Mathematics Goes Ballistic
Benjamin Robins, Leonhard Euler, and the Mathematical Education of Military Engineers

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ABSTRACT

Efforts to understand the trajectory of cannonballs are an interesting example of the tensions between practical and theoretical knowledge. Although Galileo's 1638 parabolic trajectory was an important theoretical step forward, field gunnery practice was guided by the Tartaglia's 1537 "mixed motion" model through the eighteenth century. In 1742, Benjamin Robins published *New Principles of Gunnery*, and revolutionized the study of ballistics by suggesting the projectile's initial velocity – not its range – was the appropriate parameter to consider in accounting for air resistance. In 1745, Leonard Euler produced a German translation of *New Principles*, adding his own extensive commentary. Euler's annotated translation quickly became a standard text – Napoléon Bonaparte studied ballistics from the French version – thereby influencing the education of artillery officers and, eventually, of all engineers. This paper surveys the contributions of Robins and Euler to mathematical ballistics theory, examines the influence of these developments on the education of eighteenth century military engineers, and considers the extent to which the history of ballistics theory supports the thesis that the drive to reconcile practical knowledge with theoretical knowledge can be a critical element in shaping mathematical theory. We close with comments concerning the use of this history in today's classroom.

1 Introduction

Like everyone else, mathematicians are dependent on the support of society and its institutions for their livelihood. So it is not surprising that mathematical practitioners of the early modern period undertook a variety of tasks of potential military value within the political context of expanding European nation states. For example, Turgot – in his capacity as Minister of the Marine and Controller General of France – wrote to Louis XVI on 23 August 1774:

> The famous Leonard Euler, one of the greatest mathematicians of Europe, has written two works which could be very useful to the schools of the Navy and the Artillery. One is a *Treatise on the Construction and Manœver of Vessels*; the other is a commentary of the principles of artillery of Robins … I propose that Your Majesty order these to be printed[1].

The two works Turgot proposed for translation had been completed by Euler during his Berlin period, in 1749 and 1745 respectively. As indicated by Turgot (and by its title *Neue Grundsätze der Artillerie aus dem Englischen des Herrn Benjamin Robins übersetzt und mit vielen anmerkungen*), the latter was itself a translation into German (with added commentary) of an English text, *New Principles of Gunnery*, published by Benjamin Robins in 1742. The continuation of Turgot's letter further illustrates the value European nation states placed on mathematics, as well as the financial benefits which mathematicians might reap from that state interest:

[1]As quoted in Truesdell (1984), p. 337.

> It is to be noted that an edition made thus without the consent of the author injures somewhat the kind of ownership he has of his work. But it is easy to recompense him in a manner very flattering for him and glorious to Your Majesty. The means would be that Your Majesty would vouchsafe to authorize me to write on Your Majesty's part to the lord Euler and to cause him to receive a gratification equivalent to what he could gain from the edition of his book, which would be about 5,000 francs. This sum will be paid from the secret accounts of the Navy[2].

Elsewhere, Turgot asked whether "this Euler, who lets nothing slip by unnoticed, might have treated in his mechanics or elsewhere" the most advantageous height for wagon wheels[3]. In addition to ballistics, navigation and wagon design, mathematicians of this period were called upon to assist with problems in cartography, cryptography, fortification design, and hydraulic engineering. But to what extent – if any – was the drive to reconcile practical knowledge with theoretical knowledge in these areas a critical element in shaping mathematical theory?

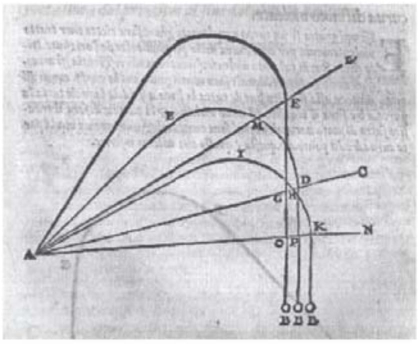
In this paper, we consider this question within the context of the history of ballistics theory, focusing particularly on the contributions of Robins and Euler. We then examine the educational context of eighteenth century military engineers in order to evaluate the suggestion of some historians that the new stage of ballistics theory launched by the work of Robins and Euler provided justification for increasing the level of mathematical studies for military engineers. We close with some comments concerning the value of this (hi)story for today's mathematics classroom.

