DIALOGISM IN MATHEMATICAL WRITING: HISTORICAL, PHILOSOPHICAL AND PEDAGOGICAL ISSUES

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ABSTRACT

The notion of dialogism was introduced by the Russian semiotician Mikhail Bakhtin. For him, every sentence or every discourse must be understood as a rejoinder in a dialogue: it is an answer to other sentences, or discourses and it is intended to be received by somebody. Our purpose in this lecture is to explore the meaning and the implications of this notion for mathematical texts. The consequences for historical works are clear, in the sense that one should pay attention to the nature of texts (letters, papers, books), to the texts known to the authors, and so on. In a philosophical perspective, the notion of dialogism leads to a reflection on mathematical proof. At the classroom level, dialogism is an interesting notion for the reading of ancient texts by pupils and thinking about the persons for whom the pupils write.

1 What is dialogism ?

Mikhail Bakhtin (1895-1975) was a well-known Russian literary critic and semotician. The notion of dialogism was mainly developed in his paper entitled "The Problem of Speech Genres" and written in the years 1952-1953, which was translated into French in 1984 and English in 1986. Here, Bakhtin explained that every utterance may be considered as a rejoinder in dialogue. Firstly, every utterance must be regarded as a response to preceding utterances of a given sphere. Secondly, every utterance is oriented toward the response of the others. This notion of dialogism is linked to three major notions named by Bakhtin; *responsive attitude of the listener*, *addressivity* and *speech genres*.

For Bakhtin, when a listener understands the meaning of speech, he takes an active *responsive attitude* towards it: "he either agrees or disagrees with it (completely or partially), augments it, applies it, prepares for its execution, and so on. […] Any understanding of live speech, a live utterance, is inherently responsive, although the degree of this activity varies extremely" [Bakhtin 1986, p. 68]. Bakhtin notes that this also pertains to written and read speech. Thus, as he explains, the speaker himself is oriented toward an active responsive attitude: he expects some response, agreement, sympathy, objection, and so on. That means that the utterance (i.e. what is said) is related not only to preceding utterances, but also to subsequent links in a chain of statements. From the beginning, the utterance is constructed while taking into account possible responsive reactions of an audience.

So, for Bakhtin, an essential marker of the utterance is its *addressivity*: the utterance has both an author and an addressee. "This addressee can be an immediate participant-interlocutor in an everyday dialogue, a differentiated collective of specialists in some particular area of cultural communication, a more or less differentiated public, ethnic group, contemporaries, like-minded people, opponents and enemies, a subordinate, a superior, someone who is lower, higher, familiar, foreign and so forth. And it can also be an indefinite, unconcretized *other*“ [Bakhtin 1986, p.95].

These considerations about the addressee determine a choice of a *genre of speech* by the author: the choice of compositional devices, the choice of language vehicles, the *style* of his or her utterance. Bakhtin gives as examples popular scientific literature addressed to a particular group of readers with a particular background of responsive understanding, special educational literature addressed to another kind of reader, and special research work addressed to an entirely different sort.

The choice of a particular speech genre “is determined by the specific nature of the given sphere of speech communication, semantic (thematic) considerations, the concrete situation of the speech communication, the personal composition of its participants, and so on” [Bakhtin 1986, p.78]. The genre of speech depends on the sphere of speech communication according as the sphere is reduced to one colleague or to the readers of a scientific review, or enlarged to contemporaries for a manual, or more when a text is intended for posterity.

So, with the notion of dialogism, we have the idea that an utterance is not only a product of an author about a certain subject. Bakhtin writes : “Each utterance is filled with echoes and reverberations of other utterances to which it is related by the communality of the sphere of speech communication. Every utterance must be regarded primarily as a response to preceding utterances of the given sphere (we understand response here in the broadest sense). Each utterance refutes, affirms, supplements, and relies on the others, or supposes them to be known, and somehow takes them into account […]. The utterance is filled with *dialogic overtones*, and they must be taken into account in order to understand fully the style of the utterance” [Bakhtin 1986, p. 91-92]. He explains that our thought itself, our philosophical, scientific and artistic thoughts, are born and shaped in the process of interaction with others.

