NAPIER'S RODS IN TODAY'S CLASSROOM

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ABSTRACT

Often students, of all ages, do not consider mathematics to have a history connected to individuals. This session will present some historical mathematics of Napier and connections, in particular his famous "Rods" or "Bones," with the current use of a lattice for multiplication. Participants will be introduced to a set of Napier Rods to do some simple calculations and then apply this same idea to that of a lattice to see the connections. If time allows, other historical connections for lattice multiplication will also be presented. Participants will take away their own set of Napier rods to be used with their students.

1 Introduction

Often students, of all ages, do not consider mathematics to have a history connected to individuals. For some, mathematics is often considered an area in which rules are just given and then memorized. Students of all ages may be fascinated to learn that much of the mathematics they learn is indeed based on people and their discoveries.

Mathematics educators often have so much material to cover that the historical connection to the mathematics their students are learning is overlooked. Some teachers often look at historical connections as a separate entity to itself and not a connection for the mathematics they are covering. Other teachers may not be aware of the historical connections to the mathematics they are teaching. The National Council of Teachers of Mathematics (NCTM), *Principals and Standards for School Mathematics* (2000), stresses that children need to understand the meanings of operations, as well as be able to compute fluently. Teachers can highlight historical connections with the mathematics there are teaching, especially in the realm of understanding meanings of operations and computations.

This paper demonstrates that, while many adults and children in the US are learning a "new" method for multiplication, the lattice method has indeed existed for centuries. This paper will connect several centuries old mathematical concepts/approaches/practices to what is currently taking place in many elementary classrooms.

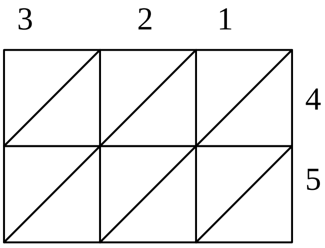
One such connection that can be made in mathematics classrooms is the historical contribution of John Napier. John Napier (1550 – 1617) was a Scottish mathematician. He devised logarithms which aided in performing calculations of large numbers in times prior to the invention of calculators and computers. Calculating with logarithms was still being taught in many high schools in the mid-1980s. The availability of affordable calculators and computers has lessened the necessity to work with logarithms.

2 Lattice or Gelosia Method Of Multiplication

In elementary school, when addressing multiplication, Napier and his invention of Napier's Bones can be discussed. Napier's Bones were created so that individuals would be able to compute quicker than having to write down the current algorithm, which happened to be a lattice. Napier's Bones were made on sticks of wood, or bone, and could be easily accessible when needing to compute. In several elementary NCTM standards-based curricula, such as Everyday Math, the lattice method of multiplication is a model shown to many children. Many adults did not learn to multiply using a lattice and find it both unfamiliar and frustrating. Napier would have felt right at home with this NCTM standards-based curriculum. The lattice multiplication was once referred to as the Gelosia method of multiplication. The Gelosia method of multiplication dates back to India sometime near the tenth century. In Italy, the Gelosia method is also referred to as the jalousia method; where jalousia can be interpreted as an iron grill that was placed over their windows to keep strangers out. "Fibonacci is credited with introducing Gelosia multiplication to the Europeans in 1202"(Richards, 2005, p. 32).

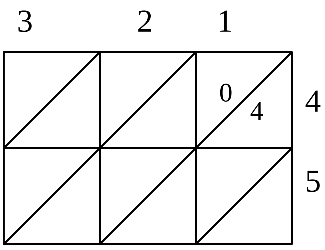
In the days of Napier, the Gelosia method of multiplication was the most commonly used algorithm. Probably the most difficult aspect of using the Gelosia method for multiplication is creating the lattice. When one wants to multiply two numbers, one must create a lattice so that each digit has its own column or row. For example, if we would like to multiply 321 x 45 we would need to create a lattice that has 3 columns and two rows (See fig. 1). Once the grid is made, the next step is to draw diagonal lines across each box from corner to corner.

Figure 1.



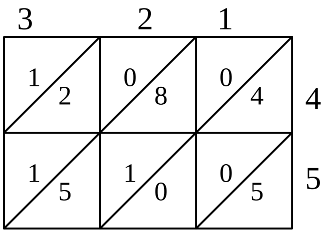
After labeling the lattice, the next step is to multiply each component section by section. If a product is less than 10, then the tens digit goes in the upper part of the square and the ones digit goes in the lower triangle of the square. For example, 4 x 1 = 4 so in the upper triangle we would place a 0 and in the lower triangle we would place a 4 (See fig. 2).

Figure 2.