2 The Early History of Mathematical Ballistics Theory

Efforts to understand the trajectory of a cannonball are particularly interesting as an example of the tensions that can exist between practical (military) knowledge and theoretical (mathematical) knowledge. For instance, although Galileo's 1638 discovery of the parabolic projectile trajectory was (and is) viewed as an important mathematical step forward, the actual practice of field gunners continued to be guided by the less sophisticated "mixed motion" model presented by Tartaglia in his 1537 *La nova scientia*. In fact, this remained the case until well into the eighteenth century for reasons to be considered below.

The mathematical construction of a cannon trajectory developed by Niccolò Tartaglia (1499 – 1557) was founded "on perfectly conventional academic ideas" of the sixteenth century; namely, the violent and natural motions of Aristotelian dynamics and the ideal constructions of Euclidean geometry[4]. Tartaglia represented the initial (violent) portion of the motion by a line with slope determined by the angle of the cannon and the final (natural) portion of the motion by a vertical line, with a circular segment on which the highest point of the trajectory occurs joining these two lines (Figure 1).

[2] As quoted in Truesdell (1984), p. 337.
[3] Truesdell (1984), p. 338.
[4] Hall (1983), p. 116.



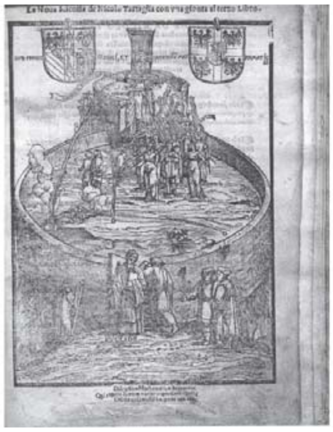


Figure 1
Tartaglia's Model of Projectile Motion

Figure 2
Cover Page of Tartaglia's *Nova Scientia*

Although the resulting image seems absurd to a modern eye, the cover page of *La nova scientia* makes it clear that Tartaglia realized his mathematical model was not a realistic depiction of a cannon trajectory (Figure 2). In *La nova scientia* and in his 1546 *Questi et Inventioni Diverse*, he evean stated that no part of the trajectory is actually a straight line. But as Büttner et al argue:

> The challenge was not to produce a realistic image of the trajectory, but rather to relate such an image to other knowledge available on projectile notion[5].

"Other knowledge" in this case was the experience-based practical knowledge of gunners, observations such as the fact that the range does not monotonically increase with a decreasing angle of elevation, but reaches a maximum at some non-zero angle of elevation relative to the horizontal. Tartaglia's model was successful in accounting for this particular aspect of practical knowledge, and implied that the maximum range is attained at a 45° angle of elevation.

Tartaglia further claimed his theory would permit the compilation of a table of ranges which field gunners could use to adjust the angle of elevation as need for a desired range. Although Tartaglia himself never compiled such a table, he described in *La nova scientia* the construction of a gunner's quadrant for measurement of cannon elevation on a twelve-point scale, a devise later revised to a conventional 90° scale and used in conjunction with range tables throughout the eighteenth century.

In traditional accounts of ballistics history, no further theoretical developments occurred until Galileo Galilei (1564 – 1642) announced that, absent air resistance, the trajectory of a projectile is parabolic. The first published account of Galileo's theory actually appeared in *Lo Speccio Ustoria*, a 1632 text on trajectories written by Galileo's student Bonaventure Cavalieri (1598 – 1647). Galileo's own more detailed development of this theory appeared in his *Discorsi e dimostrazioni matematiche: intorno a due nuoue scienze* of 1638[6]. More to the point, the parabolic trajectory was the result of an extended

[5] Büttner et al (2003), p. 13.

[6] Another of Galileo's students, Evangelista Torricelli (1608 – 1674) attempted to make Galileo's theory more useful to gunners; see Swetz (1995), pp. 97–100.

effort – by Galileo and others[7] – to produce a theoretical model of projectile motion that captures all elements of practitioner's knowledge. Interestingly, Galileo's efforts in this regard led him to link the trajectory of a projectile with the shape of a hanging curve (or catenary)[8]. Within the context of Aristotelian physics, Galileo proposed that both curves are subject to the same combination of violent action (along the horizontal) and natural action (along the vertical). This assumption led to his erroneous conclusion that – like the trajectory of a projectile free of air resistance – the curve of the catenary is a parabola, a conclusion that was not corrected until the 18th century.