2 Dialogism in mathematics: postulate 4 of Euclid

It seems a paradox to speak of dialogism about a subject as mathematics. Indeed, a mathematical text is generally thought as universal: a mathematician does not write “I” and he or she is not seen as an author, they have no addressee because they write for ‘eternity’. No author, no addressee: a mathematical text seems to belong to everybody. But we know, as teachers and as historians, that the risk is that it belongs to nobody, or only to a restricted sphere of persons (phenomena of exclusion and ethnocentrism for instance).

So, let us examine such a paradox with an example. “That all right angles are equal to one another”: it is postulate 4 of Euclid’s *Elements* [Euclid 1956, p.200]. As a postulate in Aristotle’s sense, the reader has to agree to it, as true and obvious by itself. The reader should not have any “active responsive attitude”. So, this sentence is not accompanied by other sentences: *The Elements* did not say why we should take this postulate, how it is chosen, and so on.

But, now, let us take the *Commentary of the first book of Euclid’s Elements* of Proclus written in the fifth century. Here, we learn about many readers who “refuted, affirmed, supplemented” Euclid’s postulate. Proclus begins with the purpose of

Geminus (around first century), who discuss if the Euclid's sentence can be or not taken as a postulate. Following this, Proclus gives one "proof" of the postulate. Then, he mentions other proofs given by Apollonius (third century before J.-C.), Porphyre (third century) and Pappus (fourth century). All these texts mentioned are lost, specially the commentary of Pappus about the reciprocal of the postulate, "each angle equal to a right angle is right", which is also commented by Proclus. All these readers had an "active responsive attitude".

From our point of view, the commentary Euclid's Elements by Al-Narizii is very interesting because it is given in a style of a dialogue. For postulate 4, there is a dialogue between Euclid and Simplicius: "Euclid says: All right angles are equal to one another. Simplicius says: "The truth will appear clearly to who uses a logical reasoning in this matter. If a right angle comes from a perpendicular line without any inclination, as this perpendicular line does not admit any increase or decreasing, but stays always in the same state, then the right angles are always equals". Further, there is a new dialogue about postulate 5 where a person is Al-Narizii himself. "Euclid says: That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles. Simplicius says: This postulate is not quite clear. We need a proof by lines, as Anthimathus and Diodorus did with many propositions. Al-Narizii says: we gave an explanation for that and the additions of Geminus after the proof of proposition 26 of first book" [Al-Narizii 1897, p. 23-29].

Now, if we turn to commentaries of postulate 4 by an historian like Heath, we will find commentaries of Geminus according to Proclus, but also there are other more recent purposes like those of Saccheri, Veronese, Ingrami, Enriques, Amaldi. Heath finishes with Hilbert: "Hilbert takes quite a different line. He considers that Euclid was wrong in placing postulate 4 among the axioms. He himself, […] proves several theorems about the congruence of triangles and angles, and then deduces our postulate" [Euclid, p.201]. So, Heath considers that the work of Hilbert is like a response to Euclid. Then, he comments on Euclid as if he was able to know the thought of Euclid : "It was essential from Euclid's point of view that it [postulate 4] should come before postulate 5 since the condition in the latter […]". Indeed, now the utterance "all right angles are equal to one another" is filled for us with echoes and reverberations of other utterances. Using the word of Bakhtin, we can say it is a "polyphony".

3 Dialogism and innovation in mathematics

The Bakhtinian approach offers to the historian of mathematics a way to understand the question of innovation in science, which is completely different of that of Kuhn [Barbin 2008]. Kuhn treats the question of innovation in the homogeneous context of tradition [Kuhn 1959], while the Bakhtinian approach permits us to incorporate an innovation in the heterogeneous context of the exchanges between a mathematician and his contemporaries. For instance, Bakhtin explains that in some case, an author can choose a special speech genre, which is the intimate style. He writes: "when the task was to destroy traditional official styles and world views that faded and become conventional, familiar styles became very significant in literature" [Bakhtin 1986, p.97].

Let us examine the case of Archimedes' works. Archimedes proves his results on areas and volumes by *reductio ad absurdum*. For instance, he proves that a parabolic segment is equal to a third of a certain triangle by showing that in each supposition, if the segment is either greater or smaller than the third of the triangle, this situation leads to a contradiction.

