History of The Return of Applications in Undergraduate Mathematics in the United States

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ABSTRACT

Changing university education in the U. S. is like political change in democratic countries: many people need to be persuaded and there is much argument. The advice of prestigious leaders and committees is often ignored as each department makes its own decisions. Money mostly plays the role of helping in the attractive presentation of alternatives. One of the significant changes of the last century or so is the change in emphasis given to teaching applications of mathematics to students specializing in mathematics. We outline how recent increases in this emphasis have come about and we discuss social factors that probably influenced the increase in the teaching of applications in the second half of the twentieth century.

1 Overview of Mathematics Education in United States' Universities

In the United States, full-time university students spend their first four years, typically while they are aged 18 through 21 or so, in what is called undergraduate studies. This covers many subjects but it is possible to specialize in mathematics by spending between a third and a half of one's time on mathematics and related courses. After undergraduate studies some students may go on to graduate studies in which they study only mathematics. This paper deals with the role of applications of mathematics in undergraduate mathematics courses, especially courses for students specializing in mathematics as their major subject.

Trends in mathematical research have only an indirect connection to undergraduate studies. In particular, the prestige that leading researchers have on account of their research does not translate into special influence on undergraduate curricula. Indeed, the professional concerns of research on the one hand and undergraduate teaching on the other are represented by entirely separate professional organizations: the Mathematical Association of America (MAA) for undergraduate teaching and the American Mathematical Society for research in general and the Society for Industrial and Applied Mathematics for applied research.

Neither MAA nor any other organization, whether private or governmental, has any effective control over curriculum in the individual colleges. It is true that the professional organization MAA, has a permanent committee, the Committee on the Undergraduate Program in Mathematics (CUPM), that issues advice on undergraduate curricula every few years. But this advice, and the advice of individual leaders, is often vague since it is meant to fit highly autonomous universities with different missions, different types of students and different traditions. As we shall see, this well meant advice has sometimes been ignored.

In principle, it would be possible for two universities to have undergraduate mathematics curricula that are radically different from one another. Although examples of this can be found, they are quite rare. In some way, a rough consensus usually forms about appropriate mathematics curricula for undergraduates. We are concerned in this paper with how this consensus was altered to include a wider role for applications.

2 Applications before the middle of the twentieth century

The earliest comprehensive information we have about undergraduate mathematics in the U. S. is a description of the situation around 1889 in 19 important institutions in the U. S. [Cajori, 1890]. Cajori's survey turns up frequent courses of an applied nature taught in mathematics departments, including Astronomy, Descriptive Geometry, Least Squares Correction of Observations, Mechanics, Navigation, and Surveying. The next time for which we have any data is 1960, when MAA carried out a survey of many universities [Mosteller, et al, 1961]. This MAA survey shows that none of the courses we have just mentioned were still being taught in mathematics departments except for Mechanics (but only at 8% of institutions surveyed whereas it was present at about 50% of the mathematics departments in Cajori's 1890 survey). Not only did nearly all of the applied courses of 1890 disappear, few if any new courses for mathematics majors that showed mathematics being applied to data were turned up in the 1960 survey.

3 The return of applications in the second half of the twentieth century

3.1 The extent of the comeback

By the later part of the 20th century, applications were making a comeback. One form of this comeback was the creation of courses for advanced students in mathematical modeling[1]. In such courses, the emphasis was not primarily on learning new mathematics, but on studying examples of how mathematics is applied. As we see in Figure 1, based on data collected by the Conference Board of Mathematical Sciences (CBMS), mathematical modeling courses developed a certain modest popularity [Jewett, et al, 1967], [Fey, et al, 1976], [Albers, et al, 1987], [Lutzer, et al, 2002].