In an unpublished manuscript, Galileo proposed that a chain might be used to allow gunners to predict the angle of elevation needed for a specific range. Even had his model of the catenary been correct, however, this suggestion could not have fully met the practical needs of gunners of his day – nor could his (theoretically correct) parabolic model of projectile motion. By neglecting to account for the effect of air resistance, the theoretical predictions simply could not be reproduced in practice[9]. Thus, despite the fact that Galileo's parabolic theory was incorporated into seventeenth century artillery texts, field artillerists continued to employ the Tartaglian mixed motion conception, especially for projectiles at high velocity for which the effect of air resistance is more pronounced. Clearly, this was not just due to a lack of correspondence between the parabolic model and an actual trajectory – Tartaglia's model did not correspond to actual trajectories either. But as Swetz remarked, a gunner who is primarily interested in finding range as a function of cannon barrel may find that it works just fine to simply model a trajectory by something as basic as a right triangle[10]; such a model was, in fact, proposed by Daniel Santbech in 1561. Furthermore, "something like Tartaglia's picture of mixed motion persisted among these artisanal gunners because it provided a ready guide practice in the field" until well into the eighteenth century, during which time gunnery practice remained an art (rather than engineering)[11].

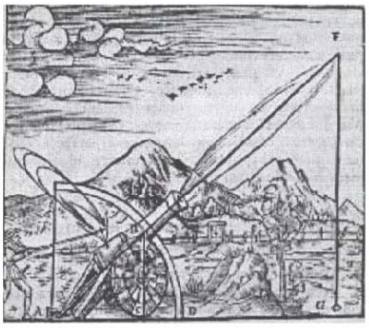


Figure 3 - Daniel Santbech's Model of Projectile Motion

In short, given the available artillery technology and the uncertainties which characterized actual field practice, the approximations provided by the mathematically less precise models fitted well with the practical needs and the working conditions of field

[7]For a discussion of Thomas Harriot's work in this area, see Büttner et al (2003), pp. 21–23.

[8] See Büttner et al (2003), pp. 23–24. Note especially their claim (p. 24) that "[Galileo's] erroneous conviction that he could demonstrate the equivalence of parabolic trajectory and the chain provided him with a strong argument form claiming, against a firmly rooted and traditional doctrine, that the projectile trajectory has a symmetrical shape, a claim he could not prove otherwise."

[9] Hall (1969), p. 42, suggests this was due in part to the greater precision of Galileo's new mechanics, which allowed for tests of a greater order of accuracy than previously.

[10] Swetz (1995), p. 10.

[11] Alder (1997), pp. 93–94.

artillerists. Galileo himself was well aware that air resistance was a factor, but did not attempt to incorporate it into his theory. Like all (mathematical) theories, his was limited by the explanatory resources available to him. Prior to the development of calculus and Newtonian physics, theorists simply did not have the means to account for air resistance. Yet Galileo's work contributed to an intellectual transformation which replaced the Aristotelian by Newtonian physics, and Euclidean geometry by calculus. Subsequent attempts to solve the problem of air resistance in projectile motion were made by Christian Huygens, Johann Bernoulli, and Isaac Newton. By assuming air resistance was proportional to the square of the velocity, however, their models required the solution of non-linear differential equations. This (new) mathematical obstacle was further complicated by the unavailability of accurate numerical data for use as initial conditions and coefficients in these equations. A breakthrough in this latter regard came, at last, in the work of English mathematician and military engineer Benjamin Robins.

3 Benjamin Robins[12] and *New Principles of Gunnery*

Benjamin Robins was born in Bath in 1707 – the same year as Euler[13] – to parents described as "practicing Quakers" who were neither well-educated nor financially well-off. Early recognition of Robins' talent for mathematics led to him becoming a pupil of Lord Henry Pemberton, editor of the third edition of Newton's *Principia*. At the age of 20, Robins' first publication *Demonstration of the 11th Proposition of Sir I. Newton's Treatise on Quadratures* appeared in the *Philosophical Transactions of Royal Society*; he was elected to the Royal Society the same year. During the 1730s, Robins developed an interest in military engineering. He also joined in the British debate over the foundations of fluxions initiated by Berkeley's *The Analyst* [14] and the British critique of continental differential techniques. His contributions to these debates earned him further acclaim in the Royal Society. Among Robins' publications in defense of Newtonian fluxions is a 1739 pamphlet including three treatises, the first of which was entitled *Remarks on Mr. Euler's "Treatise of Motion"*. In its preface, Robins declared:

> In the first of the treatises I design to examine, the author has unfortunately followed the principles of his calculus with so little caution, as even to contradict Euclide himself[15].

