

# **Mathematics and music in scientific dissemination context: an epistemological/historical approach**

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## **Introduction**

The aim of this presentation is to show the use of an exhibition as didactical instrument -- in museum “Estação Ciência” of the University of São Paulo -- to approach historical and epistemological aspects of the relationship between mathematics and music. The exhibition attempts, amongst other goals, to create an environment for professionals involved with teaching/learning processes; to experience activities that will enhance their ordinary knowledge and activities in the scientific area and its connection between mathematics and music; promote the opportunity to experience situations historically contextualized involving simultaneously mathematical, musical and physical concepts through direct or analogical reproduction of experiments that can trigger the interest and curiosity of the visitors by the study and development such concepts further; etc. The analogical thought underlying the experiments of the exhibition plays an important role inasmuch as it stimulates the establishment of conjectures as well as the creation of new problems and ways for solutions stimulating the creativity. In establishing a context for teachers to experience activities of culture and extension to their curricular activities, one values the history of mathematics particularly concerning its relationships with music, making accessible the historical context in which such relationships emerged.

The exhibition is composed by 8 modules distributed in rooms that should convey the central ideas of the relationship between mathematics and music: 1) Motivational and relevant ideas for the grasp of the acoustic musical concepts; 2) The experiment of the monochord: ratio vs. musical intervals and the mathematical systematisation of the scale; 3) The Renaissance: the birth of music as an experimental science; 4) Scales and temperament: the emergence of incommensurable magnitudes in theoretical music; 5) Harmonic Series/Fourier Series; 6) Consonance and dissonance: from the arithmetic symbolism to a physical conception; 7) The Music of the Spheres; 8) From the speculative mathematics to the empirical mathematics: a scientific revolution in music. Each module contains mechanical and/or multimedia interactive devices, and make use of multiple representation interconnected through texts and/or hypertexts with a historical and epistemological approach which aim the development and strengthening of the mathematical and musical concepts associated to each room.

In the following, I will approach brief comments about some rooms presented in the exhibition, which explore more mathematical situations related to music, and how the

exhibition is related to enhancing the mathematical creativity of students as well as how it can stimulate mathematical heuristic through mathematical-musical situations.

### **Motivational and relevant ideas for the grasp of the acoustic musical concepts**

The exhibition starts with an introductory room in which the visitant is encouraged to understand how mathematics took part in modelling different music-acoustical phenomenon throughout history. It presents some posters and experiences that lead students to understand mathematically acoustical-musical phenomenon. In order to do that, he/she are instigated with diverse questions to motivate the understanding of the situations involving mathematics and music approached during the exhibition. Some of such questions are: What is the relationship between mathematical ratios and musical intervals?; what make you recognize the voice of anyone?; What is timbre?; Could you imagine a relationship between timbre and mathematics?; Why exists some sounds that suit and other that not suit? Can you guess if an interval suits from the mathematical ratio underlying it?; Why some sounds combining sound give the sensation of one ? What is consonance?; What is the relationship between Pythagoras and string instruments?; Could you imagine the relation between musical scales and mathematics? Between musical scales and calendars? Could you imagine how to use mathematics to build a scale? Have you heard about tuning scales? What is temperament? Is there any relationship between temperament and irrational numbers? The planets emit sounds?

### **The experiment of the monochord: ratio vs. musical intervals and the mathematical systematisation of the scale**

Presenting the history of the so-called Pythagoras' experiment of monochord, this room allows the reproduction, as well as the exploration and experimentation of its main consequences -- ratios 1:2, 2:3 and 3:4 underlying the perfect consonances octave, fifth and fourth -- culminating with the construction of the Pythagorean scale. Pythagoras' discovery through the monochord experiment casts light on a large number of discussions about musical theory that have mathematical ratios as their foundations. In such an experiment, Pythagoras did not just establish a correspondence between musical intervals and ratios of a string, but connected musical consonances to simple ratios as well as compounding ratios to compounding intervals. It is quite probable that, for cultural reasons, the Greek mathematicians, along with his contemporaries and predecessors, conceived of the theory of ratio as a generalization of music, inasmuch as the proprieties of strings and comparisons between pitches, as well as calculations related to such magnitudes through ratio and proportion, were an important part of mathematics from the Pythagoreans until Euclid (Grattan-Guinness, 1996, p.367).

From an educational point of view, there are several themes on the relation between mathematics and music and especially between mathematical ratios and musical intervals which are explored starting from this experiment. In particular, the room explores basic problems involving ratios and musical intervals, as well as the idea of compounding ratios (Sylla, 1984, p.19), the latter making use of a mechanical device as well as a

software that allows the visitant making the compounding ratios and at the same time hearing the correspondent composition of musical intervals. Both experiments allow students to establish conjectures on musical contexts making use of mathematical calculations as well as on mathematical contexts through manipulations with musical intervals, making music a heuristic context for conjecturing in mathematics, opening the possibilities to solve problems with operations with ratios, and encouraging students with musical competences to create and solve problems in ratio contexts, by means of viewing the correspondent situations in musical contexts.

Such situations lead also to the impossibility of adjust fifth's with octave's cycles associated to the impossibility of solving a mathematical equation. Such an impossibility results in the Pythagorean comma, that represents the "seed of imperfection" with similar nature to the irrationality in mathematics and to the impossibility of precise adjust between natural cycles of the moon and the earth around the sun in astronomy (Abdounur, 1999, pp.12). Such observations establish a link with the room "scales and temperament", since a Pythagorean comma and the others generate from the attempts of construction of the musical scales based in rational numbers represent the motor to the development of diverse temperaments, what propitiate reflections related to the meaning of irrational numbers.

