THE ROLE OF THE REFEREE IN THE HISTORY OF MATHEMATICS

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ABSTRACT

The aim of the Royal Society of London, like in every scientific society, is to "organise" the Science, within the certain, in time and place, context. In order to do that, the "society" uses the judgment procedures and structures for every single theme it is submitted. Within the Royal Society of London the referees decide about the works which they are going to publish. The decisions of the referees are submitted to the authority of the council for the final approval. In fact, the decisions of the referees determine the "co-optation"[1] or the exclusion of proposes in the sciences field.

In particular, my study examines the judgment structures, e.g. the opinions of the referees, for the Cayley's paper: *An introductory Memoir upon quantics* published in 1854 in the *Philosophical Transactions of Royal Society.*

Introduction

The subject to be called "theory" must be based on a kind of authority or authorities from a scientific institution, e.g. Royal Society of London or Science Academy of Berlim and so on.

I study the relation between the subject itself and the "administration" of it. I use the term "administration" to mean "the structures are responsible for the subject publication", e.g. responsible to put the subject in the social[2] order.

It is clear the prestigious scientific institutions are responsible to decide about the "knowledge"[3] and they do it usually based on a collegiate (Weber, 1978) and it is clear too the recognized subject has its history or histories. What is not clear is the relation between judgment of a subject and a subject itself.

The judgment of a "pseudo-knowledge" imposes a "decision" upon it. The "decision" is linked in a regular sense to a collegiate (Weber, 1978) that means the discordant opinions "can" take part on the decision about what will be or not "knowledge", therefore the final decision must be submitted to the "committee" [45]. Sometimes the vote is needed. Obviously many structures, we usually call "institutions", they play in different places at the same time, and consequently there is the probability to appear divergent results about the same subject.

In the present work I fix Royal Society of London and Cayley's paper published on 1854 in *Philosophical Transactions* as an example of a scientific structure and a publication. This paper was the first from a series of memoir on quantics where the author developed part of the invariant theory.

I will focus to the theory, to the author and the judgment ways.

I can not answer why this theory come out in 19th century and how mathematicians in Britain turned upon it, it is not the case to describe or to find the motivation, but to

[1] D'Ambrosio (2005).
[2] Social order: when the subject becomes recognized, e.g. it does not belongs anymore only to the author, it concerns to the public sphere.
[3] It is important mention that "knowledge" must be considered only that one is submitted to be judged.
[4] Filter System (D'Ambrosio, 1989).

understand how much the "administrative" order[6] play on the recognition process[7], consequently play upon a "reason" that makes a subject becomes a theory.

Tony Crilly and Karen Hurgen Parshall have worked on the subject. Tony Crilly published Cayley's biography in 2004 and Karen Hunger Parshall published Sylverster's biography in 2006. In historical sense I base on those authors's work.

The new aspect in the present study is: "administration of the knowledge". I have based on the Statutes of Royal Society of London in regards to the publication in *The Philosophical Transaction*.

On the Philosophical Transaction of Royal Society of London

If it is reasonable to expect a connection between the subject and the "administration" of it, I would say that the referee is a connection.

According to the Statutes of Royal Society of London (1847) about the Publication Papers (Chapter XIII) it is written: "…to whom the consideration of the Publication of such papers as shall have been read, or communicated to the Society (…) shall from time to time be referred" (Chapter XIII, I, p.21). "The majority of the said Committee (…) shall be at liberty to call in their assistance (…) any Fellow (…) who are knowing and well-skilled in the particular branch of the Science to which the Paper under deliberation relates (Chapter XIII, III, p.21). The method of proceeding:

> *The entry in the Journal-book of the Society, relating to any paper, upon which the opinion of the Committee is to taken, shall be read; or, if any Member shall desire it, the paper itself shall be read; after which the question shall be put, whether that paper shall be printed in the Philosophical Transactions, unless the opinion of the majority of the Committee shall be in favour (sic.) of adjourning the consideration of it to a subsequent meeting. The question shall always be decided by ballot, and by majority of votes; but if there be an equality of votes, the further consideration of the question shall be adjourned to the next meeting of the Committee, when that question shall be taken into consideration before any other business is entered upon: and, if at the second balloting upon the same question, there be still an equality of votes, it shall be determined in the negative* (Chapter XIII, IV, p.22).
>
> The referee responsibility is to decide about the value of the subject. Mr. Graves for example, he declined on Cayley's paper, his justification:
>
> From want of such familiarity and from want of time I find myself obliged to decline the reference of the committee and accordingly return the paper without reporting on the expediency of publishing it in the Philosophical Transaction; for such a report would rightly be taken to involve more than my general confidence in Mr. Cayley's eminent ability as an accurate and profound mathematician.
>
> I can state however that (with the exception of speculations an "ideal spaces of any number of dimensions") the paper is difficult of comprehension from its abstract character (…) Persons who have studied Mr. Cayley's other works and the congregate papers of Mr. Sylvester and Mr. Hermite would be likely as I imagine to regard it as blending great simplicity with great generality I believe that Mr. Spottiswoode and Prof. W. Thomson would be able to estimate the value of the present paper (…) with respect to the fecundity of the principles which it develops (4).