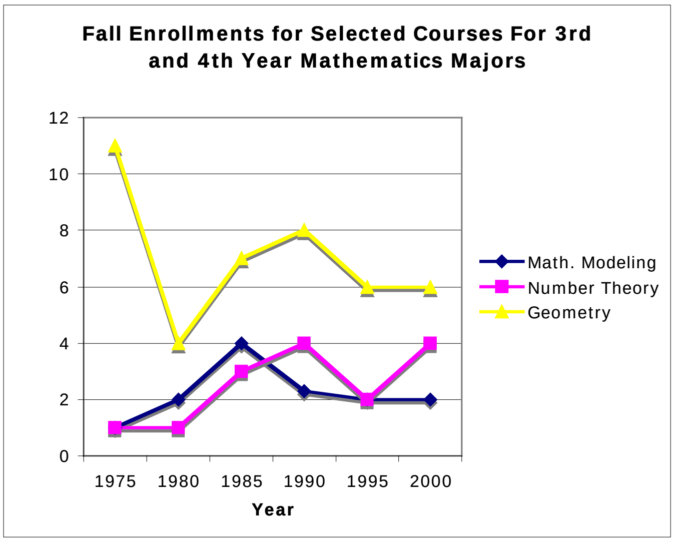

Figure 1

[1] In this era, the phrase "mathematical modeling" came to be used to describe the use of mathematics in applications, with careful attention to the data of the problem. [Pollak, 2003] "Methods" courses which present mathematics that can be useful, but without examples of its use, are in a different category.

In the last few decades of the twentieth century, curriculum revision projects often included discussions of the role of applications. Calculus is a good example. In the late 1980's and the 1990's, reform of the calculus course became a widely discussed and generously funded activity. In the years from 1988 to 1994, the National Science Foundation (NSF), a branch of the federal government, funded 127 projects involving new calculus curricula [Ganter, 2001]. One criticism of the existing calculus courses of that time was that they had too much stuffed into them and this left little time for students to explore the conceptual issues in a way that would lead to real understanding [Douglas, 1986]. "Lean and lively" became a slogan sometimes used to describe the sort of calculus course that was sought by some reformers. Having applications in a course takes up time (a student needs to be told something about genetics to understand an application in genetics) and one might suppose that this should be minimized if the objective is to avoid overstuffing the course. But, from the evidence in [Ganter, 2001], teaching of applications ranks in second place among objectives for improving calculus.

Ganter identifies 18 objectives for calculus reform that have been articulated in 5 leading papers that discuss the development of the reform movement. The five most common objectives in the 127 projects supported by NSF between 1988 and 1994 are shown in Table 1. About $1,686,000 was spent on projects that had applications as one of their objectives. We do not have any data on how these calculus reform projects affected the role of applications in the calculus courses as they are actually taught around the country. But, at the very least, the matter was discussed and written about.

Objective for improvement of calculus	**Percent of 127 projects with that as one key objective**
Use of computers	89.7%
Use of applications	73.2%
Conceptual understanding	63.7%
Laboratory experience	55.9%
Discovery learning	55.1%

Table 1. U. S. calculus reform projects of the 1990's [Ganter, 2001, p. 17.]

Another example of the greater role for applications in mathematics curricula is the existence of entire degree programs which combine some mathematics with other studies aimed at preparing students for particular kinds of jobs. Degree programs with special applied purposes which were counted in CBMS reports are: Statistics, Actuarial Science, Operations Research, Joint Computer Science and Mathematics, Joint Mathematics and Statistics, Joint Computer Science and Statistics [Albers, et al, 1987], [Lutzer, et al, 2002]. Figure 2 shows that these programs have a small but definite presence among undergraduate programs[2].

Finally, some universities provided experiences with applications in which the experiences were not in the form of a conventional course or degree program. One example is training a team to compete in the *Modeling Competition in Mathematics,* a nation-wide competition. Another example is the mathematics clinic, which simulates the

[2] We cannot be sure of the popularity of special purpose applied programs before 1985. CBMS did not collect data on these programs before 1980 (which suggests they were too rare to come to the attention of this highly professional organization) and the data collected by 1980 was collected differently and is not strictly comparable to the data we show in Figure 2.

industrial problem-solving environment and where students are confronted with knotty, realistic problems often requiring an entire semester of work.