### **Scales and temperament: the emergence of incommensurable magnitudes in theoretical music**

This room exhibits a crisis in mathematics/music which has a relevant heuristic power in the development of the trajectories of mathematics and music for instance with the temperament. The room is shaped in three parts disposed geometrically in parallel, displaying the situations that stimulated the crisis mentioned above, such as the impossibility of commensurate the side and the diagonal of the square and the impossibility of fit an integer number of fifth's cycles into an integer number of octave's cycles, as well as the impossibility of adjust an integer number of lunar months into an integer number of years. In this room students are invited to reflect on the formation of the diatonic scale and have the opportunity to verify the "harmony" between mathematical, musical and physical aspects that converge to constitute these musical notes. This room intends also to present the underlying ratios to different temperaments for the diatonic scale. Originating with the Pythagorean scale and consequently, with the Pythagorean comma – which allows a link with the room "The experiment of the monochord: ratios x musical intervals and the mathematical sistematization of the scale - -- the idea of temperament pass through Architas of Tarento in the IV century a.C (Fallas, 1992), Gioseffo Zarlino in the XVI century (Zarlino, 1562,1571,1588), Descartes (Buzon, 1987) and Mersenne Buccolini, 2003) in the XVII century. Establishing here a link with the room "The Renaissance: the birth of music as an experimental science", the dynamic that lead to the construction of such an idea is related with the concept of logarithm, developed since the beginning of the 16th century and also approached in this room in musical contexts as a essential tool for conceiving temperament.

This room also makes possible to the visitor to hear serially the relations between notes for each temperament accompanied with the correspondent underlying ratios, as well as compare such notes with those produced by the ratios of the Fourier Series. In order to make easier the perception of small differences among such temperaments, a software presented in this room allows the parallel hearing of a same note in different temperaments .

### **Harmonic Series/Fourier Series**

Equipped with mechanical-electronical devices such as oscilloscopes, spectrum analyzer, strings, etc, as well as musical instruments and musical synthesizers, the main goal of this room consists of express the relation between harmonic series of a musical note and Fourier Series of the function which represents such a note. Making use of an oscilloscope, this room presents analogically the experiment of Galileo (Cohen, 1984), which associates the musical sound to a periodical function and extends it promoting a multi-representation of the same sound by means of different devices: an oscilloscope that translates in time the wave's form; a spectrum analyzer that makes possible representations in frequency of the sound, whose partial terms are associated to their harmonics, establishing a "link" with the room "The Renaissance: the birth of music as an experimental science", concerning vibrating strings in two halves, three thirds, four fourths, etc. Reproducing the experiment of Wallis, it provides shapes associated respectively to the partial terms of the Harmonic Series of a sound produced by the vibration of the whole string, representation in time of the different senoid harmonics of a sound, etc.

In front of the electronic-mechanical device, the visitor can vary parameters of the sound such as pitch, intensity and timber, observing the consequence in each representation, such as respectively frequency, amplitude and the shape of the electronic signal expressed by the oscilloscope; in the fundamental frequency, in the amplitude of the harmonics and in the shape of the harmonics of the representation provided by the spectrum analyser

### **Consonance and dissonance: from the arithmetic symbolism to a physical conception**

In considering the mathematical-acoustical aspects of the concepts of consonance/dissonance, this room intends to present different mathematical conceptions of consonance built throughout history: ratio of simple numbers by Pythagoras, two or more notes that sound as one by Archytas (Fallas, 1992); coincidence of harmonics of two or more notes by Zarlino associated to the MCM between their frequencies (Abdounur, 1999, p.211); coincidence of pulsations generators of the related notes by Galileo (Cohen, 1984, p.90); proximity of the harmonic root in the Fourier Series; physiological conception of Helmholtz (Helmholtz, 1954) etc. This room offers the simultaneous hearing of two or more notes, whose combination one intend to evaluate the consonant/dissonant character, considering mathematical-acoustical aspects and making

use of the visualization of its harmonic shape according to the Fourier Series, which allows a link with the room “Harmonic Series/Fourier Series”.

### **From the speculative mathematics to the empirical mathematics: a scientific revolution in music.**

This room aims at showing up mathematical/musical aspects that contributed to a re-structuration of theoretical music in Renaissance which passes from a mathematical-cosmological-speculative vision to a mathematical-physical-empirical one, by means of a historical-epistemological interpretation of such a phenomenon at the light of concepts as Scientific Revolution of Thomas Kuhn (Kuhn, 1962) and Epistemological obstacle of Gaston Bachelard (Bachelard, 1996). It presents basically panels that shows aspects related to such a change, including the emergence of sound as vibrational phenomenon, the concept of consonance as coincidence of vibrations besides others that contributed to the strong transformation in the theoretical music from a speculative character to an experimental one. It propitiates an opportunity of reflection about the questions presented before from a wider point of view.

### **Last Remarks**

The use of an exhibition as a tool to approach didactical and historical aspects of the relationship between mathematics and music provides the possibility of approach mathematics in a wider sense inasmuch as music can be used throughout the exhibition as a way to establish and express analogies between both areas, stimulating the habit of finding an analogous field to an approached mathematical area considered, in which one have more familiarity to explore and make conjectures, which can be potentially transferred to the original mathematical domain in order to solve and create problems. Such an experience stimulates the use of the analogical thought, which plays an important role in the mathematical creativity.

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