This kind of proof has the advantage of avoiding any consideration of the infinite or of the composition of the continuum. But, as mathematicians of the seventeenth century notice, this kind of proof does not explain how the value of one third is obtained. So, these mathematicians thought that Archimedes had a "method of invention" but that he had hidden it to appear more admirable [Barbin, 1992]. We have known for a hundred years that it is true that Archimedes had a "method of invention", but it is false that he had hidden it. Indeed, the method of Archimedes to obtain the value of one third and others results arises in a palimpsest discovered by Heiberg. For our point of view on speech genre, it is interesting to point out that this method appears in a letter of Archimedes to his friend Eratosthenes.

In this letter, Archimedes writes: "Seeing moreover in you, as I say, an earnest student, a man of considerable eminence in philosophy, and an admirer [of mathematical enquiry], I thought fit to write out for you and explain in detail in the same book the peculiarity of a certain method, by which it will be possible for you to get a start to enable you to investigate some of the problems in mathematics by means of mechanics". So, he addresses Eratosthenes with an intimate style, which expresses his respect. Then, Archimedes explains why he writes out this method for him. "This procedure, I am persuaded, no less useful even for the proof of the theorems themselves; for certain things first became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards because their investigation by the said method did not furnish an actual demonstration" [Archimedes 1909]. So, this method is useful, but it is a mechanical procedure and not a geometrical proof. We can understand that a letter to a friend is a better speech genre to use in writing about this method than a treatise.

The mechanical procedure of Archimedes can be compared with the method of indivisibles introduced in the seventeenth century by Cavalieri and Roberval. This method is a method for inventing propositions on areas and volumes. It consists of seeing an area or a volume as a sum of its indivisibles, lines or planes. All the mathematicians used it, because it was necessary to invent new results. But it was not an orthodox geometrical procedure, and the question arose: can some result obtained by such a method be considered as proved? [Barbin 1992]. Consequently, it is noteworthy that the story of the letter above is repeated. Now the author of the letter is the French mathematician Blaise Pascal.

In 1658, Pascal found a method to obtain centres of gravity of areas or volumes defined by a cycloid. He organized a competition between mathematicians of Europe where the challenge consisted of solving four problems, but nobody found the solution. In December 1658, Pascal gave the results as an application of his method, but he chose not to write a treatise, but a letter. This method used indivisibles and the sum of indivisibles. So, the style of a letter permitted him to give a legitimacy to his method as the true way used by him to find results. He wrote in his letter to Carcavi: “I would like that you had been rewarded by this discourse I give to you: where you will see not only the resolution of these problems, but also the methods I used, and the way by which I arrived at them. As you told me, it is what you mainly wish, and what you often regret about the Ancients, that they do not do the same, giving us only their results without teaching us the ways by which they obtained them, as if they had been jealous of this knowledge” [Pascal 1963, p.131]. The style of a letter permits Pascal to have a direct addressee when he introduces the indivisibles: “I would not make any difficulty to use the language of indivisibles, […] which seems not geometrical only to those who do not understand the doctrine of indivisibles, and imagine that it is to sin against geometry to use it “ [Pascal 1963, p.135]. This kind of utterance is not possible in a treatise. Further, Pascal explains why it is not a sin to use indivisibles, appealing to the intelligence of his reader. We can see here that Pascal chooses the familiar style of a letter “to destroy the traditional official style”, as Bakhtin writes, of the Archimedean style of proof of *reductio ad absurdum*. Pascal did not know the other letter, the one of Archimedes to Eratosthenes.

4 Dialogism and history of mathematics: pedagogical issues

There are many works and papers on the Bakhtinian approach in education (but not too much in France). They particularly concern the English-teaching field. Caryl Emerson writes about these numerous works: “Here as elsewhere in the humanities, Bakhtin was applied like a talisman, quoted, misquoted, paraphrased, carnivalized? into a legend, and soon he became a familiar presence.” [Emerson 2000, p.21]. Some works also concern interactions in the classroom. For instance, Elsie Rockwell develops a Bakhtinian approach in her ethnographic research on a lesson observed in a rural classroom in central Mexico. She wrote about dialogism: “teaching is inherently dialogical, in the Bakhtinian sense; response is constantly required, awaited, elicited, expected. Even when there is no immediate verbal response, the teacher’s utterance begs reaction or remains pending another moment when it may again be taken up, admitted or subtly refuted by students” [Rockwell 2000, p.264].