[6] It belongs to the British mathematics network from 19th century.
[7] Value-sign (Baudrillard, 1972).

He considered as appropriate names Mr. Thomson or Mr. Spottiswoode to be referees. He criticized the use of word quantic as not necessary. As a consequence of Graves's declination Cayley's paper was postponed[8].

Based on the referee authority, Thomson declared "I have no hesitation in recommending that it should be published in the Transaction" (1).

Thomson was involved with physics research as possible to verify in the Philosophical Transactions of Royal Society and Graves name is related to the octonions, according to Parshall (2006) "Arthur Cayley and John Graves almost immediately extend Hamilton's ideas to eight-dimensional octonions that satisfy neither the commutative nor the associative laws of multiplication" (p. 187).

Graves (1854) mentioned about the invariants, facients and covariants in his report:

In ready over Mr. Cayley's paper I find that he does not (as I had unopposed when I first heard the word) treat of some new kind of function (...) that he denotes by the name Quantic any rational and integral function considered with respect to dimension or degree.

In discussing certain kinds of notation functions and their algebraic transformations the employment of this nomenclature and notation become almost indispensable, (...) in the present instance I doubt the [necessity][9] for introducing the term quantic.

This paper supplies extremely general methods of finding the "covariants" of functions and reduces to (...) the principles in which depend (...) of hyperdeterminants, invariants, discriminants.

Then in few readers who cant grasp the full meaning of it without being familiar with previous researches of the same kind (4).

Thomson (1854) reported that he contacted the author about one or two points that would make it easier to the reader, that makes us to consider Thomson's familiarity with the subject.

I have written to the author on one or two points where it appeared to me that some additional explanation on illustrations (...) [to] make his work more easily read.

Should the Committee decide on printing his paper, it will not be necessary to make any delay on this account unless Mr. Cayley (...) makes application to have his manuscript returned. The addtitions which I suggest may I believe be inserted without inconvenience on the proofs (1).

The Statute of Royal Society (1847) proposed "…in the particular branch of the Science" is necessary to be assisted by the "Fellow (…) who are knowing and well-skilled in the particular branch of the Science to which the Paper under deliberation relates" (Chapter XIII, III, p.21).

According to the documents quoted and the Statutes of Royal Society Thomson was who knew that "…branch of the Science" (1847, p.21) because he accepted to judge the paper. Graves did not give his opinion. The conclusion is based only on the formal rules of Royal Society, e.g. Statutes and referees. It is not possible to assert that Thomson was more capable than Graves to judge "An introductory memoir upon quantics". However if we focus to the work of Graves and Thomson, it sounds that Graves fits more to Cayley's

[8] Archive. *Royal Society of London.* Minutes of the Committee of Papers: 1853-1854. June 15 and 29, 1854. CMD 90d.

[9] I used "necessity" as a synonymous for the word in the document that I did not manage to identify. "Necessity" is at least a proper synonymous if we consider Crilly (2004)'s analysis from the referees of Cayley's work.

work than Thomson. Disconcertingly the documents attest that Thomson used his authority and guaranteed the publication and Graves declined on it.

Cayley's paper was communicated to the *Journal Book of the Royal Society* on 4 of May of 1854 (5).

The Committee of Papers[10] on 11 of May of 1854 refereed Cayley's paper, on 15 and 29 of June it was postponed and finally on 26 of October it was classified "to be printed".

The *Introductory memoir upon quantics* was published in the volume 144 of *Philosophical Transactions* on 1854.

Value

The subject itself called invariant theory was legitimated by the scientific societies at 19th century and nowadays by the history of mathematics. I have focused to the procedure to have the paper published and I exemplified with Cayley's paper.

Nowadays this paper is part of the history of mathematics, consequently we are able to see it as a retroactive effect from the fact it became a theory (Zizek, 1992). In any case we are able to see as well that nobody was or is free of the procedure to publish, that imply in a judgment, that imply to take in account the "name"[11] of the author.

In fact the referee authority is able to make "a study" be or not "valued" (Foucault, 1977). The most people will be able to read only the referee opinion, e.g. the "value". If A consider the work valued, B considerer the "word from A", then B consider the "work" valued.

