INFORMATICS – A SUBJECT DEVELOPING OUT OF MATHEMATICS

A REVIEW FROM 1970 TO 2007

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ABSTRACT

The use of computers and calculators in mathematical education since the late seventies has initiated a small revolution in Mathematics. Suddenly some techniques in calculation like using a slide rule or working with spreadsheets for logarithms were dispensable. Nevertheless data-processing was an important part of mathematical education at this time. Therefore some motivated teachers in Austria worked within an experiment to establish computer application in mathematics education. But the importance of computer applications was growing and so an optional course was started in some schools. Most of the teachers of these courses were mathematicians. That's the reason why a lot of mathematical topics were discussed in this earlier informatical education.

By and by programming languages and mathematical application programs were developed. Some of them, especially the graphical representation with the help of structograms and the languages BASIC or PASCAL, were implemented in mathematical education. The level of education rose through this use and the increasing importance of computer science in education was activating the awareness. So after a while, in the year 1985, a curriculum was set up for Informatics. Since then it is a school subject on its own in Austria, where special topics are taught. But the tight binding to mathematics was never lost. So it was obvious that students had to work with mathematical aspects like greatest common divisor, number systems or logical topics where you can find a close context to Informatics. Since 1989/90 another structural change occurred. Informatics became universal and all pupils at the age of 15 had the chance to take this subject for another 3 years as an optional subject. In education the programming of applications to different mathematical aspects was common for the first time. But when Computer Algebra Systems were introduced in education, they were also part of instruction because it was easy to create little imperative or functional programs.

In the last few years programming became less important. Application software like spreadsheets or picture editing software is taught more often. Hence this is not seen as a balanced informatical education K.J. Fuchs and H.-St. Siller are trying to enforce the implementation of programming paradigms with the help of hand-held calculators in Mathematics and Informatics. In my presentation I want to show the development of Informatics out of Mathematics with a strong focus on Austria over the last 37 years.

1 Prologue

Through the implementation of small electronic calculators in mathematics education slide rules and logarithm boards became obsolete. The development started in the year 1972 when Hewlett-Packard designed the electronic calculator HP-35. The producer called it an 'electronic slide rule'; this name was the only thing which was common to the former calculating methods. With the help of this calculator it was possible to calculate logarithmic and trigonometric functions by pushing one button. A revolution, also for mathematics education, was started, as it can be seen in the following apt quotation [Weiss 2005]: 'It is a pleasure for me, to work without paper, pencil and spreadsheets ...'[1]

Five years after the appearance of HP-35 the German society for Didactics of Mathematics stated in a written comment [GDM 1978]: 'Electronically calculators –

[1] All quotations are originally written in German. They are translated by me.

especially simple, not programmable calculators – have achieved as auxiliary material for calculating in everyday life. To an increasing degree they are used by pupils and students which affect tuition. This extraneous evolution requires an important reaction from experts in mathematics education in order to avoid negative effects in learning mathematics on the one hand and to gain chances for advancing positive effects to education on the other hand.' To reach these aims the society proposed four recommendations [GDM 1978]:

- 'Using calculators instead of slide rules and spreadsheets beginning at the 7th school year,
- Spreading calculators in education has to be elaborated and discussed also to the effects to methods of mathematics education,
- Aspects of the calculator should be allowed in teacher basic and advanced training,
- Consequences of calculators for mathematical learning processes should be analyzed in all ranks.'

In the same year in Austria the curricula for mathematics education made the amendment that electronic calculators are allowed to be used in classes up from the 7th school year either in grammar school or in secondary school, if pupils attend the first performance group. Additionally it states that if a calculator is introduced in class it will be necessary that enough practice is done with it. Since then the calculator displaced the old calculating facilities and started its triumphal procession which has lasted until today.