Robins' detailed critique of Euler's 1736 *Mechanica* repeatedly faulted Euler for his use of algebraic calculations as a means to solve problems via difficult equations – calculations which Robins found obscure and logically inconsistent to solve problems he believed were more simply approached via geometrical techniques.

Despite a promising beginning towards an academic career, political missteps soon halted Robins' progress. In 1739, he published three pamphlets criticizing the Whig government of Sir Robert Walpole, later to find his way barred to an appointment as

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[12] Although not well known among mathematicians, Robins has received a fair amount of attention from engineers in recent years for his work in ballistics and its accompanying contributions to aerodynamics and experimental fluid mechanics. This is due in part to a series of biographical articles published in engineering journals by William Johnson, F.R.S. A partial list of Johnson's articles appears in the references.
[13] Unlike that of Euler's, the exact date of Robins' birth is unknown.
[14] See Boyer (1959), pp. 229 –235.
[15] Robins (1739), p. iv of preface.

mathematics professor at the new Royal Military Academy in Woolwich, founded by the Walpole administration in 1741. Robins began his research in ballistics around this time, in part to bolster his application for this position. In 1747, he was awarded the Royal Society's most prestigious award, the Copley Medal, for this work. After years of striving for professional advancement, Robins accepted a position with the East India Company on 8 December 1749. H began his duties as Engineer General in India on 14 July 1750, only to die there of a fever on 29 July 1751.

Robins' ground-breaking contribution to the study of ballistics was the suggestion that the initial velocity of a projectile – and not its range – was the appropriate parameter to consider in order to account for air resistance. Critical of experiments conducted by the British Ordnance Department, Robins conducted his own experiments using an instrument of his design; his ballistics pendulum remained the most popular ballistics instrument through mid-nineteenth century[16]. Designed to measure projectile velocity, the ballistics pendulum consisted of a flat wooden plate swinging from a pendulum, which was in turn was suspended from a rigid tripod. A ribbon attached to the wooden plate allowed Robins to measure the first deflection of the pendulum following the impact of a bullet on the wooden plate. Using Newtonian mechanics and Huygens' theories of pendulum motion, Robins was then able to calculate the projectile's velocity. By measuring weight and velocity of a bullet at different ranges and applying Newtonian principles, he was also able to employ data obtained with the ballistics pendulum as a means to measure air resistance. The ballistics pendulum thus provided Robins, and later Euler, with numerical coefficients for these two fundamental parameters in the differential equations for projectile motion, thereby providing the means to reconcile (mathematical) theory with practical (experimental) results.

Robins published his results in his *New Principles of Gunnery* of 1742, a relatively short 150-page text consisting of two chapters preceded by a historical preface. Another important feature of *New Principles* was the connection it established between internal ballistics (e.g., gunpowder explosions) and external ballistics (e.g., trajectory). Robins treated internal ballistics in Chapter 1, where he sought to solve the problem of determining a projectile's muzzle velocity as a function of its mass, gunpowder quantity, and barrel geometry. The answer to this question was considered relevant to issues of eighteenth-century cannon design. Thus, for example, Robins was cited as an authority (by both sides) in the French eighteenth century military debate concerning the optimal length of a cannon barrel[17]. Robins confirmed his theoretical solution of this problem in Proposition VII experimentally using data from the ballistics pendulum[18].

In Chapter II of *New Principles*, Robins treated problems in external ballistics, including the question of the magnitude of air resistance encountered by earthbound projectiles. Again, both theoretical and experimental reasoning were employed to support his conclusions. In particular, Robins' experimental results showed the claim of Huygens and Newton that air resistance was proportional to the square of the velocity was true only at lower velocities. Once projectile velocities approach the speed of sound, his data

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[16] Robins also developed an instrument called a *whirling arm* to measure air resistance at velocities too low for the ballistics pendulum. See, for example, Johnson (1992c), pp. 305–306 for a description.

[17] Alder (1995) treats this debate at length. For a summary , see Steele (2005), p. 295, or Steele(1994a), pp. 207–212.

