TEACHING AND LEARNUING OF GEOMETRY IN JAPAN IN THE LATE NINETEENTH CENTURY

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ABSTRACT

Until early 1870s, Japanese people learned traditional Japanese mathematics, *Wasan*. Mathematics was regarded as a practical science, and they learned mathematics through problem-solving. After the Meiji Restoration (1867 – 1868), Japanese Government intended to modernize Japan by introducing Western civilization into Japan. Since 1872, mathematics has been taught in Japan mainly in Western style. Among the various branches of Western mathematics, the most difficult one for Japanese in early 1870s was geometry, as Euclidean geometry is of quite different nature from *Wasan* geometry. At first, Euclidean geometry was considered relating with mensuration. The value of Euclidean geometry, however, was recognized soon. Geometry was regarded as a subject for mental discipline. Teaching of geometry was gradually improved. In this way, teaching of geometry had got the right track by the end of the nineteenth century.

1 Introduction

This paper deals mainly with teaching and learning of Euclidean geometry in Japan in the last three decades of the nineteenth century. In the following, the names of Japanese and Chinese are written in order of surname and given name, according to the customs in Japan and China.

First, we mention briefly about learning of Western mathematics before that time. Until early 1870s, Japanese people learned traditional Japanese mathematics, *Wasan*, which had been developed in Japan from the seventeenth century based on Chinese mathematics that had been introduced into Japan before. Mathematics was regarded as a practical science. General people learned elementary arithmetic as a useful practical knowledge for their daily lives and for their occupations, especially for merchants, carpenters and craftsmen. They learned elementary arithmetic through practical problem-solving. Calculations were carried out by using *soroban*. Therefore, elementary arithmetic was often called "*soroban*". Some people studied mathematics not only as a practical knowledge but also as an art. *Wasan* was developed by them. For most of them, their primary concern was skillful solving of complicated problems, a great many of which were calculation of quantities related to geometric figures. In this way, Japanese people learned mathematics through problem-solving.

Japan closed the country to outsiders from the seventeenth century to the middle of the nineteenth century. Only very limited trade with China and Holland was permitted — only Dutch and Chinese vessels were allowed to trade only at Nagasaki. Christianity was strictly forbidden. Even Chinese translation of Western books on science, unrelated to Christianity, if these were translated by missionaries, were prohibited to import to Japan until 1720. Therefore, only a small number of Japanese had some knowledge of Western science at the period of national isolation.

As to mathematics, until the Meiji Restoration (1867 – 1868), only a very small number of Japanese learned Western mathematics. Many of them were naval officers in the middle of the nineteenth century, and they learned mathematics from Western teachers, at first from

Dutch naval officers, as a preliminary subject for learning navigation and gunnery. Some of them made significant contributions to the introduction of Western mathematics into Japan in the early Meiji era.

Until early 1870s, possible ways of learning Western mathematics were:

(1) learning mathematics under a Western teacher,

(2) learning mathematics under a Japanese teacher who had some knowledge of Western mathematics,

(3) leaning from Western books,

(4) learning from Chinese translation of Western books,

(5) learning from books on Western mathematics written in Japanese.

Among these, (1) was available only for very limited ones. Others learned Western mathematics either under a Japanese teacher at first, then by themselves from books, or by themselves from books from the beginning. Western mathematics was learned mainly from Western books, written in Dutch in the early days, later in English or French, sometimes in German. Western mathematics was also learned from Chinese translations.

Publication of books on Western mathematics written in Japanese began in 1857. Two books on Western arithmetic were published in that year. Until early 1870s, books on Western mathematics written in Japanese were not so many, and the subjects treated in these books were limited.

Among the various branches of Western mathematics, arithmetic, elementary algebra and trigonometry were not so difficult to learn for Japanese at that time, as there were no significant differences in ideas of arithmetic and algebra between Western mathematics and *Wasan* except technical terms and notation. The most difficult one was geometry, for, Euclidean geometry is of quite different nature from *Wasan* geometry.

Chinese translation of the first six books of the "Elements" of Euclid by Matteo Ricci and Xù Guāngqǐ, which was based on Clavius' edition, was published in 1607. It was introduced into Japan not later than 1720. Japanese mathematicians, however, did not pay any attention to it. They thought that they treated complicated figures and carried out complicated calculations in their *Wasan* geometry, but Euclid's Elements dealt only with simple geometric figures and argued about obvious facts deliberately, so they considered Euclidean geometry of no use for their studies and researches. In this way, they misunderstood that the level of their *Wasan* geometry was higher than that of Euclidean geometry. They could not recognize the value of Euclid's Elements. They could not understand the idea of Euclidean geometry as a deductive system based on axioms (Japan Academy, vol. 4, 1959, 160 – 161).