In the same year that the first electronic calculators came on the market the Austrian government started experimental schools for data-processing in mathematics education. For some unknown reasons this experiment didn't stand the test and was stopped. Instead pupils were able to choose an elective called 'EDV', which means the same as data-processing. They were able to choose it from the 9th to 12th year in school. Since at this time there was no adequate study for teachers implemented, courses for teachers interested in this subject were created. Most of the teachers who were interested in this type of thinking were mathematics teachers. Reichel describes it like this [Reichel 1995]: 'Teachers for mathematics have been the natural solicitors for problems and (at first) the 'natural experts'.' So it is not remarkable that a great deal of mathematics teacher were allowed to teach data-processing.

Because of the increasing importance of computers in everyday life the Austrian government paid attention to this development through a big education and training-program called 'Computer – Education – Society' and established a binding exercise for the 9th school year, with an own curriculum, in the year 1985/86 in grammar schools. In the following years, 10th to 12th school year, pupils were able to choose it as an elective again. The curriculum for data-processing accentuates three points:

(1) Problem-oriented education,
(2) Project-oriented education,
(3) Application-oriented education.

In the same year projects for additional data-processing education in other subjects were implemented in some grammar schools. Especially in Mathematics a lot of exercises were done with computers. Because of the fact, that lot of mathematics teachers were teachers for Informatics and because the importance of points (1) and (3) this is not very astonishing. But at the same time those teachers also taught mathematical contents in Informatics for the same reasons. The students or pupils coded the mathematical problems in a certain programming language and tried to get a correct output which they had also produced manually. First measures for modeling, a pivotal idea in both subjects were becoming visible.

Through this usage in teaching specific contents, the computer was applied as:

- Data medium, for demonstration, construction or exemplification of specific mathematical phenomena,
- Instrument, for adaptation of some techniques and skills, aiding the appreciation of mathematical processes or terms and definitions, abatement of the calculating effort in some specific examples,
- Tutor, as a teaching aid for special learning processes.

2 Mathematics and Informatics – Similarities or Extremes

2.1 Informatical contents in Mathematics

As we have seen above the roots of Informatics were strongly connected to Mathematics. On the one hand through the persons who introduced calculators in education on the other hand through the contents and concepts which were taught. The breakthrough for teaching with the help of electronical auxiliary in the German-speaking countries, Germany and Austria, can be found in the year 1977. In this year a lot of material for teaching with calculators or computers was published [Ahrens 1978], [Hayen 1978], [Wynands 1978], [Stein 1978], [Sieber (1) 1978], [Schönwald 1978], [Hainer 1978], [Sieber (2) 1978], [Sieber (3) 1978] and [Jäger 1978]. From then on the expansion of calculators and computers in mathematical education started. This fact can be proven in the university calendar of the University of Salzburg. The Department of Mathematics considered the new development with an own lecture [Parisot 1979] in teacher-training.

Especially in the first years of introducing the electronic calculating auxiliary exciting discussions for introducing programmable or non programmable calculators took place, based on the natural history of data-processing. Inventiveness and individual work characterized the first years of data-processing. After a base was established, structured programming was found to be more profitable. So two paradigms were designed:

- TOP-DOWN-Programming, which means a stepwise refinement of the problem and the way of solving a problem till the solution is found,
- BOTTOM-UP-Programming, where a specific difficulty of a problem was taken up and the problem was solved from this point on.

Because of barely available equipment in schools it was necessary for the problem which should be solved with the help of a programmable calculator to be described in an adequate way. Therefore two ways were common:

- Flow charts [DIN 66001],
- Structured charts [DIN 66261], also called Nassi-Shneiderman-diagrams [Nassi/Shneiderman 1973].

The treatment of such problems always took place in mathematics. This was the subject where these abstract problems could be discussed in a way that was in common with the curriculum. Furthermore mathematical contents were ideal for processing by computers because certain questions, like descending the runtime of problems or improvement of iteration methods could be discussed in a deep going way. Wolgast

[Wolgast 1977] edits typical examples, like nested intervals, method of Heron or calculating extreme values, which could be handled in mathematical educations.

