

## History of Mathematics in Mathematics Education: Recent developments

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**Abstract:** This is a survey on the recent developments (since 2000) concerning research on the relations between History and Pedagogy of Mathematics (the *HPM domain*). Section 1 explains the rationale of the study and formulates the key issues. Section 2 gives a brief historical account of the development of the *HPM domain* with focus on the main activities in its context and their outcomes. Section 3 provides a sufficiently comprehensive bibliographical survey of the work done in this area since 2000. Finally, section 4 summarizes the main points of this study.

### 1. Introduction

This is a survey describing the state-of-the-art on the themes of the ICME 13 *Topic Study Group* (TSG) 25: ***The Role of History of Mathematics in Mathematics Education***. It gives a brief account of the developments since 2000<sup>1</sup> on the relations between History and Pedagogy of Mathematics, in order to illuminate and provide insights on the following general questions:

- Which history is suitable, pertinent, and relevant to Mathematics Education (ME)?
- Which role can History of Mathematics (HM) play in ME?
- To what extent has HM been integrated in ME (curricula, textbooks, educational aids/resource material, teacher education)?
- How can this role be evaluated and assessed and to what extent does it contribute to the teaching and learning of mathematics?

These are the key issues explicitly addressed in and/or implicitly underlying what we call the *HPM perspective* as detailed below.

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<sup>1</sup> The year of publication of *History in Mathematics Education: The ICMI Study*, edited by J. Fauvel & J. van Maanen (Kluwer 2000); a highly collective work that has been a landmark in the area (see section 2, p.4).

## 1.1 The HPM Perspective

Mathematics is a human intellectual enterprise with a long history and a vivid present. Thus, mathematical knowledge is determined not only by the circumstances in which it becomes a deductively structured theory, but also by the procedures that originally led or may lead to it. Learning mathematics includes not only the “polished products” of mathematical activity, but also the understanding of implicit motivations, the sense-making actions and the reflective processes of mathematicians, which aim to the construction of meaning. Teaching mathematics should give to students the opportunity to “do mathematics.” In other words, although the “polished products” of mathematics form the part of mathematical knowledge that is communicated, criticized (in order to be finally accepted or rejected), and serve as the basis for new work, the **process** of producing mathematical knowledge is equally important, especially from a didactical point of view. Perceiving mathematics both as a logically structured collection of intellectual products and as processes of knowledge production should be the core of the teaching of mathematics. At the same time, it should be also central to the image of mathematics communicated to the outside world.

Along these lines, putting emphasis on integrating historical and epistemological issues in mathematics teaching and learning constitutes a possible natural way for exposing mathematics in the making that may lead to a better understanding of specific parts of mathematics and to a deeper awareness of what mathematics as a discipline is. This is important for ME, helping to realize that mathematics:

- is the result of contributions from many different cultures;
- has been in constant dialogue with other scientific disciplines, philosophy, the arts and technology;
- has undergone changes over time; there have been shifting views of what mathematics is;
- has constituted a constant force for stimulating and supporting scientific, technical, artistic and social development.

This helps to improve ME at all levels and to realize that although mathematics is central to our modern society and a mathematically literate citizenry is essential to a country’s vitality, historical and epistemological issues of mathematics are equally important. The harmony of mathematics with other intellectual and cultural pursuits also makes the subject interesting, meaningful, and worthwhile. In this wider context, history and epistemology of mathematics have an additional important role to play in providing a fuller education of the community: not being a natural science, but a formal science closer to logic – hence to philosophy – mathematics has the ability inherent in itself to connect the humanities with the sciences. Now that societies value and want young people educated in the sciences, but have a hard time finding out how to get people to “move” from humanities to the sciences, integrating history and epistemology in ME can make this connection visible to students. This is most important, especially today when there is much concern about the level of mathematics that students are learning and about their decreasing interest in mathematics, at a time when the need for both technical skills and a broader education is rising.

