculators in Mathematics and Informatics. In my presentation I want to show the development of Informatics out of Mathematics with a strong focus on Austria over the last 37 years.

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***Theme* 1**

Teachers' conceptions of history of mathematics

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In 1997, history of mathematics was included in the curriculum goals for elementary and lower secondary schools (ages 6-16) in Norway. However, studies suggest that history of mathematics did not get the attention that the curriculum mandated.
To learn more about how this 1997 change in the curriculum may have been approached by mathematics teachers, I did an interview study of four Norwegian secondary and high school teachers. This is a phenomenological study where the goal is to gain more knowledge about teachers' conceptions, to supplement the findings of my earlier studies.
The study shows that the teachers differ in what they consider to be history of mathematics and in what the goal of incorporating it in teaching may be. They include history of mathematics in very different ways and to different degrees, and have different opinions on how that works out.
The knowledge gained in this study may contribute to the discussion of how to successfully integrate history of mathematics in the average mathematics classroom – that is: how to engage "ordinary" teachers in this endeavor.

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***Theme* 2**

Defining math teachers' knowledge: discussion about examinations for elementary and secondary teachers in Brazil in the nineteenth century

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Educational reforms require an evaluation of all the elements which have to do with the school environment, such as students, textbooks and teachers, among others. It is common sense that a good teacher is essential to the improvement of any subject. But how can one measure the teacher's "quality"? In Brazil, one of the conditions for the recruitment of teachers for primary and secondary public schools is the public contest. The first actions in this sense date from 1759 and substituted the religious' classes for isolated ones given by improvised teachers, with no diploma requirement. Throughout the century teaching reforms adopted different selection processes, training and admission of future teachers. As years went by the examinations became more comprehensive, thus demanding more specific knowledge besides written and oral exams from the teachers. This text aims at discussing the teaching of Mathematics in Brazil during the 19th century through contests for teachers taken place in Rio de Janeiro (Brazil) and raise further questions about the selection of teachers and the contents which are required in order to occupy such positions, thus providing other researchers with materials for comparisons with models originated in other countries.

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***Theme* 5?**

Geometría y Análisis en la Historia Temprana de las Integrales Elípticas

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Este trabajo enfrenta el estudio del concepto de integral elíptica a partir de los detalles de su surgimiento y devenir en la historia de las matemáticas durante los siglos XVII y XVIII. En él se presenta una forma de enseñar este concepto, que parece haberse olvidado en los nuevos textos de Cálculo Integral. Se trata pues de un trabajo interdisciplinario en el cual se intervienen las teorías de la Educación, la Historia y las Matemáticas mismas.
Desde la Educación, el trabajo se sustenta en La teoría de la Transposición Didáctica. En lo histórico se apoya en un método hermenéutico basado, en la lectura, análisis e interpretación de algunos escritos originales de finales del siglo XVII y del siglo XVIII, así como en algunas reinterpretaciones posteriores. Para lo matemático, se usa la formalización contemporánea del Análisis en términos del Algebra Lineal y la Topología. Con el fin de dar unidad a la presentación, se usa la simbología moderna estándar y se da una organización axiomática y demostrativa.
El concepto de Integral elíptica surge de manera natural al intentar calcular la longitud de arco de algunas curvas elementales tales como las secciones cónicas, la espiral parabólica o la lemniscata, entre otras. En los albores del Cálculo Infinitesimal se utilizaron procedimientos geométricos y analíticos algebraicos para estudiar las propiedades más evidentes de dichas integrales, dada la imposibilidad de un cómputo exacto de su valor numérico.
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***Theme* 2**

The Math Wars: A Cultural View

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In the 1990s, the mathematics curriculum in the US became the object of intense conflicts known as "the Math Wars." We will examine this conflict, some of its historical roots and continuing consequences, and we will consider a cultural interpretation of these battles.
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***Theme* 4**

Seki-Takakazu's memorial year

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Seki Takakazu is the most notable person in the history of mathematics in Japan. He was admired as Holy Master Seki Takakazu by his successors. He was born about 1640, and was dead 1708. This year is the tercentennial memorial year after his death.
We may admire him as a genius. He was born nearly at the same time with Newton. But compared with Newton, the mathematical environment was not favorable for him. The mathematics studied by his contemporaries were, so-to-say, the studies of the counting techniques of lengths, areas, volumes, etc.,and very few of the real mathematics. The literature he could refer to were only two or three books brought from China. So he had to engage, he alone, with the exploitation of mathematics by his own hands.
He made many ingenious works.