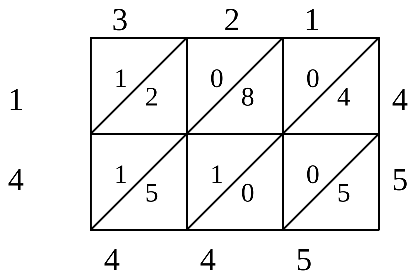
One may continue to fill in the Gelosia as appropriate. A strength of this method is that someone needs to only know their basic multiplication facts and how to add in order to use the Gelosia method to solve complicated multiplication problems. See figure 3 for a completed lattice describing 321 x 45.

Figure 3.



Now that the Gelosia is filled in, one needs to find a solution to the problem 321 x 45. The lattice has separated the various place values of the problem. The right most diagonal is the ones place, the next diagonal is the tens, the next the hundreds, then thousands, and ten thousands. If the numbers were larger, additional columns and/or rows would be added to the lattice. The last step in finding the solution is to add the digits in the various diagonals. If the sum on any diagonal is a value more than 10, then one should carry the appropriate value to next diagonal (See fig. 4).

Figure 4.



To obtain the final answer one reads the digits along the left hand side of the lattice. In our example, we see that 321 x 45 = 14, 445.

3 Napier's Bones

To assist the general public with their calculations, Napier created rods, sometimes referred to as bones, that could be used to multiply in a fashion similar to the Gelosia method. Napier created a rod for each digit 0 through 9 and on the rod would be the multiples of that number (See fig. 5 next page). There also was an index rod that assists in solving.

An example of using the Napier Rods would be to multiply 321 x 4. To do this one would take the individual rods for 3, 2 and 1 and place them side by side. Next place the index rod next to the others and look at the fourth row. While reading across the row, if the digits in a given diagonal add up to a value more than 10 then adjustments must be made. See figure 6 for a close up of the fourth row of 321 x 4.

Figure 5. Index Rod

0	1	2	3	4	5	6	7	8	9		
0/0	0/1	0/2	0/3	0/4	0/5	0/6	0/7	0/8	0/9		1
0/0	0/2	0/4	0/6	0/8	1/0	1/2	1/4	1/6	1/8		2
0/0	0/3	0/6	0/9	1/2	1/5	1/8	2/1	2/4	2/7		3
0/0	0/4	0/8	1/2	1/6	2/0	2/4	2/8	3/2	3/6		4
0/0	0/5	1/0	1/5	2/0	2/5	3/0	3/5	4/0	4/5		5
0/0	0/6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4		6
0/0	0/7	1/4	2/1	2/8	3/5	4/2	4/9	5/6	6/3		7
0/0	0/8	1/6	2/4	3/2	4/0	4/8	5/6	6/4	7/2		8
0/0	0/9	1/8	2/7	3/6	4/5	5/4	6/3	7/2	8/1		9

Figure 6.

3	2	1	
1/2	**0/8**	**0/4**	fourth row
1/5	1/0	0/5	

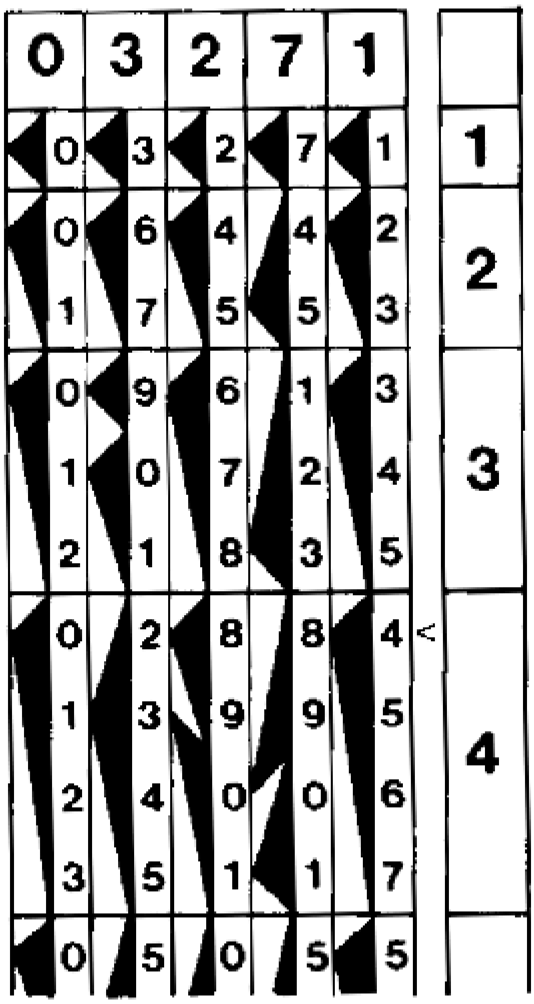
By looking at the bolded numbers (the fourth row) on this collection of Napier's Bones (figure 6), one can see that 321 x 4 = 1284.