## 1.2 Summary of the content of the present survey

The rationale underlying this perspective has formed the core and main concern of the approaches adopted towards integrating History and Epistemology of Mathematics in ME (the **HPM domain**), especially in the context of the ICMI affiliated *International Study Group on the relations between the History and Pedagogy of Mathematics* (the **HPM Group**) since its

formation in 1972.

What follows consists of four sections:

**Section 2** gives a brief historical account of the development of the *HPM domain* with focus on the main activities in its context and their outcomes since 2000 (§2.1); a short presentation of journals and newsletters (§2.2); and an outline with comments on the key issues mentioned in section 1 and references to the literature for details (§2.3).

**Section 3** constitutes the major part of the survey. It provides a sufficiently comprehensive bibliographical survey of the work done since 2000: Collective works in §3.1 (collective volumes, special ME journal issues, conference proceedings, resource material); individual works in §3.2 (books & doctoral dissertations, papers in scientific journals, collective volumes and conference proceedings).

Though the emphasis is on research results of an as broad as possible international interest, due attention is paid to nationally-oriented implementation of the *HPM perspective* as well.

*Remarks:* (a) Next to each reference the TSG 25 themes to which it is related are indicated, numbered as in the Appendix below.

(b) Collective works exclusively on the *HPM perspective* (i.e. those in §3.1) contain several important contributions. However, in order to keep this survey to a reasonable size, these contributions are not included as separate items in §3.2, though some of them are quoted in section 2 (hence, they appear in section 5). Instead, these collective works are annotated briefly.

**Section 4** summarizes the main points of this study.

**Section 5** contains all references given in section 2.

**Important remarks:** To keep this document - especially the bibliographical survey - to a reasonable size, (a) several **abbreviations** are used for the titles of journals, conferences etc, all explained at the beginning of **section 3**; (b) many lengthy URL have been integrated into the title of the reference to avoid making the text difficult to follow and non-appealing. These URLs are **not** displayed in a print-out of this document, so the reader is advised to use its electronic version available at <http://www.clab.edc.uoc.gr/HPM/HPMinME-TopicalStudy-18-2-16-NewsletterVersion.pdf>

For several references (especially those in the annotated bibliography in section 3.1), we provide hyperlinks, where one can find online additional information in the form of an abstract, review, outline of contents etc.

APPENDIX: *Main Themes of TSG 25*

T1: Theoretical and/or conceptual frameworks for integrating history in ME.

T2: History and epistemology implemented in ME, considered from either the cognitive or/and affective points of view:

- a. Classroom experiments at school, the university and teacher pre- & in-service education;
- b. Teaching material: textbooks, resource material of any kind.

T3: Surveys on:

- a. Research on the HM in ME;
- b. The HM as it appears in curriculum and/or textbooks.

T4: Original sources in the classroom and their educational effects.

T5: History and epistemology as a tool for an interdisciplinary approach in the teaching and learning of mathematics and the sciences by unfolding their fruitful interrelations.

T6: Cultures and mathematics fruitfully interwoven.

## 2. An outline of the historical development of the HPM domain

Integrating HM in ME has been advocated since the second half of the 19<sup>th</sup> century, when mathematicians like De Morgan, Poincaré, Klein and others explicitly supported this path and historians like Tannery and later Loria showed an active interest on the role HM can play in ME. At the beginning of the 20<sup>th</sup> century, this interest was revived as a consequence of the debates on the foundations of mathematics. Later on, history became a resource for various epistemological approaches; Bachelard's historical epistemology, Piaget's genetic epistemology and Freudenthal's phenomenological epistemology, at the same time stimulating the formulation of specific ideas and conclusions on the learning process (Fauvel & van Maanen 2000, p.202; Barbin & Tzanakis 2014, p.256 and references therein).