- The research about the sum of powers of natural numbers. He showed his result, based on the table of binomial coefficients, for the powers from 2 to 11. This work is just the same with the work of Jack Bernoull, affording the way to calculate Bernoulli numbers in the same way..
- The research of the solution of the indeterminate equation of natural numbers. He deeply studied the Chinese remainder theorem, and made wide generalisations to different moduli.

- The systematic research of the regular polygons from the triangle up to the polygon with 20 sides. His work is very systematic. It is not apparent from what is left, but, examining his arguments, it seems to us, he prepared several simple lemmas and established his discussion common to all these different cases.
- The rigorous and accelerated determination of the circle number π. He began with calculating the perimeter of regular polygons starting from the square inscribed in a circle and making double of the number of sides up to 131,072. Afterwards, by inventing an acceleration method, he could determine precise length of the perimeter of the circle. He also tried to give a formula to calculate the length of the arc of a circle, but he was not able to get desired result. The work is left to his disciple Takebe Katahiro.
- Other than these, the most important and most influential work was the development of the ameliorated method of the celestial element method, the writing aside method. This is, so-to-say, formation of algebraic equations with variable coefficients. According to this ameliorated method, he and the mathematicians after him could establish many fruitful results. He also discussed to eliminate variables from several equations, and established the formation of determinants.

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***Theme* 1**

La integración de la historia y las nuevas tecnologías en el proceso de formación de matemáticos

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Concebir la integración de las tecnologías informáticas y la historia en el proceso de enseñanza aprendizaje de la matemática universitaria, es aún un reto que debemos reconocer, por lo que se requerirá de los docentes en este nivel educativo adaptar la utilización de las mismas a las exigencias y peculiaridades de los procesos educativos que en este ámbito se desarrollan, desde una perspectiva innovadora.

Presentamos en el trabajo algunas de las experiencias desarrolladas en el proceso de enseñanza aprendizaje en el primer año de la carrera Licenciatura en Matemática, integrando el uso de Internet y la Historia de las Matemáticas de manera interdisciplinar a través del Análisis Matemático y la asignatura Seminario de Problemas, con el objetivo de desarrollar la motivación por la especialidad en estos estudiantes.

Compartimos la idea de que la integración real de las tecnologías de la información y en específico Internet en el papel motivador hacia el aprendizaje matemático necesita de un replanteamiento de la práctica desde la óptica de un modelo sistémico, el cual debe potenciar el uso de la reflexión en el proceso enseñanza aprendizaje, la estimulación a la autonomía del aprendiz donde cada cual describa su propio camino hacia el saber, así como los enfoques centrados en el aprendizaje, lo que guiará al estudiante a seguir su proceso de aprender a lo largo de la vida, sin necesidad de utilizar nuevos modelos.

Por lo que en la experiencia que se describe en el trabajo se acomete el proceso docente desarrollando un modelo didáctico donde el aprendizaje del estudiante es el centro del problema a resolver, radicalmente distintos a cualquier modelo tradicional, cambiando las formas organizativas del tiempo y el espacio de las clases, así como las modalidades y estrategias de evaluación. Reconociendo además que la historia de las matemáticas puede utilizarse en el aula también como elemento motivador para los alumnos, pero el profesor sale beneficiado también, ya que el conocimiento de los hechos históricos de la matemática lo sensibiliza sobre las posibles dificultades que pueden surgir en los alumnos para la comprensión de algunos objetos matemáticos.

Se concibió dentro de la asignatura Seminario de Problemas, que estaban recibiendo los estudiantes al unísono con el Análisis Matemático, una tarea investigativa de manera individual sobre el desarrollo de las matemáticas y sus principales artífices, profundizando en los temas estudiados en este curso en sus asignaturas básicas.

