A historical overview of analysis exams in Romania

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ABSTRACT

In the present paper we give a historical review of the mathematical analysis problems given at admission exams to the Faculty of Mathematics of Bucharest, Romania. The University of Bucharest was founded in 1864 and since 1866 till 1962 the mathematics career was part of the Faculty of Physical-Mathematical sciences. Admission exams were introduced in 1947 and till 1976 consisted of a written and an oral examination; since then admission is based on three written exams: algebra, geometry and mathematical analysis. The Universities had the authority to elaborate the admission subjects, however between 1980 and 1990 these were established centrally, by the former Ministry of Education. We present a qualitative evaluation of the analysis problems given at the admission exam between 1947 and 1995. Our analysis is made along three aspects: coverage of the analysis curricula by exam problems, problem types and problem difficulty. The study reveals five problem types, usually present in an analysis exam. These problems gave a fair coverage of the curriculum. Their difficulty varied over time, but after 1980 problems became more algorithmic. One of the most interesting conclusions is that, in some periods, exams seemed to reflect more political purposes than educational ones.

1 Introduction

High stakes testing is a central theme for (often quite heated) discussions among students, parents, educators and officials. Beyond the public interest stirred by the subject, research also has its interest. One of the research lines analyses particular aspects of the testing - like policies (Heubert and Hauser, 1999); question difficulty (Fisher-Hoch and Hughes, 1996) or impact on teaching practices (Jesper, 2006). Another research line consists of comparative analysis of exams in different countries or subjects (Dossey, 1996). A third line can be identified as the historical analysis of the exams (Madaus et al., 2003; Karp, 2007). Our interest lies in a historical analysis of the mathematical exam problems and not in the ways in which testing was carried out. Schubring (1988) mentions two benefits of historical analysis. On one hand, it can shed light on the “administrative history” of the exams, by tracing changes in official directives and school curriculum. On other hand, it allows identify the “real history” that actually occurred in schools. Under such consideration, an analysis of the problems administered at exams can reveal subtle changes on the view of what it should be an examination and how it should be carried our. As Karp (2007) stated “purely mathematical problems offered on exams can be interpreted as manifestations of specific social views, and the changes in their types and structures can be seen as expressions of—or at least wishes for—social changes”.

In the present paper we give a historical review of the mathematical analysis problems given at admission exams to the Faculty of Mathematics of Bucharest, Romania. The University of Bucharest was founded in 1864 and since 1866 till 1962 the mathematics career was part of the Faculty of Physical-Mathematical sciences. Admission exams were introduced in 1947 and until 1976 consisted of a written and an oral examination; since then admission is based on three written exams: algebra, geometry and mathematical analysis. It has to be said that each Faculty has its own admission exams and the nature of the problems given at these exams reflect the specialty of the institution.

Therefore, as far mathematics problems are concerned those given at the admission to the Mathematics career were the most difficult ones. The Universities had the authority to elaborate their subjects, however between 1980 and 1990 these were established centrally, by the former Ministry of Education (Rogai and Modan, 1996). In Romania, in December 1989 the communist regime came to an end, meaning also that Universities regained their authority regarding admission policy.

Exams for entering an educational institution were common in that period; there was an exam for high school and another one after two years (to continue high school studies). The number of available places at these institutions was established centrally and usually remained the same over long periods, so there was a considerable competition between students. The number of students competing for a place at a University fluctuated during the years and from faculty to faculty. It has to be mentioned the mathematics curricula showed small changes during the analyzed period.

We present a qualitative evaluation of the analysis problems given at the admission exam between 1947 and 1995. Our analysis is made along three aspects: coverage of the analysis curricula by exam problems, problem types and problem difficulty. In the first part we present an overview, after which we examine the evolution of each problem type and, in the fourth section, present a more detailed analysis of sequence problems. We end the paper with conclusions and future lines of research.