This interest became stronger and more competitive in the period 1960-1980 in response to the *New Math* reform, when its proponents were strongly against “a historical conception of ME,” whereas for its critics, HM appeared like a “therapy against dogmatism”, conceiving mathematics not only as a language, but also as a human activity. In 1969, the NCTM in USA devoted its 31<sup>st</sup> Yearbook to the HM as a teaching tool (NCTM 1969) and in the 1970s a widespread international movement began to take shape, greatly stimulated and supported by the establishment of the *HPM Group* at ICME 2 in 1972 and its scope in 1978 (HPM Group 1978).

Thus, during the last 40 years, integrating HM in ME has evolved into a worldwide intensively studied area of new pedagogical practices and specific research activities and a gradually increasing awareness has emerged of what was described in §1.1 as the *HPM perspective* (Fasanelli & Fauvel 2006 for a historical account and references prior to 2000; Furinghetti 2012, Barbin 2013, Barbin & Tzanakis 2014 for a concise outline of later developments, and references there in).

The rising international interest in the *HPM perspective* and the various activities of the HPM Group worldwide, led to the approval by ICMI in 1996 of launching a 4-year *ICMI Study* on the relations between HM and ME. After a *Discussion Document* drawn up by the *Study* co-chairs (Fauvel & van Maanen 1997) and a *Study Conference* in 1998, at Luminy, France, the Study culminated in the publication of a 437-page volume written by 62 contributors working together in 11 groups (Fauvel & van Maanen 2000). This was a landmark in establishing and making more widely visible the *HPM perspective* as a research domain in the context of ME and greatly stimulated and enhanced the international interest of the educational community in this area.

Below we give an account of the main regular activities and their outcomes concerning educational research and its implementation in educational practice, relevant to the *HPM domain* and mainly realized in the context of the *HPM Group*.

### 2.1 Meetings and related collective volumes

#### 2.1.1 ICME Satellite Meetings of the HPM Group

These quadrennial meetings are a major activity to bring together individuals with a keen interest in the relationship between the HM and ME; researchers in ME interested in the HM in relation to mathematical thinking, mathematics teachers at all levels eager to gain insights into the *HPM perspective*, historians of mathematics wishing to talk about their research, and mathematicians wanting to learn about new possibilities to teach their discipline. and all those with an interest in the *HPM domain*.

They are organized just after, or before the ICME:

1984 Adelaide; *ICME 5*, Adelaide  
 1988 Florence; *ICME 6*, Budapest  
 1992 Toronto; *ICME 7*, Quebec  
 1996 Braga, *HEM Braga 96* conjointly with the 2<sup>nd</sup> *ESU*; *ICME 8*, Seville  
 2000 Taipei, *HPM 2000*; *ICME 9*, Tokyo-Makuhari  
 2004 Uppsala, *HPM 2004*, conjointly with the 4<sup>th</sup> *ESU*; *ICME 10*, Copenhagen  
 2008 Mexico City, *HPM 2008*; *ICME 11*, Monterrey  
 2012 Daejeon, *HPM 2012*; *ICME 12* Seoul  
 2016 Montpellier, *HPM 2016*; *ICME 13* Hamburg  
 2020 cancelled due to the COVID-19 pandemic (originally planned to take place in Macau, China as *HPM 2020*); *ICME 14* Shanghai (as a hybrid conference)

The books published as a result of these *HPM* meetings are listed below using the [abbreviations](#) introduced in section 3:

Swetz *et al* 1995 (after *ICME-6*); Calinger 1996 (after *HPM 1992*); Lagarto *et al* 1996 (during *HPM 1996*); Katz 2000 (after *HPM 1996*); Horng & Lin 2000 (at *HPM 2000*); Bekken & Mosvold 2003 (before *ICME 10* & *HPM 2004*); Horng *et al* 2004 (before *HPM 2004*); Furinghetti *et al* 2004 (at *HPM 2004*; revised edition Furinghetti *et al* 2006; see §3.1.3); Cantoral *et al* (at *HPM 2008*); Barbin *et al* 2012 (at *HPM 2012*; revised edition in progress); Radford *et al* 2016 (at *HPM 2016*).