2 Western mathematics in the early Meiji era

After the Meiji Restoration, Japanese Government intended to modernize Japan by introducing Western civilization, especially Western science and technology, into Japan. Modern educational system was introduced in 1872. As to mathematics, the Department of Education decided to teach only Western mathematics at all school levels — teaching of traditional Japanese mathematics was abolished. It was too radical to be carried out, however. Many teachers were unfamiliar with Western mathematics. So the curriculum was revised within a few months. Since then, mathematics has been taught in Japan in Western style, with some consideration on the traditional way of calculation, the use of *soroban*, in elementary arithmetic.

Since early 1870s, textbooks of Western mathematics written in Japanese have been published. Many of these books published in 1870s and 1880s were translations or adaptations

of Western books into Japanese.

The most popular textbooks of Western mathematics in Japan in early 1870s were textbooks by Charles Davies. Soon after, textbooks by Horatio Nelson Robinson were used widely. The most popular textbooks of mathematics in 1870s and early 1880s were Robinson's ones. Japanese translations or adaptations of Robinson's textbooks were published, and some of the textbooks were reprinted in Japan: for instance, (Robinson 1883).

Davies' Geometry was based on English translation of Legendre's Geometry, and was edited to be used as a textbook of geometry in the United States. Robinson's Geometry was also edited to be adapted for a textbook of geometry at schools in the United States then. Robinson used algebra in proving propositions, and laid emphasis on mensuration and practical problems.

3 "Geometry is mensuration"

As mentioned above, Japanese mathematicians paid no attention to Euclidean geometry until the middle of the nineteenth century. The value of Euclidean geometry, however, was gradually acknowledged by Japanese mathematicians since the middle of the nineteenth century.

In 1871, Fukuda Han, a mathematician and military officer, wrote in the introductory remarks of his translation "*Dai Biseki Shūkyū Yakukai*" (Fukuda 1872), with revision and annotation by his father Fukuda Riken, as follows:

> Geometry is measurement. This does not mean that geometry deals with only with measurements of astronomical quantities and land surveying, however. Students should not confuse geometry and such practical measurements.

This view of geometry is due to Fukuda Riken, as is seen from Riken's annotation in page 1 of (Fukuda 1872). Fukuda Riken (Fukuda Izumi, 1815 –1889) was a Japanese mathematician who studied *Wasan* at first, then Western mathematics. He was a mathematician in a period of transition from *Wasan* to Western mathematics.

The expression "Geometry is measurement" is an explanation of geometry based on the origin of the word "geometry" ("meetkunst" in Dutch). At the same time, it explains an idea of *Wasan* geometry. As the quote from Fukuda above, many Japanese mathematicians at that time comprehended Euclidean geometry relating with mensuration — mensuration of quantities related to geometric figures such as lengths, areas, angles and volumes. Books on geometry by Japanese authors or editors published in 1870s show their such comprehension of Euclidean geometry. Many of the introductory books on geometry by Japanese authors published in 1870s laid emphasis on mensuration and geometric constructions rather than proofs of propositions. For instance, a book published in 1873 as the first step towards geometry treated only definitions, constructions without proof and simple calculations relating with geometric figures.

Robinson's geometry, which laid emphasis on mensuration and practical applications, played a role of a textbook in a period of transition from *Wasan* geometry to Euclidean geometry. Use of Robinson's geometry as a textbook made the transition easier. In this sense, Robinson's geometry was a "good" textbook for Japanese at that time.

Japanese translations at that time also show the difficulties of the translators. First, it was necessary for them to choose appropriate words for translation. To find a proper Japanese equivalent of a technical term was a difficult task. As to geometry, their translations also show their difficulties in understanding geometry: the differences between the definition and characteristic properties, between proof and explanation, the meaning of reasoning and proof, to

give a proof of intuitively obvious facts, and so on. Some of the translators of Western mathematics were, though they had fair knowledge of foreign language, unfamiliar with Western mathematics. So, there were some mistakes or incorrect expressions by their translations.

