Teaching Big Ideas in mathematics through history

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In almost every university that educates future secondary school teachers, history of mathematics appears a part of the curriculum. In this paper I will describe possible ways to integrate the history of mathematics in other upper division courses, especially in a capstone course and in courses related to mathematics teaching methods. One of the mottos in teaching mathematics is Abe Shenitzer's maxim:

> "One can *invent* mathematics without learning much of its history.
> One can *use* mathematics without knowing much - if any - of its history.
> But one cannot have a *mature appreciation* of mathematics
> without a substantial knowledge of its history**."**

Episode 1: The notion of proof in mathematics

Students are shown a way to calculate the area of a quadrilateral as it appears in the Rhind Papyrus (Eves, 1982, p. 14). This activity allows the students to expose their own conceptions about the nature and the significance of a mathematical proof. The discussion of what constitutes evidence in mathematics and how truth is established is also fostered by this activity. A pilot study of such an experience is described in Winicki-Landman (2002).

Episode 2: Mathematical definitions: What? When? Who?

Students are asked to read some definitions of solids as they appear in Euclid's Elements. Specifically, they are asked to read the definition of *cone, pyramid, cylinder and prism*. They are asked then to look for other definitions of these solids and to compare these definitions. This compare-contrast activity fosters a discussion of the some key issues related to mathematical definitions: the existence of equivalent and non-equivalent definitions of the same concept, the notion that a definition establishes sufficient and necessary conditions, the fact that mathematical definitions are dramatically different from dictionary definitions, the symbiotic relationship between proving and defining and some pedagogical issues related to the defining process. A brief description of possible interactions is provided in Winicki-Landman (2004). To emphasize the evolutionary aspect of definitions in mathematics, the students are asked to read Kleiner (1989).

Episode 3: Aesthetics in mathematics

Students are exposed to three different versions of a special case of Fermat's Little Theorem: For every natural number n, n^5-n is a multiple of 10.

They are asked to analyze them, compare them and choose their favorite one. Via this activity, the students are taught how to analyze proofs using criteria like generality, complexity, naturalness, clarity, security and elegance (Barbeau, 1988). They are also asked to read Kleiner (1991) who describes rigor in mathematics not as a dogmatic idea but as an evolving one.

Episode 4: Open-ended questions in mathematics

Students are exposed to Leibniz's triangle and using it, the idea of open-ended questions is modeled. These types of questions involve significant mathematics, lead to may different approaches, use of different representations, may have different solutions, foster mathematical communication among the students and they may lead to the formulation of new significant questions. Such an experience is described in Winicki-Landman (2005).

Final Remarks

There are many creative ways to introduce the history of mathematics into its teaching. The special kind of knowledge that teachers need can definitely be developed and deepened when historical perspectives are added to the class. This kind of knowledge is different from just content knowledge: it is *pedagogical* content knowledge and mathematics teachers need specific and concrete experiences that enable them to look into the mathematics content deeper and with "different eyes".

References

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