2 General overview

Mathematical analysis it taught in the last two years of high school and the curricula contains all elements from sequence limits to integral applications. We found that since the separate analysis exam was introduced, this, it presents an almost standard structure with each problem corresponding to large sections of the high school material (the same can be said about algebra exam). Typically, analysis exams consisted of five problems: a sequence problem, a study of derivability /integrability / continuity of a given function, graphical representation of a function, proof of an inequality (by applying monotony properties of the functions) and primitive or integral calculations. These problems give a fairly good coverage of the curricula.

From time to time, an unusual problem appears in the exam (like exception from the above mentioned exam-structure). By unusual we mean that techniques used for the solution or problem formulation are not very common. These cases were isolated and didn’t repeat in consecutive years. An interesting episode in the exams occurred between 1965 and 1972, when problem fields (Pehkonen, 1992) were common. In these cases the exam consisted from 2 or 3 problems with many sub points related to each other. The clear disadvantage of this exam type is that if someone does not know the very first question to answer, he would, probably, not be able to continue through.

A second observation is that problem of the same type increase somehow in difficulty over time. An illustrative case for this is the case of *graphical representation* problem type. Such problems appear from 1960, mostly employing trigonometrically functions in their expression; however it is only in 1967 when for the analysis the second order derivate is required. Problem expressions stars to complicate slowly, first by the use of parameters (mostly in 60-71), then by using other types of expressions (fractions of polynomials, etc.) Later on, all combinations are present.

Third, in the period of 1980-1990, when problems were proposed centrally, there is a change in the style of the exam problems. At the beginning of this period, problems went back to the basics, their solution consisting mostly of maximum two steps with direct applications of theorems, criteria or lemmas. During this period, exams consisted, almost invariably, of five problems with their types repeated over the years. A characteristic of these exams is that they put more accent on algorithmic performance then on mathematical understanding and situation analysis.

Fourth: as far difficulty is concerned, there were variations in the overall difficulty of the exam, but also in the difficulty of some particular problems. An especially interesting period is the one framed by 1970 and 1973 when the overall difficulty of the exam is considerable. Then a period of more accessible problems comes, followed by the period of the 80's marked of almost routine problems. However, has to be underlined that until 1990 it was necessary for the student to solve problems beyond the textbook. One who would only know the problem types presented in textbooks could not pass the exam. Textbooks give basic knowledge, but could not treat the use of this knowledge in all situations. Problem collection books are specially designed not only for training on already known problem types, but also to introduce new ones. There was also a social aspect for the need to use problems beyond textbooks. Until 1990, only public Universities existed with a limited number of places. Fluctuations in the number of applicants were mainly due to national politics in the public health sector. Meanwhile having normally 1 to 3 applicants per place until 1985, in 1986 there were 9 pupils competing for a place. Another type of explication could come from the long tradition of mathematics in the country. Rumania was between the first organizers of the international mathematics Olympiad and for the decades organized local, county and national level contests. Also, journals on mathematics - addressing not only mathematicians, but also alumni from secondary level up - were published regularly since late 1800's. Schools promoted participation in mathematical contests, organized by educational authorities or specialized journals and, also, often counted with *mathematics clubs* as extracurricular activity. All these factors contributed to the fact to consider problems beyond textbooks as natural part of the exams. However, after the change of the communist regime there was a gradual reduction in interest for mathematics. On one hand, new professions were offered and, on other hand, a reorientation in professional choices occurred when the once centralized market was opened up for free competition. Mathematics, as study option, falls out of interest leading some faculties to struggle to fulfill the available number of places. Some universities opted to gradually reduce the complexity of admission exams. Subsequent curricular modifications are another factor that contributed to the changes in admission problems. The analysis curriculum was significantly reduced, not only in terms of extent but also in concepts. Altogether, these circumstances are reflected on the admission problems after 1991: less and easier problems.

3 Problem types in analysis exams

The main types of problems are: sequence problems, graphical representations of functions, derivability and continuity of functions and equations / inequalities that require the use of functions in their solutions. These are broad categories, each of which includes several subtypes of problems (see table 1).