### 2.1.2. The European Summer University on the History and Epistemology in Mathematics Education (ESU)

The initiative of organizing a *Summer University* (SU) on the *History and Epistemology in Mathematics Education* belongs to the French ME community in the early 1980s. The French IREMs organized the first interdisciplinary in 1984, in Le Mans, France, followed by another three in France. The next one was organized in 1993 on a European scale; the 1<sup>st</sup> *European Summer University on the History and Epistemology in Mathematics Education*, (a name coined since then, abbreviated as *ESU* since 2004), though many participants come from outside Europe. Since 2010, *ESU* is organized every four years to avoid coincidence with the *HPM* meetings.

Since its original conception, *ESU* has been developed and established into one of the major activities in the *HPM* domain. It mainly aims to: provide a school for working on a historical, epistemological and cultural approach to mathematics and its teaching, with emphasis on actual implementation; give the opportunity to mathematics teachers, educators and researchers to share their teaching ideas and classroom experience related to a historical perspective in teaching; and motivate further collaboration along these lines among teachers of mathematics and researchers on the *HM* and *ME* in Europe and beyond, attempting to reveal and strengthen the *HPM perspective*. Below is a list of the *ESUs*:

1993, *ESU 1* Montpellier, France  
 1996, *ESU 2* Braga, Portugal (conjointly with *HEM Braga 96*)  
 1999, *ESU 3* Leuven & Louvain-la-Neuve, Belgium  
 2004, *ESU 4* Uppsala, Sweden (conjointly with *HPM 2004*)  
 2007, *ESU 5* Prague, Czech Republic  
 2010, *ESU 6* Vienna, Austria  
 2014, *ESU 7* Copenhagen, Denmark  
 2018, *ESU 8* Oslo, Norway  
 2022, *ESU 9*, Salerno, Italy

The following works were published after the *ESUs*: Lalande *et al* 1995; Lagarto *et al* 1996; Radelet-de-Grave & Brichard 2001; Furinghetti *et al* 2004; Barbin *et al* 2008; Barbin *et al* 2011a; Barbin *et al* 2015, Barbin *et al* 2019.

### 2.1.3. The *HPM* domain at ICMEs

Activities related to the *HPM perspective* have always been present in the ICMEs; [Fasanelli & Fauvel 2006](#) for ICMEs before 2000. Since 2000, such activities have formed an established part of the ICMEs' scientific program:

(a) *ICME 9, Tokyo, Japan, 2000*

WG for Action 13: *History and Culture in Mathematics Education* coordinated by J. van Maanen, W.S. Horng.

Summary in H. Fujita, Y. Hashimoto, B.R. Hodgson, P.Y. Lee, S. Lerman & T. Sawada (Eds) (2004). [Proc. of the 9<sup>th</sup> ICME](#) (pp.287-291). Dordrecht: Kluwer Academic Publishers, in CD (accessed 18/2/16)

(b) *ICME-10, Copenhagen, Denmark, 2004*

TSG 17: *The role of the history of mathematics in mathematics education*, organized by M.K. Siu, C. Tzanakis, A. El Idrissi, S. Kaisjer, L. Radford.

Summary in M. Niss (Ed) (2008). [Proc. of the 10<sup>th</sup> ICME](#) (pp.363-367). IMFUFA, Roskilde University, in CD (accessed 18/2/16)

This TSG led to a post-conference publication (Siu & Tzanakis 2004)

(c) *ICME 11 Monterrey, Mexico, 2008*

TSG 23: *The role of the history of mathematics in mathematics education*, organized by A. El Idrissi, A. Miguel, F. Furinghetti, A. Garciadiego, E. Barbin (accessed 18/2/16).

(d) *ICME 12 Seoul, Korea, 2012*

TSG 20: *The role of history of mathematics in mathematics education*, organized by R. Chorlay, W-S. Horng, M. Kronfellner, K. Clark, A. El Idrissi, H. Chang (accessed 18/2/16).