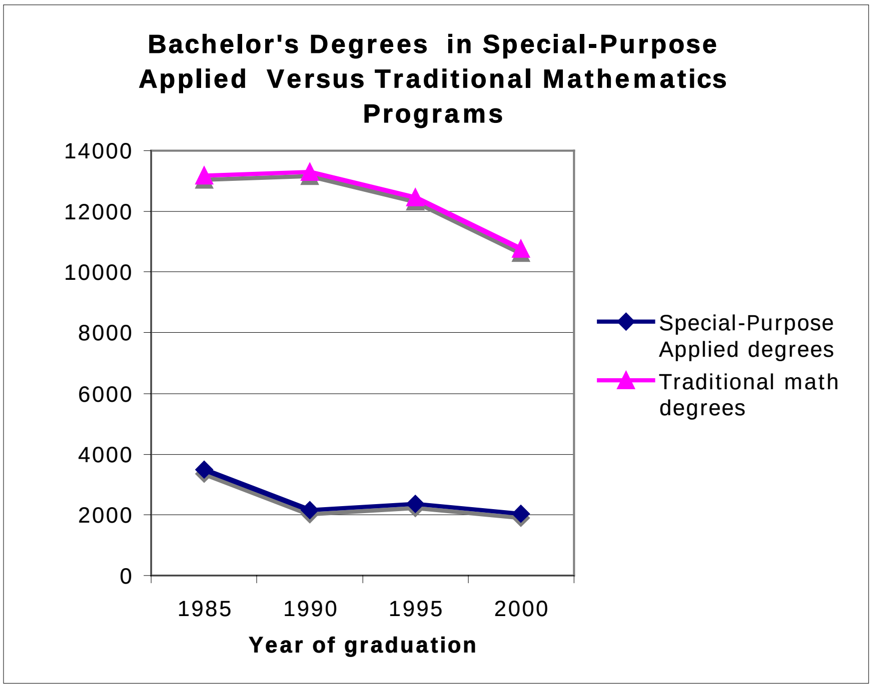

Figure 2

3.2 How applications returned to the curriculum

We come now to the steps by which applications became a widely discussed and sometimes practiced element of the world of undergraduate mathematics. In the period from 1950 to 1958, a systematic census made by my colleague Jack Winn shows that there were 10 articles in the *American Mathematical Monthly*[3] making favorable remarks about teaching the applications of mathematics [Betz, 1950] [Committee on the Mathematical Training of Social Scientists, 1952] [Richardson, 1952] [Hull, 1953] [Bush, et al, 1954] [Hohn, 1954] [Joint Committee, 1955] [Tukey, 1955, 1958]] [Fry, 1956]. A census of articles from the following two decades has not yet been done, but it would undoubtedly turn up many more articles favoring applications. These articles just cited were just the opinions of individuals, but MAA also added its prestigious organization's voice as early as 1955 when its Committee on the Undergraduate Program (CUP, the predecessor to CUPM) published the two volume set *Universal Mathematics*. These books were meant to be used for a proposed new course for all college students, including mathematics majors [Price, 1955, p. 10]. The second volume [Davis, 1955] contains substantial material on mathematical modeling in the social and biological sciences. This book, and its intention of being usable for all students, represents an official recommendation of the value of teaching applications to mathematics majors on the part of MAA, the most important organization that concerned itself with undergraduate mathematics.

The *Universal Mathematics* volumes were never used in teaching to any great degree. The leader of the committee that wrote them, William Duren, had this to say [Duren, 1956]: "The book is not yet suitable as a textbook and caused considerable difficulty to students and instructors. The main trouble is that students cannot read it." However, a

[3] The main journal aimed the undergraduate teaching faculty at that time, published by MAA.

related textbook, *Introduction to Finite Mathematics* [Kemeny, et al, 1957], became a great and enduring success and presented mathematics for the social sciences and, in later versions, business management. But it is important to note that this book, as declared in its preface, was intended for social science majors and not those with mathematics as their major subject.