[18] Steele (2005), p. 86, argues that Robins' resolution of this problem represents an early connection between 19th century engineering thermodynamics and 17th century mechanics.

suggested, air resistance increased by a factor of three. Robins derived an equation of air resistance as a function of velocity which agreed with these experimental results. In doing so, he provided theoretical justification for a fact well-known to artillerists: a parabola satisfactorily models projectiles with low velocities (such as mortars), but fails at higher velocities.

In Proposition VI of Chapter 2, Robins formally announced the demise of Galileo's model:

> The Track described by the Flight of Shot or Shells is neither a Parabola, nor nearly a Parabola, unless they are projected with small Velocities.

His supporting data included, for example, a range predicted to be 16 miles according to Galileo, but which was in fact less than 3 miles. The final proposition of Chapter 2 considered the possible cause of observed lateral deviations from a parabolic path. Robins (correctly) hypothesized the cause as random spinning of the projectile, an explanation known today as the Magnus effect or Robins-Magnus effect. Perhaps because Euler disagreed (claiming instead that lateral deviations were due to irregularities in the projectile), Robins' explanation was not accepted until the work of Gustav Magnus (1802 – 1870) a century later. In an unpublished manuscript of 1747, Robins advocated the use of rifled barrels with egg-like (rather than spherical) bullets as a corrective for this effect[19].

Although Robins did not include field gunnery tables in *New Principles*, he presented a table to the Royal Society in 1746; a paper submitted to the Royal Society in 1750 provided experimental evidence of his table's usefulness for mortar and cannon of the time. The table itself remained unpublished until 1761, a decade after his death. Perhaps surprisingly, its use required only knowledge of Galileo's ballistics theory, and no knowledge of calculus. Even in *New Principles*, Robins had minimized the use of calculus. Dr. James Wilson, Robins' close friend and the editor of his collected works following his death, suggested Robins did so in a deliberate effort to gain patronage by articulating the benefits of his work for artillery practice, while minimizing its mathematical complexity[20]. Despite the failure of his *New Principles* in this regard – which Dr. Wilson attributed to on-going personal opposition caused by his polemics against Walpole – Robins did gain recognition for his work from another of his polemic targets: Leonhard Euler (1707 – 1783).

4 Ballistic Contributions of Leonard Euler

The general details of Euler's biography are well-known. Less well-known is his first work in ballistics theory: *Meditatio in Experimenta explosione tormentorum nuper institua* written in 1727. Frequently mentioned as the first known appearance[21] of the symbol *e*, this manuscript remained unpublished until 1862. Despite its apparent lack of appeal to

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[19] See Johnson (1992c), pp. 313–314 and Steele (1994a), pp. 115–116.

[20] Steele (1994a), quotes Wilson to this effect on p. 102, and discusses Robins' derivation of his gunnery table on pp. 109–110.

[21] The first published occurrence of *e* appears in Euler's 1736 *Mechanica*, the work which was the subject of Robins' 1739 critique. A partial translation is included in Smith (1959), pp. 95–96. The Latin version of *Meditatio in Experimenta* appears in Volume 14, Series 2 of Euler's *Opera Omnia*, along with his other works on ballistics.

the 1727 Russian artillery corps which conducted the seven experiments in question, *Meditatio* provides evidence of Euler's early interest in ballistic studies. The experiments were also observed by Daniel Bernoulli, who reported on them in his *Hydrodynamica* of 1738. The model of gunpowder explosion employed by Bernoulli was also used by Robins, but without acknowledgment by Robins of Bernoulli's earlier work. This fact led to one of Euler criticisms of Robins' work in his German translation of that work.

Euler's translation of *New Principles* appeared in 1745, four years after his move to Berlin. Secondary sources typically report that Euler completed the translation in response to an inquiry from Frederick regarding the best available artillery text[22]. In 1744, Euler wrote to Frederick requesting permission to complete a translation of *New Principles* which he had already begun. Although Frederick's written response to this letter is missing, one surmises the requested permission was granted; Euler not only translated Robins' work, but added his own extensive (250 page) commentary on the (150 page) original work. In his commentary, Euler corrected Robins' analytic errors and provided a critique of certain of his assumptions, as well as treating new topics not considered by Robins. But his commentary was not entirely critical; throughout, Euler praised Robins' work, and especially its contributions towards obtaining the necessary experimental results for development of a valid model. Regarding the ballistics pendulum, Euler declared it "one of the most ingenious and useful discoveries in artillery[23]." Even more telling is the implicit praise Euler grants Robins in his acceptance of Robins' experimental results. Perhaps even more than Robins, Euler realized the need to reconcile the theoretical results of analysis with practical results which could now, thanks to Robins, be obtained through experimentation.