When mathematical problems were taught with the help of computers it was common that tuition followed a certain procedure. Hertsch [Hertsch 1978] displays it in his article 1978:

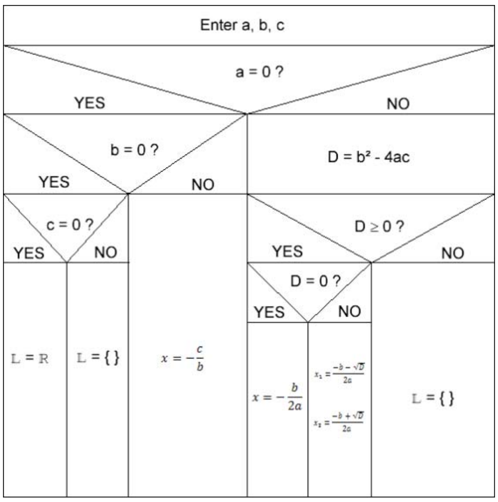
1. Phenomenological treatment of the syntax of a certain programming languages
2. Introduction to algorithms
3. Graphical representation of algorithms
4. Algorithms for solving the problem in graphical representation
5. Assignment into a programming language

Without a Computer/ Calculator

With a Computer/ Calculator

This schematic representation which displays a modern way of introducing computers in education is still used since today for teaching contents in Informatics.

Most of the schools in Austria which offered computers for teaching used the language BASIC. So it is natural that conceptual formulations in mathematics school books tell to write a code in BASIC for a certain problem, like solving the quadratic equation $ax^2 + bx + c = 0$ [Szirucsek 1985], [Reichel 1992]. Up to 1996 you can find such examples in school books for mathematics, although Informatics was established as an own individual subject in the year 1985. If computers weren't available in schools it was possible to solve such examples through building Nassi-Shneiderman-diagrams. For solving the equation $ax^2 + bx + c = 0$ it could look like this:



The advantage of this form of representation is that it is independent of the programming language used for the solution. Until today Nassi-Shneiderman diagrams are an important representation in Informatics. Especially if imperative programming languages are used.

Another important point for using computers is that the calculating effort can be minimized. Sieber [Sieber (1) 1978] has published a paper, 1978, about advancing the run-time for the fixed points of a given function. It shows impressively what can be done with the help of informatical contents in Mathematics. These changes in mathematics education can be seen up to today.

Between 1978 and 1989 the development of introducing calculators or computers has stagnated. After 1990 a lot things in education as well as in teaching Informatics and Mathematics changed and informatical subject matters in Mathematics received a lot of renewed attention because of new developments, like introducing CAS in education and especially in Austria because of a new curriculum for Mathematics. In the commentaries to the curriculum [Müller 1991] the topic 'Informatics and Mathematics education' has its own chapter, in which is stated:

'Pupils should [...] become familiar with the application of capable mathematical texts and means for work, in particular computers. Computers [...] have to be introduced according to the didactical basic principles of the curriculum as working appliances. [...] Addressing problems of algorithmic aspects: algorithmic conditioning of problems [...] Algorithms have to be prepared as far as they are executable on programmable processors (programmable calculators, computers). As the case may be numerical examination can be taught (e.g. effects of calculating with machine-numbers), [...] execution of simulation, above all with the help of computers, [...] calculating approximated values of integrals or anti-derivatives (with upper and lower sum), also using computers.'