En criterio emitido por los estudiantes estas actividades los han motivado para el aprendizaje de los temas de las asignaturas y por la carrera. Les ha cambiado la percepción de la matemática, mostrando el lado humano y social de la misma.
Concluimos que debemos contribuir con los resultados de nuestras investigaciones pedagógicas a fundamentar porque las nuevas tecnologías por si solas no contribuyen a un desarrollo ascendente de nuestras misiones educativas, cuestión que nos precisa reflexionar en un uso correcto de los recursos que disponemos. Y la experiencia que se muestra es de integrarlas con la historia en el proceso de formación de matemáticos.

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***Theme* 6**

Las Matemáticas en la Biblioteca Palafoxiana: un acercamiento a sus libros y a sus temas.

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La Biblioteca Palafoxiana de Puebla, fundada en 1646 con la donación de 5 mil volúmenes de la biblioteca particular del Obispo Juan de Palafox y Mendoza a los Colegios de San Pedro y San Juan, es la única biblioteca antigua conservada en América que mantiene su edificio, mobiliario, estantería y su acervo formado por 42 mil 556 volúmenes y 5 mil 345 manuscritos, siendo uno de los acervos bibliográficos antiguos más importantes del mundo. Desde sus orígenes esta biblioteca tuvo una clara intención educativa, al permitirle su consulta a cualquier lector, considerándose por esto la primera biblioteca pública en América. Las matemáticas están representadas en su acervo por autores como *Euclides, Euler*, *Copérnico*, *Newton, Descartes,* y *Clavio*. Desde los tratados para el uso de instrumentos científicos hasta la gnomónica, la perspectiva o las artes bélicas; los temas de los libros y manuscritos de matemáticas de esta biblioteca permiten conocer algunas características de los lectores poblanos novohispanos que hacían uso de estos libros y del conocimiento que tenían sobre las matemáticas. El presente trabajo pretende ser un primer acercamiento sobre estos temas y sobre la importancia que, desde la primera donación de libros, tuvieron las matemáticas como materia de estudio en la *Biblioteca Palafoxiana de la Ciudad de los Ángeles de la Nueva España*.
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***Theme* 5**

Un estudio socioepistemológico del discurso Matemático escolar. El caso de la probabilidad elemental .

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En este artículo se presenta una experiencia sobre un estudio de la probabilidad elemental inmersa en la práctica de juegos de azar, que considera el discurso matemático escolar (dme) como una práctica social generadora de saberes. La experiencia contiene una socioepistemología de la probabilidad centrada en el dme, que mira las condiciones de construcción, difusión y uso social del conocimiento matemático. Contiene además, una descripción del juego de canicas que se expone en las ferias, como una práctica en la que saberes sobre probabilidad tienen sentido y, resultados de una encuesta a los dueños del juego de canicas, otra para alumnos y una entrevista a profesores. La entrevista a profesores y la encuesta a alumnos revela un discurso matemático escolar de corte formal.
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***Theme* 3**

