SOCIAL PRACTICE OF THE VARIABILITY NOTION
AN EPISTEMOLOGICAL APPROACH

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ABSTRACT

In the navigation practice and the geography of the middle XVII century, the variable notion was recognized like a “mistake” that made the needle of the compass when deflect it about the magnetic north, in regard to true north [Juleu, 1723]. The deviation of this angle called “magnetic declination”. During that time the geometers gave to the variable for giving a mean to the magnetic declination, taking it like a “mistake”, was fundamental for the transition which followed the “variability” concept in the definition the a “theory of mistakes”, for one hand, and of the derivative concept, for the teaching, for the another hand. This paper establishes a brief historical development of the definition of variability, considering its genesis like a social practice such as suggest the socio-epistemology.

1. Socio-epistemology and social practices

The socio-epistemology is a theoretical approach of systemic nature that allows dealing with the phenomena production and diffusion of the knowledge from a multiple perspective. With this the study dimensions: epistemological, cognitive and didactic have been revalued incorporating one "sociocultural" component, and taking in account the conditions epistemological that prevail in the social environment [Cantoral & Farfán, 2004]. From the sociocultural dimension, we think possible to do a study of the notion of "variability" recognized in the domain of social practices: procedural (1) and observation, these social practices happened throughout the centuries XVIII and XIX. Since it is assumed that the development of ideas is not produced only in internal form, but it is a social process, which is demonstrated through social practices, procedures and skills. We think that the notion of "variability" to determine elements that give a meaning to the concept of function that is taught in classroom.

2. The social practice

2.1 The combined method

In 1760, Euler was describing small irregularities observed from the movement of the planets and that were caused by mutual attraction; opposite to the astronomers´ idea about conceiving these irregularities as the cause of the attraction of the sun towards the proper planets only [Euler, 1760, p. 164]. Likewise, he was worried for giving a complete explanation of the lunar and planetary disturbances exemplifying with the lunar eclipses. The lunar eclipses which were predicted placing to the planets up to a difference hour in regard to the proper phenomenon. In consequence, he wished to be precise in the position of the planets centering his attention initially on the Moon. These small forces with which the planets act one with each other were called them "disturbances". Such irregularities did not allow determine accurately the position of the planets in presence of events such as the eclipses. Before this description, Euler approached the problem of the "lunar disturbances" to Mayer [1723-1762], astronomer and German geometer from the Gotinga's University (Göttingen).

In 1750, Mayer defined a method with which was possible to combine diverse "equations of condition" that he was getting from the observation or experimentation of the procedural practices and observation, and then he could solve the problem of the disturbances that Euler informed him.

The method in study, begun from the principle of doing a large number of repetitions of experiments so that establishing a "equation of condition" for each experiment in every stage was feasible. He presented the equations of condition of the following way:

Condition equations

$$\left.\begin{array}{c} a_1x+b_1y+c_1z+...+m_1=v_1 \\ a_2x+b_2y+c_2z+...+m_2=v_2 \\ a_3x+b_3y+c_3z+...+m_3=v_3 \\ \dots\dots\dots\dots \\ \dots\dots\dots\dots \\ a_nx+b_ny+c_nz+...+m_n=v_n \end{array}\right\}$$

Instrumental mistakes

$$\left.\begin{array}{c} a_1x+b_1y+c_1z+...+m_1-v_1=e_1 \\ a_2x+b_2y+c_2z+...+m_2-v_2=e_2 \\ a_3x+b_3y+c_3z+...+m_3-v_3=e_3 \\ \dots\dots\dots\dots \\ \dots\dots\dots\dots \\ a_nx+b_ny+c_nz+...+m_n-v_n=e_n \end{array}\right\}$$

The right system shows the instrumental mistakes, that are the difference between the observations of every experimentation exposed from the variables x, y, z, … m in regard to the dependent variable v, that means, if the mistakes of this type did not exist, they must be conceived null, which in the practice is false.

As it is possible to observe, the number n of observations is larger than the number of unknown quantities. The system of values that result from the combination of all the equations was considered to be the more "acceptable", that means, the more independent system of the effect of the small "instrumental mistakes". Mayer's approach to solve the system consisted of changing the signs of equations in such a way that all the terms that contain x become positive, and to add later all equations. The same adding operation had to be repeated respect of each one of other unknown quantities getting as result so many equations as unknown quantities existed. For better understanding of the method of Mayer's combination, we consider necessary to approach the next example:

We suppose that straight line is defined by BC in the figure 1, and we want "experimentally" to determine the values of the constants a and b from the equation: $y=ax+b$.