Analyzing the new curriculum [BMUKK Math 1989] to informatical subject matters in mathematics education shows lots of text passages with clear informatical subject matters which are listed in the following table:

Informatical contents	**Effects to education**
Computation rate and computational accuracy	Clarification of the necessity of theoretical considerations to existence and calculability
Recursions /Iterations	Through the insertion of trial and error-learning, detection of interrelationships and the finding of formulas can be simplified
Retention capacity	Creating a bigger data-base for inductive conclusions (e.g. divisibility-rules), models drawn from life (e.g. statistical problems)
Working with modules	Accessing a known solution for problems and working with it in new problems leads to a module-bibliotheca which allows the pushing back of microscopic thinking to macroscopic thinking
Software	Application-oriented examples can be discussed in class.
Programming	Thinking ahead instead of ad-hoc decisions. Implementing data-structures in education
Interactive Working with attendant failure diagnostic	Possibilities for an independent, automatic and controlled (but not necessary monitored) working atmosphere for pupils. Introducing computers as 'number crunchers'
Graphics	Showing important mathematical coherences through a clear and descriptive graphical instruction

Through this analysis it is possible to find three fundamental points for upgrading mathematic lessons:

- Computer is a connector between (naive) thinking and (formal) acting. With its help it is possible to reach a stepwise specification of ideas and methods in the sense of the spiral principle.

- It allows an increasing movement towards creative, macroscopic and anticipatory thinking.
- It allows more problem-oriented and application-oriented education.

These are the arguments for working with computers or programmable calculators in mathematics education today and the contents in the curriculum have also changed a lot. Unfortunately teachers often do not pay attention to new developments and some contents, which would be a great enrichment for education through the informatical sight, like functional thinking [Fuchs 2007], [Fuchs, Landerer, Micheuz 2007] are hardly realized.

2.2 Mathematical contents in Informatics

Because of the fact that most teachers of Informatics were Mathematics teachers it was obvious that a lot of problems which were discussed in the Mathematics education were again discussed in Informatics.

Topics such as quadratic equations, greatest common divisor and lowest common multiple or random numbers were discussed in Informatics, too. But no one wanted to calculate such problems by hand. The aim was to find algorithms for solving such examples with an electronic auxiliary. Students who were able to attend such education got a deep and clear understanding for such mathematical problems. Instead of calculating examples they developed a method for solving such problems as a whole. Especially for mathematical topics this was a big challenge. On the one hand they had to understand the mathematical backgrounds; on the other hand they had to know the computer-syntax or the syntax of a programming language.

One of the biggest problems teachers were confronted with, was the fact that most students were bored having to discuss contents twice. Because of an increasing influence of parents on teachers and because of the ascending importance of user software, mathematical contents were displaced and instead attendance to programs for graphical manipulation of pictures and software for making films or creating web-sites were enforced.

But after a short time of working with such programs it became evident that it was necessary to discuss mathematical problems in class again. For the explanation and understanding of certain informatical problems mathematical knowledge was essential [Fuchs (1) 2005]. So a lot of the contents which were not necessarily discussed in mathematics education moved to Informatics. In the Austrian curriculum for Informatics of 1989 you will find in the part concerning methods for problem solving [BMUKK Inf 1989]:

‘Life circle of problem solving:

- Phase of definition
- Phase of conceptual design
- Phase of implementation
- Phase of documentation
- Improving the model

Primarily this life circle shows the process of modeling as Blum [Blum 1985] had drawn it the first time in 1985 and improved it in 2005 with Leiß [Blum,Leiß 2005]. So a very basic and fundamental idea, the basic idea of modeling [Siller 2006], establishes the link between the two subjects today. Mathematical contents from former times, like the solution of quadratic equations or divisibility-rules are discussed in the sense of the life circle. Also mathematical topics such as