El pensamiento Matemático De Leonardo Da Vinci

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Un número significativo de anotaciones de Leonardo da Vinci, consignadas en sus cuadernos personales en diversos momentos de su vida, permiten formarnos la idea de que el gran artista florentino se consideraba a sí mismo como un matemático. "No lea mis principios quien no sea matemático" (Royal Library, Windsor, 19118b), o "Ninguna humana investigación puede ser denominada ciencia si antes no pasa por demostraciones matemáticas" (Codex Urbinas, Biblioteca Vaticana, 1b) son sólo algunas de ellas. No obstante, no ha faltado la crítica que, apoyada en estudios comparativos y provista de fino discernimiento, como es el caso del destacado historiador de la Física, C. Truesdell, afirme que Da Vinci, a diferencia de los babilonios que lo precedieron en más de dos milenios y medio, "jamás pudo resolver correctamente una ecuación cuadrática", observación que el mismo autor de los *Ensayos de la Historia de la Mecánica* (la edición original, de la Springer-Verlag, es de 1968; las citas aquí presentadas son de la versión de 1975 para la Editorial Tecnos) acompaña en forma estridente con los nombres de Tartaglia, Cardano, Ferrari, A. Fior y Scipione dal Ferro, es decir, los de esa "gran escuela de algebristas" que se formó en "la época y el país de Leonardo".
En los tiempos actuales, en los que la abundancia de investigadores e investigaciones, así como la importancia concedida a la publicación, obligan a reportes mesurados del ingente número de frentes de atención, nos resulta cada vez más difícil remontar el camino que lleva a un investigador a dar con un resultado. El interesado en conocer la forma como la mente de un investigador realiza sus tanteos previos a un descubrimiento o a una creación, casi nunca cuenta con los recursos para observarlo. En ese sentido, los cuadernos de Leonardo da Vinci, quien nunca llevó a la imprenta un trabajo suyo, pero quien, a la vez, no dejó nunca de consignar en forma manuscrita sus paulatinas observaciones sobre tantos campos que atraían su atención, se constituyen en un material precioso para observar cómo procede una mente genial *en borrador*.
En este trabajo nos proponemos revisar el material textual de Da Vinci, tanto el relativo a sus reflexiones acerca de la Geometría como el atinente a la puesta en práctica de su capacidad de observación para encontrar *leyes de la Naturaleza*, a fin de apreciar en pleno movimiento un atributo que ha acompañado desde su origen la evolución de las Matemáticas: el pensamiento analógico.
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***Theme* 5**

Elementos para la validación de una generalización matemática. Un analisis epistemológico a la evolución histórica del método inductivo

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La generalización es un proceso al que algunos investigadores en Educación Matemática, han dedicado varios trabajos. Mason (1999:16) afirma que la generalidad es la vida de las matemáticas y que el álgebra es lenguaje con el que se expresa dicha generalidad; Arzaquiel, (1993:, p. 8) hace una reflexión sobre dicho proceso y los errores que se pueden presentar dentro del mismo; además sugiere cierto tipo de actividades para el trabajo en el aula. Por otro lado, Radford (1996, p.108) llama la atención sobre los procesos de validez en el proceso de la generalización.
Una de las características fundamentales en el proceso de generalización es el establecimiento de principios y reglas generales que se pueden inferir de unos cuantos casos particulares a un conjunto de casos no incluidos en la muestra inicial. Es esta característica la que marca básicamente el carácter inductivo de una generalización matemática. Por esta razón y ante la necesidad de buscar elementos que permitan argumentar y validar dichas reglas, se hace necesario profundizar en las características del llamado método inductivo.
En este trabajo se realiza un análisis epistemológico de la evolución histórica del método inductivo, se establecen periodos en su desarrollo que inician en los trabajos de Aristóteles, se consolida con Bacon y se refuta y complementa con los trabajos de Popper y Russell. En cada uno de los periodos se realiza una extrae la visión que se tienen de la validación y se proponen estrategias didácticas que pueden ser de utilidad a la hora de validar los procesos de generalización que se realizan en el aula de clase.

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***Theme* 1**

Teaching Big Ideas in mathematics through history

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In almost every university that educates future secondary school teachers, history of mathematics appears a part of the curriculum. In this paper I will describe possible ways to integrate the history of mathematics in other upper division courses, especially in a capstone course and in courses related to mathematics teaching methods. One of the mottos in teaching mathematics is Abe Shenitzer's maxim:

> "One can *invent* mathematics without learning much of its history.
> One can *use* mathematics without knowing much - if any - of its history.
> But one cannot have a *mature appreciation* of mathematics
> without a substantial knowledge of its history**."**

Episode 1: The notion of proof in mathematics

Students are shown a way to calculate the area of a quadrilateral as it appears in the Rhind Papyrus (Eves, 1982, p. 14). This activity allows the students to expose their own conceptions about the nature and the significance of a mathematical proof. The discussion of what constitutes evidence in mathematics and how truth is established is also fostered by this activity. A pilot study of such an experience is described in Winicki-Landman (2002).

Episode 2: Mathematical definitions: What? When? Who?