The subject itself and its "administration" are linked with the "value upon the theory" and, of course, to the names related to it. Both are connected by the referees. But the "value" itself, what does it means? The answer would be "you see it in the work". Who would doubt it? But who is able to "see" it? The most people must believe in "the word" of few people that is supposed to know the subject. The proper "knowledge" of a "…particular branch of the Science" (Statutes of Royal Society, Chapter XIII, III, p.21) is not a responsibility of ordinary people or even all the mathematicians, it is just addressed for a few of them. The "value" appears as the published work.

The "administrative" focus is described above. There is another focus I will develop, e.g. the History of mathematics particularly the invariant theory history. The "value" in this context appears related to the subject itself, despite, we can not forget the objects that the historians work are mainly papers from important journals, consequently based on the "administrative order".

The Invariant Theory

Now the "value" is directed to a Boole's (1841) paper as the beginning of the development of invariant theory in Britain. I fixed Boole (1941) based in Parshall (2006, p. 188; 1996, p.23) and Crilly (1986).

According to Parshall (2006,1996) and Crilly (1986) George Boole's paper published in 1841 entitled "Exposition of a General Theory of Linear Transformations" was the motivation to Cayley and Sylvester. Sylvester

… had an even greater motivation to concentrate anew of these topics, given that Cayley – taking his mathematical lead from George Boole – had done and was doing

[10] Royal Society of London. *Minutes of the Committee of Papers: 1853-1854*. 11 May, 15 and 29 June, 26 October, 1854. CMD 90d.

[11] Graves' referee (4).

related work in the so-called theory of forms, and given that Sylvester's earlier work had clear and direct applications to that involving theory (Parshall, 1998,p.23).

George Boole (1815-1864) was an influence on the young Cayley. Cayley published two papers one in 1845 and another in 1846 "…which have been regarded by generations of mathematicians as laying the foundations of the subject" (Crilly, 1986, p. 242).

According to Crilly (1986) in 1850 Cayley worked on the series of memoir on quantics. He turned to the development of invariants from partial differential equations. One of his reasons was "…to calculate linearly independent and *irreducible* invariants and display them in tabular form" (Crilly, 1986, p.242).

According to Parshall (2006) James Joseph Sylvester and Arthur Cayley, especially during 1850 to 1856, build the groundwork for the British theory of invariants. In fact, they "…put the Britain on the international map algebraically" (p.186) (…) "… the area of algebraic research that dominated the British mathematical scene was invariant theory"(p.188).

Usually *Invariant Theory* is linked to the Arthur Cayley's name and the series of memoir on quantics.

An introductory memoir upon quantics

In Cayley's manuscript[12] paper:

The term quantics is used to denote the entire subject of rational and integral functions and of the equations and loci to which these give rise. The word quantic is an adjective meaning of such a degree and may be used substantively the noun understood being (unless the contrary appear by the context) function; so used the word of the plural quantics.

The quantities or symbols to which the expression degree refers or (what is the same thing) in regard to which a function is considered as a quantic will be spoken of as 'facients'. A quantic may always be considered as being in regards to its facients homogeneous since to render it so, it is only necessary to introduce as a facient unity or some symbol which is to be ultimately replaced by unity; and in the cases in which the facients are considered as forming two or more distinct sets, the quantic may, in like manner, be considered as homogeneous in regards to each set separately.

According to Crilly (2004) Cayley used the word quantic to mean homogeneous polynomial (rational and integral function). "This introduction was criticized by the referees, but Cayley felt a need for replacing the traditional term. His formal style is, in part, due to his legal training"(p.201).

By defining the extent of the word "quantics", his opening paragraph parallels the preamble of Cayley's legal draft on "Family Settlement". Both placed limits on the projected field and explained terminology and both are analytical in tone. He saw the need to place limits and make a start on its reorganization. (...). For Cayley, "quantics" were fundamental in invariant theory as "quantities" were in ordinary algebra (p.201).

Cayley (1854) remodeled "…the whole basis for invariant theory" (Crilly, 2005, p.201), in fact the purpose was to characterize "…invariants and covariants as algebraic forms annihilated by differential operators" (p. 201).

Graves (1854) as referee considered "…the paper is difficult of comprehension from its abstract character" (RSL,RR.2.42, 15 of May of 1854) and according to Crilly (2005)

[12] Royal Society of London. *Philosophical Transaction.* London: April 20 1854, PT. 49.3.

Graves "…believed the introduction of the new terminology unnecessary". Boole considered the term quantic poor on etymological sense, which express nothing (p. 202).