Later, Kemeny and Snell wrote the more advanced textbook *Mathematical Models in the Social Sciences* [Kemeny, Snell, 1962] hoping that it would lead to the creation of courses that might fit naturally into the curriculum for students who were mathematics majors. But this did not happen at many institutions. Influential mathematicians expressed doubt that mathematics had achieved any success in the social sciences. For the story of the finite mathematics course, *Universal Mathematics*, and related matters, see [Meyer, 2007].

CUPM was undaunted by the failure of CUP's work on *Universal Mathematics* to promote applications for mathematics majors. CUPM recommended the teaching of mathematical modeling in 10 widely widely circulated reports between 1962 and 2004 [Committee on the Undergraduate Program in Mathematics, 1962, 1963, 1965a, 1965b, 1966, 1972a, 1972b, 1981, 1991, 2004]. With the exception of the 1991 and 2004 reports, these reports gave clear support to the institution of mathematical modeling courses as a central mechanism for having applications in the curriculum. In the early years, these CUPM reports met the same fate as *Universal Mathematics*: they did not stimulate immediate action. The first mathematics departments to institute mathematical modeling courses seem to be Washington State University and Clemson University in 1971 [Pollak, 2003]. The first evidence that this might be the start of a movement rather than the actions of a few unusual schools appears when CBMS reports one thousand students enrolled in mathematical modeling courses in Fall 1975[4]. The lag time between the start of advocacy in 1955 till significant curricular action around 1975 was no doubt partly due to the fact that, except for [Kemeny, 1962], with its unpopular restriction to the social sciences, there were no textbooks for mathematical modeling courses until 1973 when [Maki, Thompson, 1973] appeared. On the other hand, the lack of a textbook could be ascribed to the lack of demand.

There were two other forces that were perhaps just as influential as the ones already mentioned: federal funding for mathematics education, and the creation of the Consortium for Mathematics and its Applications (COMAP).

Before the 1950's, the federal government in the U. S. had played little role in funding or influencing mathematics in the universities. In this period before federal support, the U. S. had elevated its mathematical level from mostly second-rate (in the nineteenth century) to world-class. But in the 1950's a number of political factors, largely connected to the "cold war" between the U. S. and the Soviet Union, came together to create a feeling that science and mathematics in the U. S. needed even more strengthening and that extensive federal support was essential. The main agency dispensing such funds for academic mathematics was the NSF, founded in 1950 but without significant funding till 1957.

The movement toward applications clearly benefited from NSF funds. In the first place, much of the activity of CUPM was funded by NSF and, as we have seen, CUPM recommended mathematical modeling in all of its many reports. In addition, in 1976 NSF provided funds for a large project, named Undergraduate Mathematics and Its Applications (UMAP) that was dedicated to producing applications-related modules that

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[4] That is about the same number as enrolled in the elective Number Theory, but far fewer than the 13 thousand enrolled in the required Modern Algebra course (also known as Abstract Algebra).

could be used as text material for a few days of classroom instruction. This project proved so successful that, in 1980, it evolved into the nonprofit corporation COMAP which continued module development and began other activities, mostly centering around applications of mathematics. These activities included: creation of textbooks, publication of a journal oriented toward applications, and creation of television programs on elementary mathematical modeling. COMAP is still in existence today and, over the years, it has been able to gain significant support from NSF for its activities [Garfunkel, 2005].

1981 may be regarded as a year in which the strength of the sentiment for mathematical modeling is fully expressed. By 1981, COMAP listed 265 teaching modules on applications of mathematics in its catalog, each one typically between about 10 and 50 pages long. Textbook writers unconnected to COMAP were also active. By 1981, 12 texts had appeared which could be used in upper level undergraduate mathematical modeling courses [Kemeny, Snell, 1962] [Maki, Thompson, 1973] [Melzak, 1973, 1976] [Singleton, Tyndall, 1974] [Roberts, 1976] [Chartrand, 1977] [Haberman, 1977] [Bender, 1978] [Olinick, 1978] [Boyce, 1981] [Saaty, Alexander, 1981]. In that same year of 1981, CUPM again issued curriculum recommendations, this time more emphatically in favor of mathematical modeling.