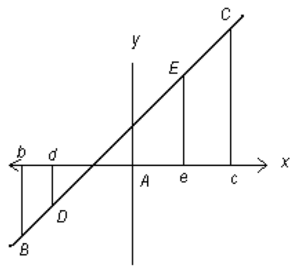


Figure 1

If the axis of the abscissas is divided in any number of parts Ad, db..., Ae, ec, and we measure these abscissas and its ordinates correspondents with a graduated arbitrary rule, we will have the following equations of condition:

$$\begin{array}{ll} x=+3 & y=-3.75 \\ x=+2 & y=-2.35 \\ x=+1 & y=-0.90 \\ x=0 & y=+0.55 \\ x=-1 & y=+1.95 \\ x=-2 & y=+3.45 \end{array} \qquad \text{Plugging in} \quad y=ax+b: \qquad \begin{array}{c} 3a+b=-3.75 \\ 2a+b=-2.35 \\ a+b=-0.90 \\ +b=+0.55 \\ -a+b=+1.95 \\ -2a+b=+3.45 \end{array}$$

Since b is positive in every equation, it is not necessary to change any sign. The addition is done and $3a+6b=-1.05$ is gotten.

Next, we change the signs of some equations, in such a way that every a is positive, and adding these equations we obtain: $9a+2b=-11.85$. From this, we get a system of two equations with two unknown quantities, that when it is solved, it gets: a=-1.43750, b= +0.54375.

When these values are plugged in the equations of condition, small differences raise because of the mistake of measurements called residues which are showed:

-0.0187	+0.0063	+0.0312
+0.0188	-0.0062	-0.0313

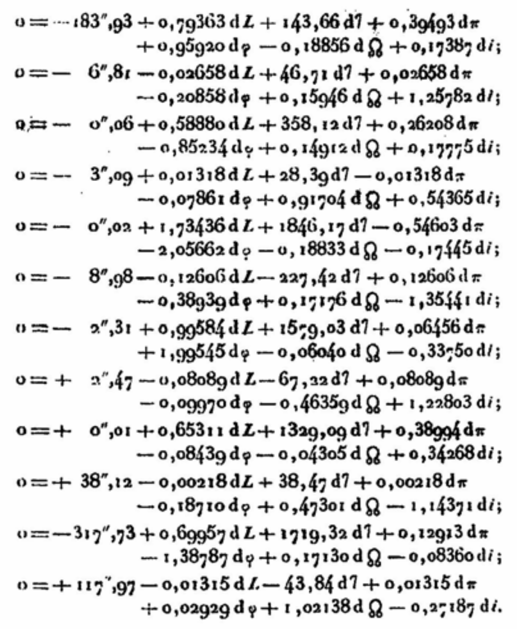
Comparing the equation $y=ax+b$ with the equations of condition, we observe that

$$ax+b=y$$

$$ax+b-y=e$$

$$a_1x+m_1-v_1=e_1$$

The residues, called $e_1, e_2, ... e_n$, are analytical mistakes caused by irregularities or instrumental mistakes in practice. Mayer conceived the mistakes $e_1, e_2, ... e_n$ as a "variation" that happens for every experimentation; so that e_1: it is the first variation, e_2: the second variation, etc. So that the set of variations or residues: $e_1, e_2, ... e_n$ were recognized as the "variability" of the experimental observations. Therefore, the notion of variability takes an analytical meaning.