4 Japanese translation of Euclid's Elements

(1) Yamada Masakuni (tr.), *Kikagaku* (Geometry), Tokyo and Sapporo, Kaitakushi, 1872 – 1873.

This is the first Japanese translation of the "Elements" of Euclid, though only Book 1. It was published in three parts from 1872 to 1873. The original text was an edition of Euclid in English, affected by English translation of Elements by Robert Simson. At the end of the book, 53 exercise problems (propositions and constructions) were given. Later, Yamada edited "English and Japanese Mathematical Dictionary", and it was published in 1878. It was the first English-Japanese dictionary of mathematical terms.

(2) Arakawa Jūhei and Nakagawa Masayuki (tr.), *Kika Mondai* (Problems in Geometry), Tokyo, 1874.

This is a Japanese translation of all of the "geometrical exercises" (588 problems in all) included in 1872 edition of "Euclid's Elements of Geometry" by Robert Potts.

Potts' book was designed as a textbook of geometry. It was based on Simson's translation, with explanatory notes and many questions and exercises of geometry from various examination papers. The first edition was published in 1845, and revised editions were published. Potts' book was also reprinted in the United States: (Potts 1876).

Both translators were naval officers, and were active members of the Tokyo Mathematical Society, the predecessor of the Mathematical Society of Japan, in its early days. Though learning mathematics through problem-solving was the traditional way in Japan, learning geometry through solving problems in this book, however, was very difficult for Japanese students at that time, as there was no Japanese translation of the first six books of Euclid's Elements then. Outlines of solutions to the exercises of the first two books of Euclid were published in 1879. Potts' "Euclid's Elements" was not translated into Japanese.

(3) Kawakita Tomochika and Yamamoto Seishi (tr.), *Kikagaku Genso* (Foundations of Geometry), Shizuoka, Bunrindō, 1875 – 1878.

This is a Japanese translation of the lectures on geometry, given by Edward Warren Clark at Shizuoka School in 1871 – 1873. Clark gave lectures on plane and solid geometry following Euclid's Elements. Publication of Japanese translation of all the lectures was intended. Actually published one, however, was Japanese translation of the first six books of Euclid, the part of plane geometry, and was published in six parts. Translation of other books, the part of solid geometry, was not published. Introduction by Clark (in English) was included in the first part. At the end of each book of Euclid except Book 5, problems for exercises were given. The main original text may be Todhunter's book, but there are some differences between Japanese translation of Clark's lectures and Todhunter's Euclid. "*Kikagaku Genso*" is the first Japanese translation of the first six books of Euclid. Kawakita (1841 –1919) was a *Wasan* mathematician and made researches in *Wasan*, he also studied Western mathematics and endeavored to popularize Western mathematics in Japan. Solutions to the problems at the end of each book were published in five parts in 1880 – 1884, where new problems were given. This is just like the style of "Problems left to be solved" in *Wasan*.

(4) Japanese translations of Todhunter's Euclid

Two Japanese translations of Todhunter's Euclid were published in 1884 in Tokyo, namely,

Sone Tatsuzo (tr.), *To-shi Kikagaku* (Todhunter's Geometry)

and

Nagasawa Kamenosuke (tr.), *Yūkuriddo* (Euclid).

Sone translated Book 1 to Book 4, and Nagasawa made a complete translation of Todhunter's book.

Sone was graduated from Kōbu Daigakkō, the Imperial College of Engineering in Tokyo, and was an architect. Nagasawa (1860 – 1927) translated vigorously many books on Western mathematics with Kawakita's encouragement, and also wrote many textbooks.

In this way, Japanese translation of "textbook edition" of Euclid's Elements was published in 1884. Japanese translation of all the thirteen books of Euclid's Elements was published at last in 1971.

5 Other books on geometry

First we mention briefly about French textbooks. French books were used as textbooks in some educational institutions. For instance, there was a course of physics in French, taught by French professors, at the University of Tokyo until 1881. As to elementary geometry, books by A. M. Legendre, A. Amiot, and E. Rouché & Ch. de Comberousse were used as textbooks or reference books in this course. Textbooks at the Military Academy were Japanese translations of French books. Some of the French books on mathematics, such as geometry by Amiot and by Rouché and Comberousse and algebra by C. Briot, were translated into Japanese.