Yet the true trajectory of a projectile remained elusive. Euler acknowledged the difficulty in his first remark on Proposition VI of Chapter 2, concerning the failure of Galileo's model:

> Here, again, Mr. Robins gives us farther expectations of discovering the real track of a canon ball. It is some years since his book was published, and nothing more, that I know of has appeared on the subject. This enquiry is so difficult, that the author was in the right to require a longer time to complete it[24].

Euler also required a longer time to resolve this problem, and his attempts to simplify the analysis in *Neue Grundsätze* met with limited success. Not until 1753 did he publish *Recherches sur la veritable courbe que decrivent les corps jettés dans l'air ou dans un autre fluide queloconconque,* the first complete analysis of equations for ballistic motion in a resisting medium.

Unlike his earlier work, the second-order differential equations used by Euler in 1753 have a familiar look to them – undergraduate physics and engineering students would certainly recognize them. Abandoning an earlier attempt in *Neue Grundsätze* to more

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[22] The source of this report appears to be the eulogy of Euler read at the Imperial Academy of Sciences of Saint Petersburg by Nicolaus Fuss on 23 October 1783.

[23] As quoted in Johnson (1992b), p. 672. Euler concluded this quote with the assertion "Whatever had been delivered on [artillery experimentation] before [Robins], was not only uncertain but erroneous."

[24] All English quotations from *Neue Grundsätze* in this paper are taken from the Brown (1777).

accurately model air resistance as proportional to both the square and the fourth power of velocity, Euler now assumed proportionality to velocity squared only, obtaining equations representing the range, altitude and velocity during both the ascending and descending portions of the trajectory. These equations predicted that, in the limiting case of a certain constant *n* generated by integration, the descending branch approached a vertical asymptote and the ascending branch an asymptote of positive slope. This prediction, reminiscent of pre-Galilean models of projectile motion, was in keeping with the observations of both eighteenth-century Prussian artillery experimenters and sixteenth-century gunners that the descending portion of a trajectory has greater curvature than the ascending portion. Euler calculated values of *n* for 18 angles of the ascending branch asymptote, and then used the trapezoidal rule to produce numerical tables for both branches of the trajectory at one particular combination of *n* and muzzle velocity. Used together, the ascending and descending branch tables permitted the calculation of the length, range and flight time for the entire trajectory for this one particular ‘species’ of trajectory. As a demonstration of the ability of analytic methods to solve difficult physical problems of practical interest, the work was a masterpiece.

4 The Education of Military Engineers in the Eighteenth Century

Although Euler produced tables for only one species of trajectories, at least two sets of tables existed by 1764[25]. The first, a set of tables for 36 different species developed by Prussian artillery officer Paul Jacobi, was presented to the Berlin Academy only to be lost after Jacobi’s early death in 1758, despite efforts by Euler to locate them. The second, a set of tables for eighteen different species developed and published by infantry officer Hennig Friedrich, Graf von Grevnetiz in 1764, proved sufficiently valid for mortar fire that they remained in use at least until World War II.

In his doctoral work, Brett Steele provides extensive documentation in support of his thesis that the ballistics work of Robins and Euler ushered in a “military revolution” in Europe not only in ballistics research and the design of military hardware, but also in the education of the military engineers and artillery officers who provided the accompanying “software” to operate that technology[26]. Kenneth Alder also notes the influence of *Neue Grundsätze* on both ballistic practice and educational practices in eighteenth century France, where Robins’ scientific ideas first spread to the educational institutions[27]. By 1751, an (unpublished) French translation of Robins by Charles Le Roy was circulating within the French scientific-military community. A series of ballistics experiments based on *New Principles* were conducted by Patrick d’Arcy (1725 – 1779), a French army captain and close friend of Le Roy, the results of which also appeared in 1751[28]. The first published French translation of Euler’s annotated translation was completed in 1783 by Jean-Louis Lombard (1723-1794), who served as artillery instructor to Napoléon Bonaparte at Metz. Napoléon himself studied ballistics from the French version *Artillerie*

[25] Steele (1994a), pp. 164–165.
[26] Steele (1994a), pp. 169–246.
[27] Although Alder (1997) agrees with Steele that Robins and Euler together “put ballistics on a new basis” (p. 104), he is also critical of certain of Steele’s historiographical assumptions (pp. 91–92). Note especially Alder’s argument that Steele’s assumptions lead him to overestimate Robins’ scientific contributions.
[28] d’Arcy (1751).