3.3 Intellectual and social influences

We turn now to ask what circumstances may have led to, or facilitated, the increased discussion, writing and teaching of applications that existed around 1981. To begin with, one must keep in mind that applying mathematics is a centuries-old and successful activity. Therefore, the intellectual argument for including applications in mathematics curricula has been strong for a long time. Since that is true, then why do we see lots of applied courses in the late nineteenth century, nearly none around 1960, and an increase starting only in the 1970's, despite CUP and CUPM urgings starting in 1955? Evidently there are other factors that play a role. We will discuss three likely social influences favoring the return of applications in the 1970's.

In the first place, in the 1970's Ph.D's in mathematics started having much more difficulty finding jobs than had been the case in the previous decades. According to data published in the *Notices of the American Mathematical Society* all Ph.D's of the 1969-1970 academic year wound up working either in academia (84%), industry (10%), government (4%), or research institutions (2%) [Editors, 1970]. These percents add to 100% and so there is no unemployment. One year later, in Oct. '71 the *Notices* reports that 80 out of 1350 (about 6%) Ph.D's who earned their degrees in 1970-1971 were unemployed [Editors, 1971]. Similar articles reporting unemployment can be found in the *Notices* in subsequent years. By 1976 the comparable unemployment rate had increased to 11% [Editors, 1976].

The fact that not every new Ph.D mathematician could find a job must surely have led professors to the idea that giving young undergraduates experience with applications of mathematics might make them more employable, and to the realization that offering such experiences might increase student enrollments in undergraduate mathematics. And this brings us to the second social influence favoring applications: improvements in enrollments were definitely needed. As we see in Figure 3, undergraduate mathematics enrollments were plunging after 1970[5].

[5] The reasons for this plunge have not been satisfactorily explained. Competition for students from growing computer science programs have been mentioned to this author in informal conversations, but the matter deserves more careful examination based on evidence.

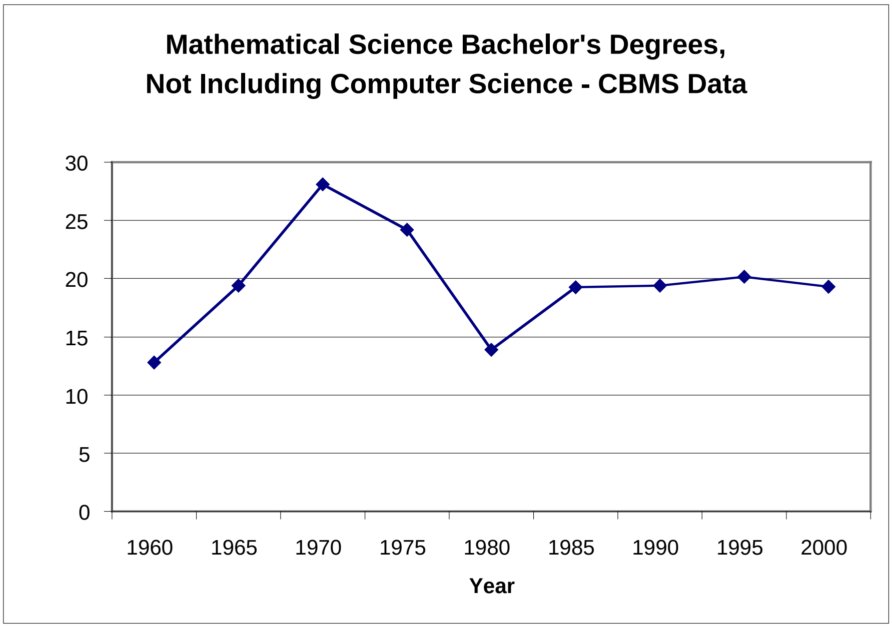

Figure 3

CUPM makes direct reference to enrollment problems in its 1981 report and points out an apparent paradox it perceives: "Yet while the number of mathematics majors is decreasing, the demand for broadly trained mathematics graduates is increasing in government and industry." [Committee on the Undergraduate Program in Mathematics, 1981, p. 10]. CUPM's report seeks to resolve this paradox by curricular approaches that can best be characterized, in its own words, as ". . . a unified view that respects the content and teaching of pure and applied mathematics equally." [Committee on the Undergraduate Program in Mathematics, 1981, p. 8]. This is a strong, even revolutionary recommendation, if one takes the word "equally" seriously.