$$
\begin{aligned}
0 &= -183'',93 + 0,79363\,dL + 143,66\,d\iota + 0,39493\,d\pi \\
&\quad + 0,95920\,d\varphi - 0,18856\,d\Omega + 0,17387\,di; \\
0 &= -\ \ 6'',81 - 0,02658\,dL + 46,71\,d\iota + 0,02658\,d\pi \\
&\quad - 0,20858\,d\varphi + 0,15946\,d\Omega + 1,25782\,di; \\
0 &= -\ \ 0'',06 + 0,58880\,dL + 358,12\,d\iota + 0,26208\,d\pi \\
&\quad - 0,85234\,d\varphi + 0,14912\,d\Omega + 0,17775\,di; \\
0 &= -\ \ 3'',09 + 0,01318\,dL + 28,39\,d\iota - 0,01318\,d\pi \\
&\quad - 0,07861\,d\varphi + 0,91704\,d\Omega + 0,54365\,di; \\
0 &= -\ \ 0'',02 + 1,73436\,dL + 1846,17\,d\iota - 0,54603\,d\pi \\
&\quad - 2,05662\,d\varphi - 0,18833\,d\Omega - 0,17445\,di; \\
0 &= -\ \ 8'',98 - 0,12606\,dL - 227,42\,d\iota + 0,12606\,d\pi \\
&\quad - 0,38939\,d\varphi + 0,17176\,d\Omega - 1,35441\,di; \\
0 &= -\ \ 2'',31 + 0,99584\,dL + 1579,03\,d\iota + 0,06456\,d\pi \\
&\quad + 1,99545\,d\varphi - 0,06040\,d\Omega - 0,33750\,di; \\
0 &= +\ \ 2'',47 - 0,08089\,dL - 67,22\,d\iota + 0,08089\,d\pi \\
&\quad - 0,09970\,d\varphi - 0,46359\,d\Omega + 1,22803\,di; \\
0 &= +\ \ 0'',01 + 0,65311\,dL + 1329,09\,d\iota + 0,38994\,d\pi \\
&\quad - 0,08439\,d\varphi - 0,04305\,d\Omega + 0,34268\,di; \\
0 &= +\ 38'',12 - 0,00218\,dL + 38,47\,d\iota + 0,00218\,d\pi \\
&\quad - 0,18710\,d\varphi + 0,47301\,d\Omega - 1,14371\,di; \\
0 &= -317'',73 + 0,69957\,dL + 1719,32\,d\iota + 0,12913\,d\pi \\
&\quad - 1,38787\,d\varphi + 0,17130\,d\Omega - 0,08360\,di; \\
0 &= +117'',97 - 0,01315\,dL - 43,84\,d\iota + 0,01315\,d\pi \\
&\quad + 0,02929\,d\varphi + 1,02138\,d\Omega - 0,27187\,di.
\end{aligned}
$$

Gauss used the method of the square minimums to the correction of six elements in the orbit of the planet Pallas. See note (2).

Figure 2

In 1795, Gauss would formulate the method of the "square minimums" doing modifications to the model of Mayer's combination especially in the systems of condition equations. The obtained mistakes for every observation of variable and independent from the result circumstances (as the produced ones for external and irregular reasons such as the effect of the air producing a less clear vision, or those due to the instruments of measurement), were called "irregulars" or "fortuitous", and able to be calculated.

He defined in contrast "constant" or "regular" mistakes that in observations of the same nature produce an identical mistake depending on circumstances and due to observations, which remain excluded from the research realized by Gauss and of the application of his method.

He showed his "Theoria Motus Corporum" to the Royal Society of Gotinga; he proved the application of the method of the minimums squared for the correction of six elements (mistakes) in the orbit of the planet Pallas utilizing twelve equations (see figure 2).

The variations or corrections were attributed differential coefficients: $dL, d\iota, d\pi, d\varphi, d\Omega, di,$. Because of the difficulty to satisfy every equation, Gauss looked a way to reduce them as much as possible according to the same principle expressed by Mayer (Gauss, 1855).

If the linear functions are considered:

$$
\begin{aligned}
&n + ap + bq + cr + ds + ... = w, \\
&n' + a'p + b'q + c'r + d's + ... = w' \\
&n'' + a''p + b''q + c''r + d''s + ... = w'', \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned}
$$

According to the fundamental principle of the square minimums, every unknown quantity must be determined so that the sum of the squares of the "mistakes" could be minimal $a = w' + w'^2 + w''^3 + ...$

Recapturing the notation of the model of Mayer's combinations, and with an adjustment to the method of the Gauss's square minimums, we can obtain:

$$\left.\begin{array}{c} a_1x+b_1y+c_1z+...+m_1-v_1=e_1 \\ a_2x+b_2y+c_2z+...+m_2-v_2=e_2 \\ a_3x+b_3y+c_3z+...+m_3-v_3=e_3 \\ .... \\ .... \\ a_nx+b_ny+c_nz+...+m_n-v_n=e_n \end{array}\right\}$$