Students are asked to read some definitions of solids as they appear in Euclid's Elements. Specifically, they are asked to read the definition of *cone, pyramid, cylinder and prism*. They are asked then to look for other definitions of these solids and to compare these definitions. This compare-contrast activity fosters a discussion of the some key issues related to mathematical definitions: the existence of equivalent and non-equivalent definitions of the same concept, the notion that a definition establishes sufficient and necessary conditions, the fact that mathematical definitions are dramatically different from dictionary definitions, the symbiotic relationship between proving and defining and some pedagogical issues related to the defining process. A brief description of possible interactions is provided in Winicki-Landman (2004). To emphasize the evolutionary aspect of definitions in mathematics, the students are asked to read Kleiner (1989).

Episode 3: Aesthetics in mathematics

Students are exposed to three different versions of a special case of Fermat's Little Theorem: For every natural number n, n^5-n is a multiple of 10.

They are asked to analyze them, compare them and choose their favorite one. Via this activity, the students are taught how to analyze proofs using criteria like generality, complexity, naturalness, clarity,

security and elegance (Barbeau, 1988). They are also asked to read Kleiner (1991) who describes rigor in mathematics not as a dogmatic idea but as an evolving one.

Episode 4: Open-ended questions in mathematics

Students are exposed to Leibniz's triangle and using it, the idea of open-ended questions is modeled. These types of questions involve significant mathematics, lead to may different approaches, use of different representations, may have different solutions, foster mathematical communication among the students and they may lead to the formulation of new significant questions. Such an experience is described in Winicki-Landman (2005).

Final Remarks

There are many creative ways to introduce the history of mathematics into its teaching. The special kind of knowledge that teachers need can definitely be developed and deepened when historical perspectives are added to the class. This kind of knowledge is different from just content knowledge: it is *pedagogical* content knowledge and mathematics teachers need specific and concrete experiences that enable them to look into the mathematics content deeper and with "different eyes".

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***Theme* 1**

Title mising

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We would like to pay attention to pedagogical effect of using well-known books of the beginning of the twentieth century for teaching contemporary mathematics courses on undergraduate level: Loria, "Spezielle algebraische u. transscendente ebene Kurven", 1902; Teixeira "Traite des courbes speciales remarquables planes et gauches", 1908-1909; Wieleitner, "Theorie der ebenen algebraischen Kurven hoherer Ordnung", 1905; "Spezielle ebene Kurven", 1908. Shortly characterising the importance of historical context in teaching mathematics courses, we would like to note that from a constructivist perspective it is easier for a student, under appropriate arrangement of teaching, to act as an architect, to reveal the truth and construct new knowledge, comparing it with the findings, which had been done by the famous mathematicians long before.

We will focus on some topics from Advanced Calculus, first of all, theory of high order curves on plane. For conducting students' activities teachers have to possess the knowledge about these books. It can be achieved through integration in teaching process such old invaluable mathematical books. We will show examples how to use brilliant material of these books in a classroom in the scope of corresponding historical-mathematical environment. We will trace the links between different curves from the historical point of view. We will consider different properties of the curves through their

historical significance and didactical value. It is interesting to note that studying these books teachers become “learners” together with the real learners. In particular, we analyse their influence on teachers’ understanding the role of history and didactical implementations to the learning process. We hope that using such material in teaching process will give opportunity for teachers as well as students to grasp the idea of evolutional development of high order curves throughout the centuries. We will present a flexible structure of units for students’ collaborative and individual work in a classroom using one-component and multi-component tasks, which are powerful didactical tools for teachers in their practice work. As one-component task we call the one, which aimed at students’ inquiry work for finding properties of a certain high order curve, i.e. students have a clear formulated direction for their activities in a classroom. As multi-component task we call the one, which aimed at students’ inquiry work for finding properties of different high order curves connected with each other, i.e. students investigate curves without clear indication the direction for their activities in a classroom: what properties and for what curves are to be suggested. We are going to consider Newton parabola punctata, Descartes curve, Steiner hypocycloid, Doppel-Herz curve and many others. Some of them are famous and well-known, others are forgotten or almost unknown.

We would like to emphasise that historical dimension in teaching mathematics has invaluable importance. In our opinion, it is a kind of innovative approach – to learn and teach new mathematical content through constructing new knowledge from old books, which serve as bridges between nowadays and past.