Here the purpose is to use the notion of dialogism about history and pedagogy of mathematics, and specially on reading original sources in classroom. I accorded three functions to this reading: replacement, reorientation and culture [Barbin, 1997]. As the historian Paul Veyne wrote, history has the virtue to astonish us with what goes without saying [Veyne 1971], and it is the reason for which reading original sources can have a function of a reorientation (“dépaysement” in French). Reorientation is interesting because it is a possibility for teacher and pupils to think about mathematics, not only as a part of the curriculum or as a scholarly task, but as a human activity.

As we see above, a simple utterance like “all right angles are equal to one another” can be filled with echoes and reverberations of other utterances. So, history of mathematics can produce a reorientation: now, we are astonished with what is not said. But to obtain this effect, we had to read Euclid’s utterance with a dialogical approach. To read an original source with a dialogical approach means to think of this source not as written by the author for us. For instance, if we read a source in the classroom only to translate it into a modern mathematical language, we immediately kill all effects of reorientation. Instead, to make the familiar unfamiliar, you made the unfamiliar familiar.

An original source has to be read as a rejoinder in a dialogue. What dialogue? Firstly, it is a dialogue between author and his or her contemporaries. To take dialogism into account is a good means for pupils to understand that mathematics is not a “long quiet way”, but that mathematics is a struggle for spirit. We have to read the author as somebody explaining something to somebody else. So, it is also a means to establish a second dialogue, a dialogue between the teacher and his or her students. In this case, an original source could be filled also with utterances between the teacher and the students. Perhaps, it explains the reason why students remember working on an original source in classroom for a long time, as many teachers notice.

I would like to give an example about a famous text I use in a course for students, *La géométrie* of Descartes. I begin to explain to students that contemporaries of Descartes did not understand this book and that Descartes wrote to his friend Mersenne to express his sadness about that. Here, I like to notice for students that some persons understood the purpose of Descartes, and that one of them was a woman, Princess Elisabeth.

In reading the first pages of Descartes’ book, it is necessary to understand that each sentence is a rejoinder, opposed to other utterances. In the first sentence, Descartes writes: “All problems in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for their construction” [Descartes 1987, p. 333]. For him, in geometrical figures we have only to consider simple things, which are straight lines. While in ancient Greek geometry, figures like triangles or squares are seen as areas and not by their simple edges. After this first sentence, Descartes explains that these simple things can be added or multiplied like arithmetical numbers are added or multiplied. This is also opposed to Greek mathematics where Geometry and Arithmetic are separated fields. Here a special comment has to be made about the introduction of a unity in Geometry by Descartes. Unity in geometry is a very good example of a familiar thing, which Descartes’ text made unfamiliar. We continue until the statement of the Cartesian method to solve problems. Here it is interesting to oppose Descartes, who solves geometrical problems by resolution of algebraic equations, with algebraists, who legitimate the resolution of algebraic equations by geometrical proofs.

When it is clear that the purpose of Descartes is not orthodox, it is interesting to examine with students whether or not he anticipates the difficulties of his reader. Each mathematical text is always a pedagogical text, and it is particularly important to read *La géométrie* with this idea in mind. Teacher and pupils can discuss about a mathematical text; it is not a common situation. This situation can continue as a reflection with students about mathematical writings.

Indeed, the mathematical writings of pupils are special in terms of dialogism. They are answers to only one person, their teacher, and there is only one addressee, the teacher. We know an effect of this: pupil writes a text for his teacher, a text written as his teacher wants and accepts as good. So, we often obtain texts that look like mathematical texts but which do not have any mathematical sense. So, it would be interesting to vary the correspondents of the pupils. In a group of Nantes IREM, the group ECCE maths, we work on correspondences between mathematicians and we try to establish real correspondences between pupils and students of the university. A real correspondence means not by Internet but by letters sent by the post.

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