Summary in S. J. Cho (Ed) (2015). *Proc. of the 12<sup>th</sup> ICME: Intellectual and attitudinal challenges*. New York: Springer (pp.485-487) [http://www.icme12.org/sub/sub02\\_05.asp](http://www.icme12.org/sub/sub02_05.asp)  
12 papers & 13 posters available at  
[https://www.mathunion.org/fileadmin/ICMI/Conferences/ICME/ICME12/www.icme12.org/sub/sub02\\_05.html](https://www.mathunion.org/fileadmin/ICMI/Conferences/ICME/ICME12/www.icme12.org/sub/sub02_05.html) (accessed 2/7/20).

(e) *ICME 13 Hamburg, Germany 2016*

TSG 25: *The role of history of mathematics in mathematics education*, organized by C. Tzanakis, X. Wang, K. Clark, T.H. Kjeldsen, S. Schorcht (accessed 4/4/2016)

### 2.1.4. The *HPM* domain at CERME

CERME is a regular activity of the *European Society for Research in Mathematics Education* (ERME), organized every two years in the form of presentations, discussions and debates within thematic WG. Though relatively new, the *HPM perspective* has exhibited great potential at CERME and is expected to play a central role in the future:

(a) *CERME 6, Lyon, France, 2009*

WG 15: *Theory and research on the role of history in mathematics education*, organized by F. Furinghetti, J-L. Dorier, U. T. Jankvist, J. van Maanen & C. Tzanakis

A new WG structured along 7 themes; 13 papers & 1 poster accepted and included in the proceedings (see §3.1.3).

(b) *CERME 7, Rzeszów, Poland, 2011*

WG 12: History in Mathematics Education organized by U. T. Jankvist, S. Lawrence, C. Tzanakis & J. van Maanen.

Structured along 9 themes; 13 papers & 1 poster accepted and included in the proceedings (see §3.1.3).

(c) *CERME 8*, Antalya, Turkey, 2013

WG 12: History in Mathematics Education organized by U. T. Jankvist, K. Clark, S. Lawrence & J. van Maanen.

Structured along 9 themes (the same as CERME 7); 12 papers & 3 posters accepted and included in the proceedings (see §3.1.3)

(d) *CERME 9, Prague*, Czech Republic, 2015

Thematic WG 12: History in Mathematics Education, organized by R. Chorlay, U. T. Jankvist, K. Clark & J. van Maanen (accessed 15/6/2016).

The WG themes were modified considerably, becoming more specific. This reflects further deepening of research in this area, with emphasis both on empirical work and its assessment and on sharpening theoretical ideas and developing conceptual frameworks adequate for describing and understanding phenomena relevant to the *HPM perspective* (see §3.1.3).

(d) *CERME 10, Dublin*, Ireland, 2017

Thematic WG 12: History in Mathematics Education, organized by R. Chorlay, S. Lawrence, K. Clark & Katalin Gosztanyi (accessed 15/6/2016).

Structured along 4 main themes with focus on empirical studies, surveys of existing uses of HM in ME, and relationships between such empirical studies and theories, frameworks and studies in other parts of ME research.

(e) *CERME 11, Utrecht*, Netherlands, 2019, organized by R. Chorlay, A. Bernardes, T. Hamann & A. Oler-Marcén

Thematic WG 12: *History in Mathematics Education*

(f) *CERME 12, Utrecht*, Netherlands, 2022, organized by R. Chorlay, T. Hamann, J. Krüger & A. Oler-Marcén

Thematic WG 12: History in Mathematics Education

## 2.2 Journals and Newsletters

### 2.2.1. *Convergence: Where Mathematics, History and Teaching Interact*

Since 2004, the MAA has published [Convergence: Where Mathematics, History and Teaching Interact](#), a free online journal in HM and its use in teaching.

Aimed at teachers of mathematics at both the secondary and collegiate levels, *Convergence* includes topics from grades 8-16 mathematics, with special emphasis on grades 8-14. Its resources for using the HM in mathematics teaching include informative articles about the HM, translations of original sources, classroom activities, projects and modules, teaching tools such as its [Mathematical Treasures](#), reviews of new and old books, websites, *Problems from another time*, and other teaching aids that focus on utility in the classroom.