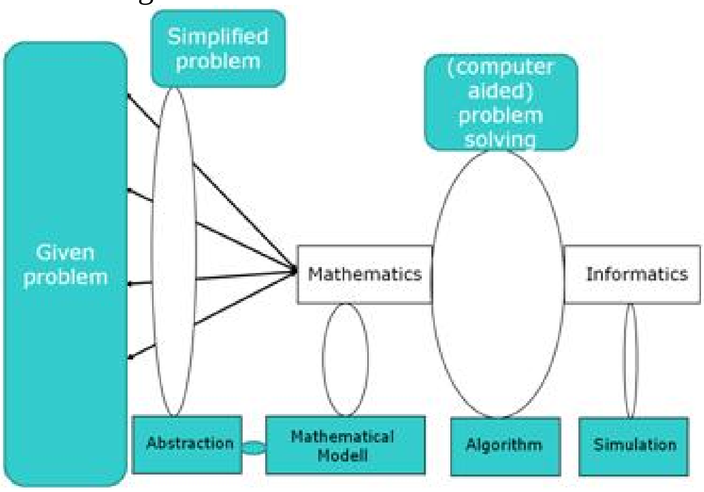
- Probability and random numbers,
- Arithmetics and strategies for calculating equations through special techniques (Regula Falsi, Newton's Method),
- Matrices,
- Cryptology,
- Curve-fitting, interpolation through splines or least square method,

are discussed with students in Informatics, as Sedgewick shows [Sedgewick 1994]. Through the new curriculum for Informatics [BMUKK Inf 2004], valid since 2004, teachers have the necessary legal regulation:

'Teachers should discuss ...

- Data-bases
- Concepts of programming languages
- Artificial intelligence
- Basic algorithms and data structures.'

Through this new curriculum both subjects Mathematics and Informatics are closely connected, as is shown in the figure:



The connection of Mathematics and Informatics through the central idea of modeling is told by Fuchs [Fuchs (2) 2005]: 'Even though Modeling is also a main concept in other natural sciences, a characteristic informatical approach can be justified. For instance the mathematician is interested in the relationship between the parameters influencing the model and in forming equations describing relations. The computer scientist is interested in the implementation.'

2.3 Mathematics/Informatics – two subjects, one idea?

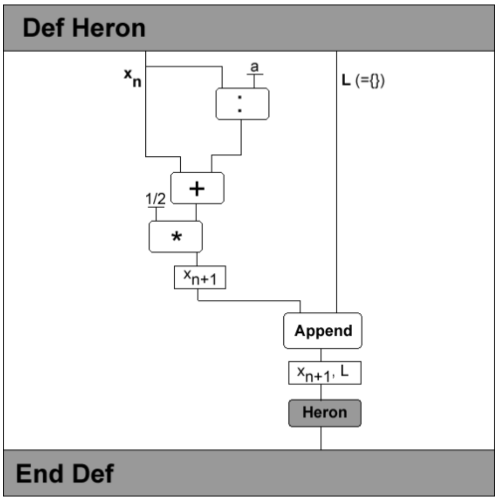
Mathematics and Informatics are closely connected through several contents. One of the most central ideas is the pivotal idea of modeling. Through a closer contemplation of the process of modeling it becomes obvious that the modeling of real situations means modeling through the pivotal idea of the function [Vollrath 1989]. Therefore it is obvious that the dependencies between values are described through functions.

So functional thinking [Fuchs 2007] gets a new significance in both subjects. The character of assignment and the character of variation are discussed in Mathematics and Informatics under this perception. Functional thinking makes it possible to cover the whole environment of Mathematics and Informatics with the help of functions and functional connections. The life circle is the central initial point for the realistic problems discussed in education. All important levels, the haptic, the iconic and the symbolic level will have to be addressed, in order to succeed teaching collected topics interdisciplinary. The parts of the life circle shown above and the process of modeling are the directional indication for successful teaching. So the idea of functional modeling is pivotal to both subjects.

A prototypical example for the different situations a teacher has to cope with when using functional modeling is the finding of roots by Heron's iteration.

In the first step of problem solving it is necessary to know the syntax of PROGRAPH-diagrams [Matwin, Pietrzykowski 1985], in the second step the syntax of the computer-algebra-system DERIVE for implementation.