(1) Rikugun Shikangakkō (ed.), *Sangaku Kōhon* [1] (A Course of Mathematics), 5 vols., Tokyo, Naigai Heiji Shinbunkyoku, 1876 – 1880.

This is a Japanese translation of a course of mathematics at the Rikugun Shikangakkō (Military Academy) given by French teachers Vieillard and Kreitman, both were French military officers. The translator was Jinbo Nagaoki, who also was a teacher at the Military Academy. This is a course of mathematics based on the course at lycées in France. Volumes 1 and 2 deal with arithmetic and algebra, volume 3 deals with plane geometry, volume 4 solid geometry, and volume 5 deals with trigonometry and indexed plan, an introduction to descriptive geometry as well as of practical use. The original text of volumes 3 and 4 is "Éleménts de Géométrie" by Amiot. Some sections of solid geometry are abridged translation of the original text. These five volumes were put on sale from a publisher to meet the demands from people outside the Military Academy, as these books constituted a course of mathematics in French style and the translation was good.

(2) Kyōdōdan Daisanka (ed.), *Heimenkika Kyōjusho* (Textbook of Plane Geometry), Tokyo, Rikugun Bunko, 1877.

This is a textbook of geometry at the Military Academy in Japan. Kyōdōdan was an educational institution in Japanese Army. The original text is "Elements of Geometry and Conic Sections" by Elias Loomis. Some revisions are made, and problems on mensuration are added. Most textbooks used at the Military Academy were Japanese translation of French

[1] Here *sangaku* means mathematics (*san* means calculation, mathematics and *gaku* means study). It is different from *sangaku* which means wooden tablets presented to shrines and temples by writing mathematical problems.

books. This book, however, was a Japanese translation of a book written in English and published in the United States.

(3) Anon., *Heimenkikagaku* (Plane Geometry), Yokosuka, Yokosuka Zōsenjo, 1880.

This is a Japanese translation of a course of plane geometry given by French teachers at the school attached to the Yokosuka Zōsenjo (Yokosuka Shipbuilding Yard) of Japanese Navy. "Plane Geometry" were based on books by Legendre and by Rouché and Comberousse, with some emphasis on practical items which would be useful in working at shipbuilding yard and with reduction of some difficult topics such as complicated constructions by ruler and compass.

(4) Akagi Chikayuki (tr.), *Jōyō Kyokusen* (Usual Curves), Tokyo, 1882.

This is an abridged Japanese translation with some adaptation of the chapters on "usual curves" ("courbes usuelles" in French) from 1875 edition of "Éléménts de Géométrie" by Amiot and 1868 edition of "Traité de Géométrie" by Rouché and Comberousse. "*Jōyō Kyokusen*" was intended to be a textbook of secondary education at that time. Conic sections and helix were treated in this book. In 1880s, the curriculum of geometry in secondary education was plane geometry, solid geometry and usual curves. Akagi studied physics in French course at the University of Tokyo.

(5) Tanaka Naonori (ed.), *Kika Kyōkasho* (Textbook of Geometry), 5 vols., Tokyo, 1882.

This is a textbook of geometry, based on Todhunter's Euclid, with consideration of books by J. M. Wilson and W. Chauvenet. The first three volumes deal with plane geometry, volume 4 treats solid geometry, with an appendix on loci (in plane and solid geometry), and volume 5 deals with conic sections by synthetic methods, with an appendix on a brief introduction to "Modern Geometry", where projective properties of figures in Euclidean space are treated.

6 Teaching of geometry — two cases

In this section, we consider teaching of geometry at two educational institutions, namely, Kōbu Daigakkō in 1870s and Osaka Chūgakkō in 1880s.

6.1 Kōbu Daigakkō

Kōbu Daigakkō, or the Imperial College of Engineering, is a predecessor of the College of Engineering of the University of Tokyo. This college was planned in 1871 by Kōbushō, the Department of Public Works of the Government, as a college to train students to be engineers serving as government officials in that Department. Main factories in Japan were under government management then. The actual start of the college under the name of Kōgakuryō was in 1873. All professors were invited from the United Kingdom, most of them from Scotland. The principal of the college was Henry Dyer, Professor of Civil and Mechanical Engineering. He was a graduate of the University of Glasgow, and at this College until 1882. Professor of Mathematics (from 1873 to 1878) was David H. Marshall, a graduate of the University of Edinburgh. Among the professors were William Edward Ayrton (Professor of Natural Philosophy and Telegraphic Engineering, from 1873 to 1878) and John Perry (Professor of Civil and Mechanical Engineering, from 1875 to 1879).