A third factor that probably aroused interest in applications among undergraduate mathematics instructors was the free-wheeling spirit and encouragement of potential[6]change that existed in the U. S. in the late 1960's and the 1970's. During and after the Vietnam War, the universities resounded with alternative ideas about politics, education and culture in general. Some of this was noisy and destructive, but quiet, constructive experiments also took root and some still survive today. An example would be Hampshire College, which opened its doors in 1970 and, from the outset, did not give students grades, preferring instead written evaluations. Curriculum at Hampshire is mostly centered on student interests and includes an unusual emphasis on projects rather than courses. Elsewhere, at Colorado College, the schedule of instruction was reorganized in 1970 so that students would study just one course at a time. The first semester of calculus, normally requiring 4 hours of class per week for 15 weeks, was pursued in 3.5 weeks of intensive study with no other subject competing for student attention.

[6] It is not so much that the country changed substantially and permanently in those years but that so many new ideas and ways of living were given a tryout.

Experimentation with new ideas reached into the most improbable areas of education. For example, the commandant of the U. S. Army War College became a devotee of transcendental meditation, a practice popularized in the U. S. by the Maharishi Mahesh Yogi, an Indian mystic who lived in a cave for two years developing his philosophy [Campbell, 1974].

A number of details of the COMAP story reflect the unusual receptiveness to new ideas in the U. S. in the late 1960's and early 1970's. COMAP's founder, Solomon Garfunkel, did his graduate work, during the Vietnam War, at the University of Wisconsin, which was noted for political protest and exploring alternative ways of living. Garfunkel earned his Ph.D, with a dissertation in logic, in just three years of graduate study, an unusually short time. He began a promising research career at Cornell University, one of the leading Universities in the U. S., and then at the University of Connecticut, a school of increasing size and ambition. However, starting around 1970, by slow degrees he abandoned this research career in order to work in mathematics education. He began his work in education under Prof. Jerrold Zacharias, an MIT physicist who had a major NSF grant to improve calculus instruction. The idea that a physicist would have a major grant to improve a mathematics course is itself highly unconventional and a sign of the times. By 1976 Garfukel was ready to try to persuade NSF to fund his own work in undergraduate mathematics education. His qualifications might not normally have impressed most NSF officials who were overseeing the awarding of money for mathematics education. He was, after all, a turncoat to his subject of research, quite young, and his only experience in mathematics education was as an apprentice to someone who was not even a mathematician. But in a further sign of the times, the NSF program officer he needed to persuade was not a mathematician, but someone who called himself a futurist, Gregg Edwards [Garfunkel, 2005].

The influence of NSF, with its significant financial resources, should not be overestimated. NSF had no means, financial or otherwise, to persuade individual professors or departments to use the teaching materials NSF money called forth. But portions of the mathematical community were ready to try out the teaching of applications and NSF money made experimentation easier.

4 Conclusion

Today, applications of mathematics have a foothold in undergraduate mathematics, and are a frequent part of the discussion of undergraduate mathematics curricula. This is quite different from the situation in the middle of the 20th century when applications were mostly absent. The new foothold arose partly through social factors, providing an interesting case study of how the teaching of undergraduate mathematics in the U. S. can respond to the social context in which it operates. Social factors present in the 1970's that were probably important included the difficulty Ph.D's had in finding jobs, the plunging numbers of undergraduate mathematics majors, and the unusual receptiveness to change which took hold of the U. S. in that era.

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