Graphical repr.	Deriv. /continuity	Equality/ Inequality	Sequence problem
The function's expression is given	Compute derivate, continuity, limits	Equations with parameters	Sequence explicitly specified
The function's expression must be identified from properties or has to be modified	Find some object related to the initial function: inverse, properties	Inequalities	Sequence is not given (for example: general term is given as the solution of an equation)
A function has to be proposed	Primitives: existence and computing	Integral equalities	Sequences described by properties
Maximum-minimum values of the function	Integrals: compute the value of a given integral		Construction of seq.
	Limit of functions		Term = function
	Volume, length of a segment		
	Asymptotes		

Table 1. Main problem types and sub problems

3.1. Graphical representation

This type of problem was present almost in all of the analyzed admission exams. As a matter of fact, after 1990, began a period of easier problems and graphical representation was slowly replaced by the study of some particular property of a function. The most common problem subtype was the one in which the function's expression is given. In overall, these problems have an average difficulty level. As far solution is concerned, there is a well defined algorithm that helps determining the elements required for a graphical representation; therefore, additional difficulty could arise from algebraic complexity related to the derivation process and equation solving. In the same time, these problems cover a considerable part of analysis curriculum by employing derivates and limit computing. Over years, function's expression got more complicated by involving new concepts in their expression. It is interesting to mention that only in 1967 was necessary to use the second order derivate of a function in order to sketch its graphics. In the period of 1969-1972 parametrical definitions dominated and it was only in 1983 oblique asymptotes had to be identified. After 1990, the function's expression became very simple. In the same time, most problems do not require anymore the graphical representation, but rather the study of some isolated property. In table 2, we give four examples of such problems.

1967	Consider the function: $f(x) = \arcsin \frac{x-1}{\sqrt{2(1+x^2)}}$. Represent its graphic.
1976	Consider the function $f(x) = \arcsin \frac{2x}{1+x^2}$, $\forall x \in \Re$. a. determine the set of points where *f* is derivable b. represent graphically *f*.

1983	Study the variation of the following function: $f: R-\{1\} \to R$ given by $f(x)=\lvert x-2\rvert \cdot e^{\frac{1}{\lvert x-1\rvert}}$. Put in evidence the oblique asymptotes.
1994	Represent graphically the following function: $f:(0,\infty)\to R$ given by $f(x)=\ln x/\sqrt{x}$.

Table 2. Four examples of graphical representation problems

Only in six problems, the expression was not explicitly given or the problem's expression asked for a transformation. In conclusion, the use of this problem type suggests that its purpose was to verify if students know / perform the algorithm correctly.

3.2. Continuity / Derivability / Integrability

Another recurrent type of problem is the one about derivability, continuity of functions and their primitives. One of the reasons behind the frequency of such problems can be the fact that there are several theorems and criteria available to solve them, so in some terms the student - before solving the problem – has to choose the adequate strategy. Most of these problems involve functions defined in "branches", so it is enough to verify properties in the connecting points of the intervals. In general, these problems have average difficulty, since the solution method is quite known from the textbooks. In some years, the function's expression was not given, but had to be computed as composition of other functions. Still, the difficulty didn't pass over average level. A subtype of these problems deals with integrals; however, the main difficulty source in such cases comes not from the integrals, but algebraic complexity of the expression. Another subtype is represented by problems that ask for determining a function with given properties. These properties are in terms of derivates or continuity. The problem often is formulated as an open ended problem, asking the student to build an example or counterexample. The use of such problems was characteristic to the years from 1971 to 1974. However, after 1980, a stronger accent is put on computing integrals or function limits, instead of analyzing the problem. For a brief period, after 1990, theoretical issues are requested: definitions and equivalent criteria for continuity. It is for the first time that definitions and theorems were required. It remains a question to debate if such problems are good exam problems. In conclusion, none of these problems goes beyond the textbook, however in overall, problems that ask for determining a function's expression are more difficult. A second conclusion is that after 1990 problems were more algorithmic (table 3).