It was a six-year college of technical education. The whole course was divided into three: (1) the general and scientific course, the first two years, (2) the technical course, the next two years, and (3) the practical course, the final two years. Theory and applications,

teaching and learning in school and practical training outside school were unified together. In the general and scientific course, "Elementary Mathematics" was taught firmly — it was a standard course of elementary mathematics with some applications to practical problems, and not the application-oriented one. The contents were: geometry, algebra, plane trigonometry, mathematical tables, spherical trigonometry and geometrical conics. Coordinate geometry were treated in "Higher Mathematics" in the technical course. In the technical course, Ayrton and Perry taught applications of mathematics to science, engineering and practical problems, and they used squared papers.

Judging from the curriculum, syllabus, examination papers recorded in the "Calendar" of the college, and number of copies of books on geometry in Class Library of the college, geometry in "Elementary Mathematics" was taught following Euclid at first. Soon after, geometry was taught mainly following Wilson, with reference to Euclid. The level of lectures had gone up year by year. Practical geometry such as Perry advocated later was not taught in "Elementary Mathematics" (Kota 2005a, 2005b).

6.2 Osaka Chūgakkō

Osaka Chūgakkō (Osaka Middle School), established in 1880, was a school under the direct control of the Department of Education. The origin of this school was Yōgakusho (Institute for Western Studies) in Osaka established in 1869. Osaka Chūgakkō was the model school of secondary education in 1880s.

As to mathematics, arithmetic, algebra, geometry and trigonometry were taught firmly. Mathematics was taught by Japanese teachers using Japanese translations of Western books, mainly American books, as textbooks.

In the "Calendar" of this school in the year 1881 – 1882 (and also those in the later years), the aims of each subjects were recorded. As to geometry,

> Geometry is a subject which studies properties and relations of lines, surfaces, angles and solid bodies, and measurements of these. Geometry not only is necessary to measure lengths, volumes and so on exactly, but also makes one's thinking exactly and cultivates power of reasoning and judgement. Therefore, a teacher should make it a principle to explain elaborately and precisely.

Thus, geometry is also considered a subject for mental discipline.

Textbooks of geometry were *Kikagaku Genso* and *Jōyō Kyokusen*. Judging from the syllabus of geometry recorded in the "Calendar" of the school, Japanese translation of Robinson was also used as a textbook (Kota 2006a, 2006b).

7 Popular textbooks on geometry

Textbooks of geometry widely used in 1880s were those by Robinson, Wilson, Wright and Chauvenet. As to textbooks by foreign authors, either original texts (in English) or Japanese translations were used. In secondary education, until the end of the nineteenth century, sometimes textbooks written in foreign language, not Japanese translation, were used.

These textbooks except Robinson were also used widely in 1890s. Use of Todhunter's Euclid as the textbook of geometry was not so popular, though his algebra, trigonometry, as well as differential calculus and integral calculus were popular textbooks in Japan. According to Ogura Kinnosuke (Ogura 1932), the most popular textbooks of mathematics in

1882 – 1885, written originally in foreign languages, were algebra by Todhunter, geometry by Wilson and by Wright, and trigonometry by Todhunter.

Most textbooks on geometry widely used in Japan in 1870s and 1880s were, using Lewis Carroll's terminology, books of "Euclid's Modern Rivals", and were criticized by him (Carroll 1885).

8 Kikuchi Dairoku

The most important contribution to the development of the teaching of geometry in Japan in the late nineteenth century was made by Kikuchi Dairoku (1855 – 1917). He went to study in England twice, namely, from 1866 to 1868 and from 1870 to 1877. He studied at the University College School of London in the first time and the beginning of the second time. Geometry which Kikuchi learned at the University College School of London was a new type of Euclidean geometry, which had originated from experimental teaching of geometry by T. A. Hirst in the early 1860s, and it had a great influence on his view of geometry and of mathematics education.

Then, he studied mathematics at St. John's College at Cambridge. He was a wrangler. After coming back to Japan in 1877, he was appointed to professor of pure and applied mathematics at the Department of Science of the University of Tokyo.