In the previous systems of equations, the sum of the squares of the mistakes is given for: $e_1^2+e_2^2+...+e_n^2=[ee]$. To determine every unknown quantity, the auxiliary $[ee]$ must be null in regard to each one of these. For example, in regard to *x*:

$$e_1\frac{de_1}{dx}+e_2\frac{de_2}{dx}+...e_n\frac{de_n}{dx}=0\ \ldots(1)$$

Then:

$$\frac{de_1}{dx}=a_1 \qquad \frac{de_2}{dx}=a_2 \qquad ...\frac{de_n}{dx}=a_n$$

Substituting in (1):

$$a_1e_1+a_2e_2+...a_ne_n=0$$

called normal equation of *x*.

If we use the same procedure for every unknown quantity, we will have:

$b_1e_1+b_2e_2+...b_ne_n=0$

$c_1e_1+c_2e_2+...c_ne_n=0$

Applying the following rule for each of the unknown quantities in the equations of condition, we will obtain the normal equations that are the same number of unknown quantities:

"(…) multiply every equation of condition by the coefficient that contains one of the unknown quantities taken with its own sign, and then equal (come to an agreement) to zero the algebraic sum of the products".

$$m_1-v_1=q_1 \qquad m_2-v_2=q_2\ldots$$

$$\left.\begin{array}{l} [aa]x+[ab]y+[ac]z+...+[aq]=0 \\ [ab]x+[bb]y+[bc]z+...+[bq]=0 \\ [ac]x+[bc]y+[cc]z+...+[cq]=0 \end{array}\right\}$$

Normals equations

This shows the characterization of the method of Mayer's combination that Gauss did.

Legendre published in 1805 in French the "Nouvelles methodes pour la determination des orbites des comètes", and some years later he applied his method of the Minimums Squared in the resolution of the attraction of ellipsoids homogeneous solving several cases. From this, he obtained a system of equations of the form: $E=a+bx+cy+\ fz+etc.$, where *a, b, c, f*, etc., are the unknown coefficients that vary of an equation to other one, and *x*, *y*, z, etc., are determined for each equation for the condition of the value *E* (mistakes), which it is a null or too small quantity.

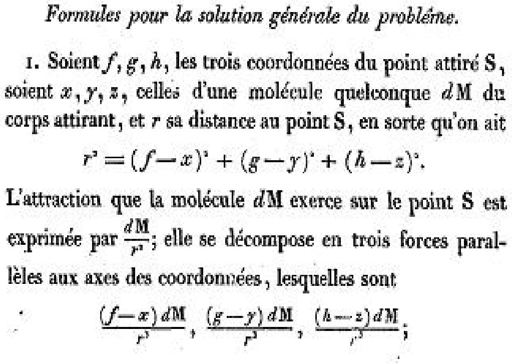
He used the sum of the square of the mistakes $E^2 + E'^2 + E''^2 + etc.$, to solve the system of equations (Legendre, 1805, pp. 79-88):

$$
\begin{aligned}
&(a + bx + cy + fz + etc.)^2 \\
&+ (a' + b'x + c'y + f'z + etc.)^2 \\
&+ (a'' + b''x + c''y + f''z + etc.)^2 \\
&+ etc.
\end{aligned}
$$

For a certain quantity *x*, the sum of the squares of the mistakes will be:
$(a' - x)^2 + (a'' - x)^2 + (a''' - x)^2 + etc.$

If we equal to zero and then clear, we obtain:
$x = \dfrac{a + a' + a'' + etc.}{n}$.

Where *n* it is the number of observations, being *x*, *y*, *z*, the common variables to the point. The sum of the square of the distances is equal to a minimum $(a' - x)^2 + (b' - y)^2 + (c' - z)^2$ (see figure 3).



Formules pour la solution générale du problème.

1. Soient f, g, h, les trois coordonnées du point attiré S, soient x, y, z, celles d'une molécule quelconque dM du corps attirant, et r sa distance au point S, en sorte qu'on ait

$$r^2 = (f-x)^2 + (g-y)^2 + (h-z)^2.$$

L'attraction que la molécule dM exerce sur le point S est exprimée par $\frac{dM}{r^2}$; elle se décompose en trois forces parallèles aux axes des coordonnées, lesquelles sont

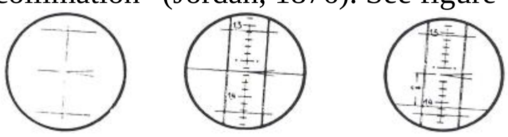
$$\frac{(f-x)\,dM}{r^3},\quad \frac{(g-y)\,dM}{r^3},\quad \frac{(h-z)\,dM}{r^3};$$

Figure 3

2.2 The variability in the procedural practices

In the procedural practices, the variability represented the totality of the instrumental mistakes that were done in the experimental observations. Nevertheless, this type of mistakes, had a treatment close to the own instruments looking for being compensated, which Mayer did not approach.