1966	Consider the function $f(x)=\arcsin\frac{1}{x}-\arccos\frac{1}{x}$. f) study the derivability of the function $F(x)=\lvert f(x)\rvert$ in the point $x=\sqrt{2}$
1971	Let $f:[a,b]\to R$ be a function such that any value of f is irrational. Can be such a function a continuous one?
1986	Compute the following limit: $\lim_{n\to\infty}\left(\lim_{x\to 0}(1+tg^2x+tg^2 2x+...+tg^2 nx)^{\frac{1}{n^3x^2}}\right)$.
1991	Define the notion of a function's continuity in a point and, then, give other definitions, equivalent to this.

Table 3. Four derivability / continuity problems

3.3. Equation / Inequality

The study of an equation' roots with the help of functions represented a type of problems present mostly in the period of 1963-1968. Afterwards, the only occasions in which such problem type was administered were in 1974 and 1987. The particularity of such problem type is that it requires the study of the variation of a function. Later on, graphical representation of functions gained terrain and it is possible that this led to the elimination of this type of problems. Another reason for not using anymore such problems in analysis exams is the introduction of a separate exam for calculus, in 1976. Therefore, the study of equations felt in the domain of algebra and became part of the algebra exams.

On other hand, problems about inequalities to prove are used, for the first time, in 1971 and on a more consistent (almost yearly) basis, from 1980 on. These problems are simple ones, since requires only a brief analysis of a functions' behavior. Integral equalities are similarly very simple, asking just for a derivation process or the use of some particular property of the function.

In all the encountered situations, the functions to derivate and study were simple ones, making this type of problems one of the most simple ones.

3.4. Sequence problems

These problems appear in each year's exam from 1969 on. Most of these problems have the sequence specified, or by giving the explicit expression of its generic term or by defining a recurrence relation between the terms. It is interesting to remark that problems after 1980 are, almost exclusively, explicitly given numerical sequences. It seems that the accent in this period is on a more algorithmic performance, therefore sequences are given by recurrent relations (computable generic term) and the most common solution involves the use of the Weierstrass' theorem. The decade before, many of the problems were open ended and often the generic term was defined by integrals or functions composition. In the same period, constructing specific sequences (with given properties) and the study of ones specified by properties was common. These problems were on overall difficult ones, going much beyond the level of textbook problems. Also, in the decade of 80's some problem types were also beyond textbook material (like second order linear recurrences). However, there is a significant difference between these two cases. In the 70's one needed to study complementary material in order to have knowledge about how to analyze the behavior of a sequence and to dispose of a more complete set of sequence examples. Later on, one appealed to problem books in order to get new algorithmically knowledge. Interestingly enough, after 1990, sequence problems are much simplified: it is enough to apply mathematical induction on the terms in order to find the sequences. In some years, sequence problems were omitted; instead, one had to compute the limit of what could be the generic term of a sequence (table 4).

1977	Determine the convergence of the sequence $(x_n)_{n\geq 1}$ with the following properties: a) $(x_{2n+1} - x_{2n})(x_{2n} - x_{2n-1}) < 0,\ \forall n \geq 1$; b) $\lim\limits_{n->\infty}(x_{n+1} - x_n) = 0$
1985	Consider *a* and x_0 two real numbers and $(x_n)_{n\geq 0}$ a sequence defined by: $x_n = \frac{2a \cdot x_{n-1}}{a + x_{n-1}}$, n>=1. Study the convergence of the sequence for a>0 and x_0 >0.

1991	a) Prove that any convergent sequence is bounded. b) Consider $x_0 \in (0,1)$ and $x_{n+1} = x_n - x_n{}^3$. Prove that $x_n \in (0,1), \forall n \in N$, that $(x_n)_{n\in N}$ is convergent and compute its limit.
1993	Compute and discuss on the parameter $a \in R$ the following limit: $\lim\limits_{n->\infty} n^{\alpha}(\sqrt{n^2+1}-n)$.

Table 4. Sequence problems from different years

The overall, as well, the detailed analysis of the problem types shows considerable fluctuations in problem difficulty and problem formulation. In general terms, problem difficulty is not related to the involved concepts, but more to the needed solution techniques. Problem formulation varies from open ended problems, through construction problems, to completely defined problems ("demonstrate", "compute", and so on). Since during the analyzed period there were no significant changes in the calculus curricula, we interpret the fluctuations as an expression of personal, subjective view of a candidate student's profile as held by the proponent of the problems. Once, centrally (by the Ministry of Education) elaborated exams were imposed, there is an increase in algorithmic problems suggesting that acquiring procedural knowledge had priority over an inquiry, analysis based approach to mathematics.