He established the curriculum of mathematics at the University of Tokyo. It was modeled after the British one. Also, on the advice of him, textbooks of mathematics at Tokyo Daigaku Yobimon, the Preparatory School for the University of Tokyo, were changed from Robinson's books to British ones, namely, algebra and trigonometry to Todhunter's books, plane geometry to Wright's book, and solid geometry to Wilson's.

The author considers that the reasons why Kikuchi's recommendation of the textbook of plane geometry was Wright's book and not Wilson's are as follows: first, he considered that the new type of teaching of geometry was better than the traditional style of "following Euclid"; next, he considered the contents and arrangements of Wright's book better than Wilson's, and, he had more familiarity with Wright's book than with Wilson's.

Kikuchi translated the fourth edition (1886) of the "Syllabus of Plane Geometry" by the Association for the Improvement of Geometrical Teaching (AIGT), (first edition in 1875) into Japanese, published as "*Heimenkikagaku Kyōjujōmoku*" in 1887. In the preface of Japanese translation, he wrote

> I am not completely satisfied with this Syllabus. I trust that the members of the Association who have contributed to prepare the Syllabus also would not regard it absolutely perfect. The Syllabus, however, is the result of discussions among learned and experienced scholars. Therefore, it is obvious that anyone who teaches geometry should refer to it. Also, students of geometry will learn a great deal from it. In our country, there are some people who have not waked from their reverie of blind acceptance of Euclid. Besides, textbooks such as Robinson's geometry are popular. Intelligent people deplore these. Therefore, I trust that the publication of a Japanese translation of the Syllabus is very valuable.

This shows his opinion on the teaching of geometry, which had been formed ten years ago.

Kikuchi wrote a textbook of geometry in two volumes in 1888 –1889:

Shotō Kikagaku Kyōkasho, Heimen no Bu (Textbook of Elementary Geometry, Part of Plane Geometry), Tokyo, Dai-Nippon Tosho,

Shotō Kikagaku Kyōkasho, Rittai no Bu (Textbook of Elementary Geometry, Part of Solid Geometry), Tokyo, Dai-Nippon Tosho,

Kikuchi's textbook was based on the Syllabus of AIGT, and was more rigorous and more nearer to Euclid than the Syllabus. According to Ogura (Ogura 1932), Kikuchi's book "was more rigorous and more polished in style than the textbook 'Elements of Plane Geometry' of AIGT". For instance, Kikuchi's book treated ratios and proportions as those in Book 5 of Euclid's Elements. Kikuchi's books were used widely in 1890s and also early in the twentieth century.

Then, Kikuchi wrote a book

Shotō Kikagaku Kyōkasho Zuihan Kikagaku Kōgi (Lectures on Geometry following the Textbook of Elementary Geometry)

in two volumes, in 1897 and 1906. In this book, he gave explanations of the aims of editing his Textbook, and of some of the topics treated in the Textbook in detail, so as to be a useful reference book for those who were studying geometry by themselves with his Textbook and for teachers. The contents of published "Lectures" were, however, only plane geometry, with appendix on a brief history of geometry.

He wrote in the Introduction of the Textbook:

> Geometry is a subject which deals with truths in shape, magnitude, and position of bodies.

In his "Lectures", he explained the aims of geometry and wrote:

> Geometry is the subject which deals with properties of space.

In this way, in the "Lectures", geometry was regarded as the science of space, not as the science dealing with properties of figures. Though this was a passage in explaining the aims of geometry, we see there a reflection of the development of geometry in the late nineteenth century, namely, from the science of figures to the science of space.

9 Euclidean geometry as a mental discipline

As mentioned above, mathematics was regarded as a practical science in Japan until early 1870s. Since the introduction of modern educational system and adoption of Western mathematics in school mathematics, mathematics has been gradually regarded a branch of pure science, with many applications in various fields, especially in physics and engineering. The main objectives of mathematics education of secondary level were regarded as students' acquisition of mathematical knowledge necessary for their studies and a mental discipline, and, in teaching of mathematics, emphasis was laid on the latter. Geometry was regarded as a subject suitable for mental discipline, and Euclidean geometry was taught rigorously. The Syllabus of Mathematics for *Chūgakkō*, boys' secondary schools for general education, officially announced in 1902, was written from this view of mathematics education.

Teaching of geometry had been gradually improved in the last three decades of the nineteenth century. Textbooks on geometry by Japanese authors were published since the late 1880s. In this way, teaching of geometry had got the right track by the end of the nineteenth century.

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