Throughout the century XIX, the procedural practices were characterized by the use of instruments of observation such as: telescopes, transit-man, levels or equi-altimeters, barometers, altimeters, etc., which suffering of precision in the capture of information due to the lack of a technology as currently. Therefore, in the levels or equi-altimeters (levels mounted on a tripod, with which it is possible to determine the vertical irregularities of the land), and in the focusing, the position of the axis of collimation (the cross recorded in the lens) was considered to be invariable. Nevertheless, since the cross, seen plane, was recorded over a curved lens, the irregularities in the focus lens produced a set of deviations in the rectilinear form of the axis of collimation. Such irregularities were known like “variability of the axis of collimation” (Jordan, 1876). See figure 4.



Disposition of the threads in the collimation axis	Aproximated reading	Final reading
	1.36	1.3648

Figure 4

For the same epoch, the aneroid barometers or mechanical barometers of pocket had the disadvantage of the readings variation when they were being compared with ones of a guide barometer of mercury. This readings variation was giving wrong lectures of the corresponding height on the level of the sea. The inevitable shakings of the field works and trips, transport, etc. were producing variations of the pressure indicated by the aneroid, which could be determined comparing them with a barometer of mercury.

In the table I we give an example of the variability in the pressure alterations that resulted from the diverse successive comparisons of aneroid, that W. Jordan used in 1873 during an expedition to Libya, with the barometer of mercury.

The aneroid Cassella No. 1641, that presented the minimal variations, was a small instrument of the size of a pocket clock, and during the trip a Jordan’s colleague took it in his pocket of the trousers.

Place and Date	**Naudet 39305**	**Goldschmid 600**	**Casella 1640**	**Casella 1641**	**Chevallier**
Cairo 5 Dec 1873	+2.8 mm	+14.0 mm	+5.0 mm.	+1.3 mm	…
Siut 12 Dec 1873	+2.5	+13.3	…	…	…
Marac 20 Dec	+2.8	+13.1	…	…	…
Fárfara 1 Jan 1874	+4.5	+13.8	+5.0	+0.7	+8.5
Dachel 10 Jan 1874	+5.0	+13.2	+7.2	+1.6	+9.6
Dachel 16 March 1874	+1.4	+11.0	+9.2	+0.9	+10.7
Charge 25 March 1874	-0.3	+9.4	+9.6	+1.0	+10.2
ssen 1 April 1874	+0.8	+8.1	+9.4	+0.4	…
Cairo 16 de Abril 1874	+0.6	…	+8.7	…	…
Minimum variation	-5.3 mm	-5.9 mm	+4.6 mm	-1.2 mm	+1.7 mm

Table I

Jordan´s colleague concluded that this way of transporting the small aneroid protects them from the irregular variations. The big and heavy aneroid, that was necessary to transport in the camels, suffered big and inevitable shakings due to the movement of these.

In 1887, Reinhertz studied the phenomenon of the variability of the aneroid. He called the variability as "remaining elasticity", and he was referring to an elastic body as a leaf of wharf holds for an end; it is doubled by other one; it does not return to its primitive position when the force that produced the deformation stops instantaneously, but it reaches its original position after several small movements. This property of the elastic bodies, Reinhertz called it the "remaining elasticity".

For example, two similar instruments with a change of pressure of 100 mm and diverse speeds produced the following remaining deviations: (See table III)

Speed of change	0.2 mm	0.5 mm	1.0 mm	2.0 mm
Remaining deviation	0.38	0.60	0.74	1.01

Table III

From here, it can be deduced that when verifying the scales it must be produced the changes of pressure to the same speed. That means, very slowly with a ratio of 1 mm. in four or five minutes approximately. The test of the aneroid scale whose graduation is 100 mm. will need, therefore, an entire day.