Step 1:



Step 2:

$$f(x) := \frac{1}{2}\cdot\left(x + \frac{20}{x}\right)$$

ITERATES(f(x), x, 1, 15)

[1, 10.5, 6.202380952, 4.713474545, 4.478314445, 4.472140217, 4.472135955, 4.472135954, 4.472135954, 4.472135954, 4.472135954, 4.472135954, 4.472135954, 4.472135954, 4.472135954, 4.472135954]

$\sqrt{20}$

4.472135954

$$f1(x) := \frac{1}{2}\cdot\left(x + \frac{20}{x^2}\right)$$

ITERATES(f1(x), x, 1, 15)

[1, 10.5, 5.340702947, 3.020944403, 2.60622991, 2.775341183, 2.685947221, 2.729107082, 2.70719127, 2.718059749, 2.712603867, 2.71532631, 2.7139
2.714644675, 2.714304115, 2.714474374]

$20^{1/3}$

2.714417616

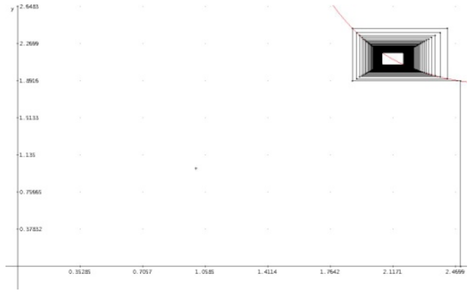
$$f2(x) := \frac{1}{2}\cdot\left(x + \frac{20}{x^3}\right)$$

ITERATES(f2(x), x, 1, 15)

[1, 10.5, 5.258638375, 2.698086188, 1.858177614, 2.487701931, 1.893389573, 2.419957021, 1.915608517, 2.38039416, 1.931598428, 2.353351859, 1.943930358, 2.333277869, 1.953867082, 2.317582116]

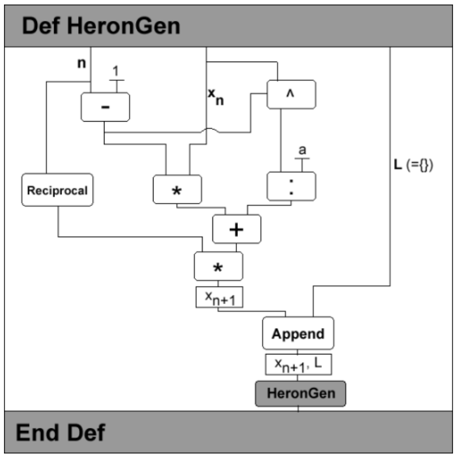
An additional aim of the example is for students to see that a simple extension of Heron's method is not well designed to calculate numerical values for all roots (it doesn't work beginning with the $\sqrt[3]{a}$ onwards). They should find a function which is able to calculate $\sqrt[n]{a}$. Therefore they have to experiment with the method of Heron first.

If students take a closer look at f2(x) they will see that this iteration is too slow. Proving this fact shows that at the fixed point $\sqrt[4]{20}$ the slope f2´(x) = -1. With the help of the Banach fixed-point theorem it is not possible to decide whether the iteration is convergent or not. But through the graphical representation with cobweb-diagrams it is possible to see the convergence:



For finding a general function they are allowed to use references [Schweiger 1995], [Fuchs2001] and the computer-algebra-system Derive for calculating numerical numbers. First they have to find the general function $f(x) = \frac{1}{n}\left((n-1)x + \frac{a}{x^{n-1}}\right)$ and draw a PROGRAPH-diagram. Calculating the slope for the fixed point $\sqrt[n]{a}$ shows f´($\sqrt[n]{a}$)=0. So the iteration will work properly.