An excellent website to consider when trying to connect the standard multiplication algorithm, the lattice algorithm and the rectangular array is the National Library of Virtual Manipulatives, http://nlvm.usu.edu/en/nav/grade_g_2.html, rectangle multiplication. This applet allows the user to see the connection between the lattice method, the standard method and a rectangular array model. The color coded sections make seeing the various components very clear. A teacher could provide sets of Napier's Bones to do easy multiplication problems, i.e. multi-digit times a single digit and, when discussing multi-digit times multi-digit multiplication, the Gelosia or lattice method can be highlighted and linked to Napier's Bones.

4 Genaille-Lucas Rulers

An extension of Napier's Bones is Genaille-Lucas rulers. These tools were developed by two French men, Henri Genaille and Edouard Lucas, in the late 1800s. These rulers allow one to solve a multiplication or division problem and simply read off an answer without having to add along the diagonals as one had to do with Napier's Bones. For multiplication one sets the rulers up to model the problem as we did with the Napier's rods, but this time the zero rod must go to the left of the number. Now one can read the answer from the ones place to the last place value by recording the digits indicated by the arrows (See fig. 7).

Figure 7. from
http://pages.cpsc.ucalgary.ca/~williams/History_web_site/time%201500_1800/Napier's%20bones.htm

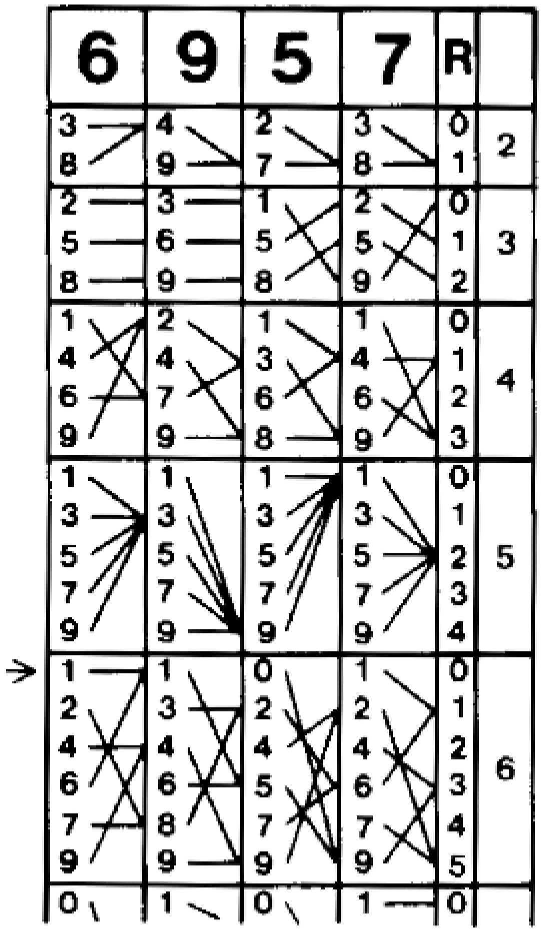


3271 × 4 = 13084

Figure 7 shows the multiplication 3271 x 4. If one starts in the fourth row with the carrot pointed at the 4 you will see the digits 4, 8, 0, 3 and 1 having arrows directed at them. Using those digits one obtains the solution of 13, 084.

Similarly, one could solve division problems using a different set of Genaille-Lucas rulers. Again to solve problems rods are lined up representing the dividend in a similar fashion to the multiplication, yet this time a remainder rod is used in case the problem at hand does not result in a whole number. As is the case with multiplication, for the quotient of two numbers one would simply have to connect the lines in order to see the solution (Figure 8).

Figure 8. from http://pages.cpsc.ucalgary.ca/~williams/History_web_site/time%201500_1800/Napier's%20bones.htm



6957 ÷ 6 = 1159 R3

Using these rods, one has the dividend, 6957 and the divisor of 6. The arrow points to the starting point in which to start to obtain the result. Following the line segments across, one will obtain the result of 1, 159 with a remainder of 3. A most remarkable and quick solution!

5 Conclusion

This paper has shown that while many adults and children in the US are confronting a “new” method for multiplication, it indeed has existed for centuries. For those familiar with Napier’s Bones, this paper has connected the topic to what is currently taking place in many elementary classrooms and has mentioned the extension of Napier’s Bones and the Gelosia method for multiplication and division to the Genaille-Lucas rulers.

REFERENCES

– National Council of of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.
– Richards, P., 2005, “The Multigrid Stencil”, *Mathematics Teaching* ***190****, 32 – 33.*