On the other hand, a large number of experimental works were carried out looking for precision of the barometric levelling. The theoretical law of the mistakes of a leveling of this type had been deduced from the fundamental expression that Laplace defined as: $h = K(\log B - \log b)(1-\alpha t)$:, where *h* it is the height of a position over the sea level, *B* and *b* are two consecutive pressures, *t* is the temperature, and α and *K* are constants. Differentiating each of the involved variables, all three partial corresponding "mistakes" are obtained, as:

$$dh_B = \frac{\mu}{K} dB(1+\alpha t),\ dh_b = \frac{\mu K}{B} db(1+\alpha t) \text{ and } dh_t = h\alpha dt .$$

2.3 The variability in the classroom

Francisco Díaz Covarrubias, Mexican engineer, quotes the method of the Mayer's combination in diverse paragraphs of his book "Infinitesimal Calculus ", written in 1873. He did indistinct use of the method of Mayer in his texts of "Topography", "Astronomy", etc., which speaks about the large knowledge of the method that Diaz Covarrubias had, and from which, surely, he took the definition of variability that added in his calculus book.

Camacho [2007] mentioned the utility of the notion of variability in the form in which Díaz Covarrubias incorporated it to the elementary definitions of "curve" and "curvature", which would support the concept of derivative. Camacho does the description of the definitions that Díaz Covarrubias approached of the following way:

“Any curve can be originated by the movement of a point (…) “

The point that describes the curve, he gave the name of "generator" [Díaz Covarrubias, 1873, 21].

It allowed inferring that the different direction changes of the generating point are diverse between the curves; all of them had the "variability" as common property. In this reflection, Díaz Covarrubias established the curvature of the curves in a completely geometric model taking it like: “(…) the representation of the variability of the directions", or the different "changes" of the generating point on the curve. Emphasizing that these changes are produced having imagined the curvature as a process that is produced on happening from a rectilinear state to curvilinear state, or from a constant to a variable. To this respect, the variability assumes two possibilities: the first one is that can be conceived in a completely rectilinear state of constancy, and the second one, more complex, is the continuity of the curve. In other words, the condition of constancy "catches", forming a part of, the variability or movement of the phenomena of study.

The approach of the variability of Díaz Covarrubias, is similar to that one W. Jordan emphasizes in the problem that he attended for the lens of the equi-altimeter from the objects in play, that means, the line and its transition towards the configuration of the curve. In fact in both cases the variability is contemplated under the same principle: "cross is conceived flat, recorded on a curved lens". While for Díaz Covarrubias the variability is defined by different changes that straight lines suffer to form the curve.

The linear mistake that is produced for the distortion of the focus, it is a variation in the context of Jordan; while in Díaz Covarrubias, the mistake or variation is found in the approximation of the polygonal of straight lines that tends to the curve, consequently, the occurrence of the variability. Nevertheless and similar to Díaz Covarrubias, the mistakes or variations assumed a meaning of variation for Gauss and Legendre in the form of the derivative $\frac{de_1}{dx} = a_1$, since one saw previously.

3. Summary

In summary, the models of combination for the compensation of the variability condensed in the mistakes determined by the above mentioned methods, it was outlined initially as a social practice arisen in the attempt for solving practical problems of the astronomy of position, and led to Gauss to re-formulate later the method of the minimums squared establishing in this way a" theory of the mistakes ", which is current at the present. Gauss gave to the method of the square minimums a bias towards the theory of the probabilities in which he would define the function of normal distribution known like "Gauss's bell”,:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{1}{2}\right)(x-\mu/\sigma)^2} \quad -\infty < x < \infty ,$$

This normal distribution is used for the study of the test of statistical hypothesis which are useful in the methods of quantitative investigation. Since it is possible to estimate, the variability was placed initially in the experimental methods with high degree of empiricism, and, gradually, was giving place to concepts that, nowadays, are a reference in the mathematics and its education.

The variability notion complements a cycle that is a central axis of the meaning associated to the function concept. It assumes the next:

Variable$\rightarrow$ Variation$\rightarrow$ Variability$\rightarrow$ Function.

This cycle is fundamental in the teaching of this concept.

Notes

1. For procedural practices we understand those civil activities and methods of the engineering such as those developed through of the topography, astronomy, etc.

2. Figure taken from the French edition of the *Méthode des moindres carrés*: (Method of the square minimums) of 1855.

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