Step 1:



Step 2:

In the second step they should implement the function with the help of a CAS:

Implementation of the general function and exemplarily calculating $\sqrt[4]{20}$ and $\sqrt{9}$:

$$f(x, a, n) := \frac{1}{n}\cdot\left((n-1)\cdot x + \frac{a}{x^{n-1}}\right)$$

```
g(x, a, n, i, j) := ITERATES(f(x, a, n), x, i, j)
g(x, 20, 4, 1, 15)
[1, 5.75, 4.338800649, 3.315315918, 2.623699935, 2.244614103, 2.125585444, 2.114825212, 2.114742531, 2.114742526, 2.114742526, 2.114742526,
  2.114742526, 2.114742526, 2.114742526, 2.114742526]

g(x, 9, 2, 1, 6)
                                   [1, 5, 3.4, 3.023529411, 3.000091554, 3, 3]
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With the help of this argumentation it is possible to direct students to a deeper and clearer understanding of mathematical coherences in mathematics education. In Informatics it is also possible to write small functional programs with mathematical software.

3 Effects of calculators/computers to mathematics education

One of the main questions when using hand-held calculators or computers is the change of mathematics education. In the past the solutions of calculations was important for teachers. Students had to know methods for solving a problem and didn't have to think about its effects to reality. Mathematics was seen as an instrument only where completing algorithms came to the fore. Important mathematical activities like demonstration and interpreting, comparing and varying, experimenting and modeling, visualizing and documenting were not considered by teachers. But modern mathematics education has to handle with these skills.

Through the usage of calculators or computers it is possible to attract notice to these points. Since today it is very difficult to persuade teachers that these points are very important. One reason therefor is that teachers don't read curricula. Their curriculum is the schoolbook. The most important point in efficient Austrian schoolbooks is calculating. Automated algorithms are used as a Black-Box by the students; the ideas aren't discussed deeply enough.

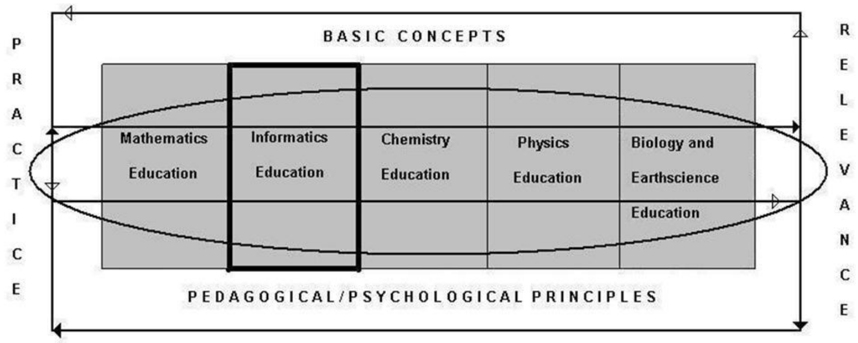
Although a lot of concepts for teaching with calculators or computers were developed by some experts, for example [Monaghan 1993], [Kayser 1993], [Fuchs 1994], [Kutzler 1996], only few teachers know about it until today.

For the reason, that mathematics education has changed that less, and because of the fact that the results of PISA 2003 in mathematics were not satisfying for Austrian pupils the Austrian government wants to implement educational standards [Siller 2007]. With their help it should be possible that demonstrating, interpreting, comparing, varying, experimenting, modeling, visualizing and documenting can be done in Mathematics.

Hence the conceptual formulations have to be changed so that a new orientation in mathematics education will be cognizable. This offers a chance that informatical concepts gain in importance in Mathematics which is necessary for a productive computer-assisted mathematics education.

4 Epilogue

Mathematics and Informatics have a lot of contents in common which is evident when we look at the historic evolution of both subjects. If I stick to the model of Fuchs [Fuchs (2) 2005] the close connections of both subjects will become evident, too:



Both subjects are the fundament for other Natural Science-subjects. The importance of both subjects can also be seen in the graphic above, although if we think in historical dimensions they have a short common history. Noticing the historical connections in both subjects it is possible to enforce a common basis through interdisciplinary teaching. This assumes that each subject retains its independence. With the help of the fundamental ideas, which are developed in Mathematics and Informatics it will be possible to strengthen the mathematical aspects in Informatics and vice versa to enforce future activities.

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