– his 1788 twelve-page summary of the work appears in his unedited papers[29] – and his genius for artillery command undoubtedly altered the nature of modern warfare. But even granting that his "scientific understanding of cannon and mortar fire was an important element in development of his victorious strategy" at the Siege of Toulon, where he first gained national attention in French Revolutionary War[30], Napoléon's frustration with the focus on mathematical theory at the expense of practical skills in French engineering schools was evident by 1802[31].

What, then, led military academies to increasingly emphasize mathematics and especially calculus in the decades following the appearance of *New Principles*, sometimes at the expense of the practical training needed for combat success? That calculus spread throughout the engineering curriculum – first in France, then in England, and eventually the United States and elsewhere – is unquestionable[32]. One is tempted to claim that this effect was, in fact, inevitable. After all, the ability to represent variation within the language of analysis provides engineers with a set of tools the the solution of optimization problems associated with applied science and technology. While acknowledging this, Alder argues that mathematics in general, and analysis in particular, served to define engineering as a profession in other ways as well[33]. For instance, he claims that military educators believed the rigor of mathematical studies encouraged desirable values and habits of thought by "impress[ing] on students the virtues of uniformity and precision[34]". Alder also proposes that mathematics granted social status and authority to those who mastered it, in part through (indirect) association with the innovative ideas of leading Enlightenment *savants*. Alder goes even further, and contends that, as the education of practitioners changed and controlled experimentation replaced experience, the very nature of their practical knowledge also changed.

In short, even if one accepts the nineteenth century mathematization of engineering as an historically inevitability triumph of science, current readings of the historical record suggest that the factors which influenced the education of artillery officers (and eventually all engineers) in the direction of increased mathematical emphasis included not only the success of Robins\Euler in reconciling theory with practice, but also the general military

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[29] Steele (1994a), p. 371.

[30] Ibid, p. 371.

[31] Alder (1997), pp. 310–312.

[32] By 1772, for example, Newtonian fluxions had become part of the mathematics curriculum at the Woolwich Academy. Woolwich mathematics professor Hugh Brown published an English translation of Euler's annotated translation of Robins in 1777. An interesting feature of his translation is its use of Newtonian fluxion notation to represent Euler's analysis.

[33] For Alder's complete argument, see Chapter 2 of Alder (1997). In summary, Alder states (p. 73): "… the analytic mixed mathematics did more than distinguish the artillery engineer from the rude cannoneers he commanded on the battlefield, or from the artisan he directed in the manufactures. It was more than the sign of a practical theory. Analysis associated the engineer with research, innovation, and a dynamic mode of thought. The geometric methods of the seventeenth century had expressed the limits of theory, and hence the limits of destruction. Analysis was accessible, open-ended, and explosive. And at the same time, it tied the creator of technological novelty to his duty to husband the capital of his (royal) employer." Alder also considers the role of the descriptive geometry curriculum developed by Gaspard Monge (1746 – 1818) in establishing a new social order for engineers, as well as its role as a tool in artillery design and as a means to objectify knowledge, in Chapter 4 of Alder (1997) and in Alder (1999).

[34] Alder (1997), p. 67.

concerns[35] of the state, and the particular political concerns of social groups and individuals (including Robins and Euler[36]) associated with the state. Furthermore, if one accepts Alder's argument, then this increased mathematical emphasis altered the very nature of practical knowledge and its relation to theoretical knowledge.

5 Conclusion

But to what extent, if any, was the drive to reconcile practical knowledge with theoretical knowledge in ballistics a critical element in shaping mathematical theory? Any effort to argue in favor of a strong connection in this regard would require a significantly deeper analysis of mathematical texts than has been presented here. But even the general outline of mathematical ballistics history presented here suggests the likely failure of such an argument. The practical success of a theoretical model may have helped to gain acceptance for the mathematical ideas on which they were based, but the development of new tools seems not to have been motivated primarily, if at all, by the failure of previous theoretical models to capture the practical knowledge of artillerists. Rather, the mathematical tools used by theorists at each stage (e.g., Euclidean geometry, fluxional\differential analysis) were already at hand, having been initially developed in contexts other than the study of projectile motion. The influence of Galileo's work on the development of Newtonian physics notwithstanding, the relationship appears to be less direct than a causal shaping of mathematical content.