Cayley (1854) defined quantic of degrees *m, m'..* in the sets (x,y,..),(x',y'..) &c. was represented by a notation:

$$(*)(x, y,..)^m (x', y',..)^{m'} ..),$$

*where the mark * may be considered as indicative of the absolute generality of the quantic; any such quantic may of course be considered as the sum of series of terms* $x^{\alpha}y^{\beta}..x'^{\alpha'}y'^{\beta'}..$*, &c. of the proper degrees in the different sets respectively, each term multiplied by a coefficient; these coefficients may be mere numerical multiples of single letters or elements such as a,b,c.., or else functions (in general rational and integral functions) of such elements* (p.247).

Edwin Bailey Elliott published in 1895 *An Introduction to the Algebra of Quantics* made a compilation that include Cayley's work. He defined as Cayley the quantics, he used examples as the binary p-ie :

$$a_0x^p + pa_1x^{p-1}y + \frac{p(p-1)}{1.2}a_2x^{p-2}y^2 + \frac{p(p-1)(p-2)}{1.2.3}a_3x^{p-3}y^3 + ... + pa_{p-1}x\, y^{p-1} + a_p y^p$$

Cayley's notation (1854):

$$(a_o, a_1, a_2, ... a_p)(x, y)^p$$

According to Elliott (1895) the *invariant* of the quantic above will be a function in the coefficients $a_o, a_1, a_2, ... a_p$ if hold an identity of the form:

$$f(A_o, A_1, A_2, ... A_p) = \phi(l, m, l', m')f(a_o, a_1, a_2, ... a_p)$$

The coefficients mentioned above comes from a linear transformation

$$x = lX + mY,$$

$$y = l'X + m'Y,$$

They are substituted in the variables of the quantic $(a_o, a_1, a_2, ... a_p)(x, y)^p$ and the quantic obtained is: $(A_o, A_1, A_2, ... A_p)(X, Y)^p$.

The typical example is the binary quadratic where $b^2 - 4ac$ is an invariant which it is the discriminant. It is an invariant because ac $ac - b^2$ hold the identity:

$$AC - B^2 = (l^2m'^2 - 2ll'mm' + m^2l'^2)(ac - b^2)$$

According to Crilly (2004) :

If Cayley's formality is evident in the "Introductory Memoir", so too are glimpses of his intuition. He took due note of Hermite's law of reciprocity, but he explained it in his own way, as mathematicians do, that Hermite's proof was identical with his own (p. 202).

Cayley (1854):

21. The number of the really independent covariants of a quantic (*)(x,y)^m is precisely equal to the order m of the quantic, e.g. any covariant is a function (generally an irrational function only expressible as the root of an equation) of any m independent covariants, and in like manner the number of really independent invariants is $\overline{m-2}$*, we may, if we please, take* $\overline{m-2}$ *really independent invariants as part of the system of the m independent covariants; the quantic itself may be take is one of the other two covariants, and any other covariant as the other of the two covariants; we may therefore say that every covariant is a function (generally an irrational function only expressible as the root* *of an equation) of* $\overline{m-2}$ *invariants, of the quantic itself and of a given covariant* (p. 256).

To find covariants of a given order and a given degree in the coefficients Cayley (1854) said:

…we may form the most general function of the proper order and degree in the coefficients, satisfying the prescribed conditions as to symmetry and weight: such function, if reduced to zero by the order of operations in question, will on account of the symmetry, be reduced to zero by the others of the operations in question; it is therefore only necessary to effect upon it, e.g. the operation $\{x\partial_y\} - x\partial_y$ *and to determine if possible the undetermined coefficients in such manner as to render the result identically zero* (p.257).

Cayley (1854) affirmed that "…if an invariant be expanded in a series of ascending powers of the first coefficient *a*, and the first term of expansion is known, all the remaining terms can be at once deduced by mere differentiations"(p.257).

In fact, my intention is not to present a new result from Cayley's work, but to include new documents, e.g. referees, to analyze the process of history recognition. However I defined quantic, invariant and I gave an example as an illustration of the subject itself. To take a wide view of its history is appropriated Crilly and Parshall's work.

Final considerations

I have spoken about two sides, "administrative", e.g., Royal Society of London and "the subject itself", e.g. *Introductory memoir upon quantics*. I have been based on Crilly and Parshall's work to introduce the reasons and the intensity around invariant theory in 19th century. The names nowadays are part of the history of mathematics are the same that were authorized to judge the subject itself. In fact, the power to judge, e.g. authorization from Royal Society, is connected to the same names are in the history of mathematics, e.g. Cayley, Sylvester, Boole, Graves, Hamilton and so on.

Disconcerting results were found: Graves declined on Cayley's paper and Thomson assumed it.

The "subject itself" that is treated by the history of mathematics follow another direction, and it can be disconcerting if related to the formal or authorized procedure to legitimate the research, e.g. the publication.

I have established the relation between the "administrative" order and the subject itself based on the documents related to a specific place, Royal Society of London, a specific name, Arthur Cayley and a specific subject, invariant theory.

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