4 Analysis of sequence problems

In the classification of the sequence problems we can adopt several point of view: the form of the question, problem content and the solving strategy. Each of these points of view can offer an interesting analysis about the historical changes in exam problems.

4.1. The form of the question

Based on this criteria we identified three problem types.

Open problems: Examples: „decide whether the sequence is convergent"; „decide the convergence of the sequence with the following properties..." In these problems, the student has to decide first if the answer if Yes or No, and then argue his answer.

Imperative problems: Examples: „Prove the convergence of the sequence"; „Compute the limit"; „Define a recurrence relation". In these problems, the student knows from the very beginning what he has to prove / verify and how.

Open/imperative problems: are the problems that require a discussion based on parameter values, but the attention is not explicitly drawn on the aspect. For example, in the problem „Compute $\lim\limits_{n->\infty} \frac{a^n + b^n}{a^{n+1} + b^{n+1}}$ " where *a* and *b* are strictly positive". In this case, we have a middle situation: the student has the question clearly formulated (*compute*), however there is uncertainty (he has to realize that a discussion is necessary and to know on which values has to be done).

	1960-1969	1970-1979	1980-1989	1990-1996
Number of open ended problems	0	7	0	1
Number of Imperative problems	3	7	26	7
Nr. of open-imperative problems	0	2	5	0

Table 5. Statistics of different sequence problem types

It can be observed that the number of the first two category problems is equilibrated in the decade of 1970-1980. In the next decade, open ended problems disappear, being replaced by imperative problems. There are three possible explanations for the situation.

A first one appeals to changes in the textbooks. At the end of the 70's, new textbooks were published. In mathematical analysis, these were coming after a period of almost 20 years during which the same textbook (with improved reprints) was used. The new textbooks, independently of the level or discipline, had a more theoretical treatment of the topic (from the point of view of rigor, making constructions from the very basis, but more difficult from didactical pint of view). We hypothesize that the new textbook, by promoting an axiomatic treatment, determined – on problem level – a switch from open-ended problems towards imperative ones.

A second factor could be the introduction, in 1980, of a unique exam. It is reasonable to think that centralization would have determined an orientation towards a kind of problem formulation that would allow verifying a basic level of preparation of the student.

The third explanation is political by its nature. The decade of 1980-1990 has been, for Romania, the period of culmination of the communist power. Probably the whole social context, in which the freedom of speech has been drastically limited, influenced in problem formulation. An imperative formulation corresponded, in fact, to the way in which daily tasks were expressed (sometimes, without even being conscious).

4.2. Solution techniques

Under this view we have identified four major types of techniques.

Problems that require a computing algorithm: Example: „Compute $\lim_{n \to \infty} n^k (\sqrt{n^2+1} - n)$". We consider this problem of algorithmic type if some experience in limit computing is enough to solve it.

Problems that require specific convergence theorem: Example: „Prove that $(a_n)_{n \ge 2}$, $a_n = \left(\frac{2}{2!} + \frac{7}{3!} + \ldots + \frac{n^2-2}{n!} \right) + \frac{n+2}{n!}$" is convergent. In order to solve such problem, it is essential to know a certain result, specific to mathematical analysis.

Problems that require the analysis of subsequences: Example: „Study the convergence of the following sequence $(x_n)_{n \in N}$ having $x_n^2 < 4, \forall n$ and $x_n^2 < x_{n-1}^2, \forall n$". These problems require studying the subsequences of the sequence in order to solve it.

Recurrent sequences: these problems ask the study of sequences given by recurrences.

These four categories are not disjoint. For example, in order to study recurrent sequences one has often to analyze the behavior of subsequences. For the studied problems, we opted to include the problem into a category based also in the solution strategy. A brief statistical analysis of the problems under this second point of view revealed a predisposition towards algorithmic and recurrence problems. After 1980 there were no more problems that would require of subsequences in their solution process. We found two possible explanations for this situation.