Does this story belongs in today's classroom? The answer depends on one's instructional goals. With respect to developing the mathematical details of the various models, one must also consider students' technical backgrounds. Although the simpler models of Galileo and Torricelli are accessible with little more than basic physics, geometry and trigonometry[37], Euler's more complicated analysis requires significantly greater technical expertise. There is also the difficult question of whether war-related topics should enter the mathematics classroom. Avoiding them, of course, will not negate the historical record; mathematics and war have been intimately connected in the past, and are even more so today[38]. Furthermore, ignoring their connection may not serve students well in making informed decisions concerning their own participation in either endeavor[39].

For those who do choose to share this story in their classroom, a number of morals can also be drawn from it concerning the practice and development of mathematics, on both the individual and societal level. Even in broad outline, the tale suggests an interplay between practical knowledge and theoretical knowledge far richer than that portrayed by

[35] The etymological origins of the French *ingénier* in the Latin *ingenium*, or engine of war, is another reminder of this fact.

[36] Truesdell (1984) has the following to say concerning Euler and politics (pp. 371–372): "In all of Euler's vast correspondence there is no mention of politics and little reference to social conditions. Evidently one country, government, or party was the same as another for him, provided it allowed free worship in the Protestant faith his father had taught him and the chance to do a mountain of mathematics for a good salary. … While obviously neither a Prussian nationalist nor a Russian one, Euler served both countries with the total loyalty which in those days was regarded as the ordinary, moral duty of a servant to his master. The personal failings of Frederick II as a candidate for God's lieutenant on earth must have been more than obvious to Euler, but it was not those that drove him from Berlin. Rather, he sought a social and financial position worthy of himself and, above all, advancement for his children."

[37] See Swetz (1995) for specific suggestions in this regard.

[38] See Booß, B., Høyrup (1984), Booß, B., Høyrup (2003a) and Booß, B., Høyrup (2003b).

[39] For more on this issue, see d'Ambrosio (1998), d'Ambrosio (2002), Shulman (2002) and Shulman (2004).

the typical textbook story about projectile motion. For instance, the role of diagrams as a mediating factor between theory and practice in the sixteenth century, and the corresponding role played by controlled experimental data in the seventeenth century, sheds new light on this interplay, and especially the need for something beyond both theory and practice in order to overcome gap between them.

This story also adds credence to the view of Rupert Hall that theorists tackled ballistic problems "because these problems were intellectually fascinating and to some extent open to solution[40]." This aspect of mathematical work is one not always revealed to undergraduates, despite the attraction it holds for many working mathematicians. On the other hand, Hall appears to be wrong in declaring "existing military art was incapable of adopting mathematical theory of projectile flight and applying it to practice, nor (so far as one can tell) did it ever attempt to do so, at least before the death of Newton[41]." Here, Robert Merton appears closer to the truth: "The effort to attain mathematical precision in artillery fire was a model for the industrial arts and a link with the current science. In any event, military needs, as well as the other technologic needs … tended to direct scientific interest into certain fields[42]". Another aspect of mathematical practice which the telling of this tale can convey to students is the "force of active tradition in guiding problem choice[43]" discussed by both Merton and Hall in relation to mathematics and ballistics.

Of course, one need not share the theories of historians to convey these ideas to students; in as much as any good (hi)story has insights to offer, the tale of how mathematics went ballistic must stand (or fall!) on its own merits.

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[40] Hall (1983), p. 115. Hall's view concerning the motivation of theorists is a rejection of an alternative view which he describes as follows: "The common supposition has been – and perhaps among the less critical still is – that these men were naively eager to solve "useful" (or at least utilitarian) problems, not so much with the object of doing good (for even in the seventeenth century to increase the accuracy of shooting could hardly be regarded as clearly an act of kindly benevolence, and we have evidence that warlike inventions were then already regarded with horror), but rather in order to prove the general utility of the scientific approach to practical problems, and to demonstrate the particular powers of the individual writer."

[41] Idid, p. 116.

[42] Merton (1970), p. 191.

[43] Hall (1983), p. 115.

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