In the first place, recurrent sequences allow testing more knowledge. If we are interested about the handling of the algebraic apparatus, we can ask about the expression of the generic term; if we are interested in analytical competences then we can ask about the convergence of the sequence. Beside, these problems extended the limits of evaluation, since in textbook there were just few problems of this type, so the student had to study problem books in order to know how to solve them.

On other hand, recurrent problems were common in the decade of 1980-1990. We consider that their intensive use in exams was related to the development of informatics, in expansion in Romania of that period.

4.2. Necessity for transfer

In sequence problems, the transfer between algebra and analysis is an important stage in solution. In some of the problems, this is realized at a minimal level. In the same time, in some others, algebraic knowledge and techniques turn out to be essential. Example: „Compute $\lim_{n\to\infty}\frac{1+2+3+...+n}{n^3}$”. This problem can turn out difficult for someone who doesn’t know the formula for the sum. The interest of a categorization based on the criteria of transfer becomes, thus, important. There are two remarks to be made.

Until 1976, sequence problems were interdisciplinary: in all of them the contribution from algebra was significant. After that year problems are more self-content. A reasonable explanation for this situation is the introduction of a separate analysis exam that year. After 1976, sequence problems vary less in type, even if they fluctuate in difficulty. The preference for recurrent sequences could be also seen as a way of distancing between the domains: not only by the formulation of the problem but also by the techniques necessary for solution. In such situation, many recurrence problems can be solved without having to obtain the generic terms of it.

5 Conclusion and future work

The purpose of the present study was to analyze problems given at the admission exams to the Mathematics Faculty in Bucharest, Romania. Our study focuses on problems of mathematical analysis, proposed between 1966 and 1995 (analysis problems before 1966 were very rare). Detailed analyses were presented on problem types and for the particular case of sequence problems.

Usually, an analysis exam consisted of five problems of four types: *graphical representation, derivability, equations and sequence problems.* The undertaken analysis shows that requested items and problem difficulty varied over time. For example, in the decade of 1980 the accent moved onto algorithmic performance in the detriment of problem situations analysis.

In case of sequence problems we proposed three classification criteria: question formulation, technique employed in solution and the type of transfer needed for solution. Based on these criteria, we found that in time there is an involution from the open ended problems, to deterministic ones and to algorithmic problems. Such phenomena can be explained by several factors.

Some of the reasons are related to the educational politics in Romania of those years. The definition of new priorities for university level education, changes in the purpose of the high school education, but also communist centralized control can explain the dominance of algorithmic problems and imperative formulations typical for the decade of 1980.

Other causes seem to root in curricular changes. The study of mathematical analysis, as a specific topic in high school, initiated in 1958. Since then, till 1999, there were only two textbooks (the first one having numerous editions, but with minor modifications). It is possible that the introduction of the new textbook, at the beginning of 80's, have imposed a new vision on exam problems.

The introduction of an unique exam per country (in 1980) can be another cause of the observed phenomena. Before that date, and also after 1990, each faculty decided the problems to be given at exams. An unique exam had to attend a big number of candidates, with varying performance from one faculty to another. Such situation could be a factor in adopting a formulation that allow verifying if the candidate has the basic preparation in the first place. By this we can also explain the growth in imperative problems and predilection towards algorithmic ones. It worth to mention, that algorithmic problem also allowed an uniform qualification of the exams, reducing differences between qualifications given by different revisers.

The above analysis is extremely important in understanding the tendencies in mathematical assessment, with a special focus on the qualities required from students and the influence of politics.

In the future, we propose to analyze problems from Mathematical Olympiads in order to see if similar tendencies can be identified. We consider that an analysis of the problems given at local phase can highlight the way in which teachers saw the exams (at local phase problem was proposed by local teachers). Such an analysis could be contrasted with the official views that admission exams suggest to be in place.

A second line of future development is on relating student's perception on the priorities regarding their mathematics preparation to teachers' perception on the correct way to prepare their students.

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