### 2.2.2. *The Bulletin of the British Society for the History of Mathematics (BSHM Bulletin)*

The [BSHM Bulletin](#) aims to promote research into the HM and to encourage its use at all levels of ME. Articles on local HM and the use of HM in ME are particularly encouraged. Originally published as a Newsletter, until 2004 when its 50<sup>th</sup> issue became Bulletin 1. Under

the influence of the late J. Fauvel, president of BSHM (1992–94), editor of its Newsletter (1995–2001) and chair of the HPM Group (1992-96) and his successor, the late J. Stedall, the Newsletter changed from providing information to members into a scientific journal with a regular Education Section directly related to issues relevant to the HPM perspective since 2002 (issue No 46).

### 2.2.3. The *HPM Newsletter*

It appears 3 times per year since 1980. Originally it was available by contacting the regional distributors; however, for the last 13 years it is also available online from the [HPM Group website](#) and its [Newsletter webpage](#)

It includes a calendar of upcoming events, a guest editorial, a ‘*Have You Read?*’ section, short reviews and announcements of meetings and activities. Furthermore, for the last 13 years it has also included short articles, reports on research projects and PhD theses, book reviews, lists of relevant websites, particular themes that are suggested for further research.

## 2.3 Comments on some key issues

From what has been presented so far and will become clearer from the bibliographical survey in Section 3, the last two decades have generated considerable research activity related to the *HPM perspective* of great variety: doing empirical research based on actual classroom implementations; designing specific teaching units; developing various kinds of teaching aids; exploring and understanding students’ response to the introduction of the HM in teaching (including teacher education); designing, applying and evaluating interdisciplinary teaching; drawing and/or criticising parallels between the historical development and learning in a modern classroom;<sup>2</sup> mutually profiting from theoretical constructs and conceptual frameworks developed in the context of other disciplines, especially philosophy, epistemology and cognitive science; and evaluating the effectiveness of all this in practice.

The key issues mentioned in Section 1 permeate all these activities as recurring themes that form the *leitmotif* of the HPM domain. Though impossible to present all the work done and the not always mutually compatible opinions and results, below a few general ideas are outlined with reference to the literature for details.

Whether HM is appropriate, or even relevant at all to the teaching and/or learning of mathematics, is an issue that, despite the extended research and the many insightful and sophisticated applications in the last few decades, has not reached universal acceptance even today. In fact, a number of objections against the *HPM perspective* have been raised (Tzanakis, Arcavi *et al* 2000, p.203; Siu 2006; Tzanakis & Thomaidis 2012, §3.4; Furinghetti 2012, §7):

#### ***A Objections of an epistemological and methodological nature***

##### *(a) On the nature of mathematics*

1. This is not mathematics! Teach the subject first; then its history.
2. Progress in mathematics is to make difficult problems routine, so why bother to look back?
3. What really happened can be rather tortuous. Telling it as it was can confuse rather than enlighten!

##### *(b) On the difficulties inherent to this approach*

1. Does it really help to read original texts, which is a very difficult and time-consuming task?

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<sup>2</sup> The old but still discussed issue of “historical parallelism” – if and to what extent “ontogenesis recapitulates (aspects of) phylogenesis”; e.g. Radford *et al.* 2000; Schubring 2006, 2011; Thomaidis & Tzanakis 2007.



2. Is it liable to breed cultural chauvinism and parochial nationalism?
3. Students may have an erratic historical sense of the past that makes historical contextualization of mathematics impossible without having a broader education in general history.

### ***B Objections of a practical and didactical nature***

#### *(a) The background and attitude of teachers*

1. Lack of didactical time: no time for it in class!
2. Teachers should be well educated in history: “I am not a professional historian of mathematics. How can I be sure of the exposition’s accuracy?”
3. Lack of teacher training.
4. Lack of appropriate didactical and resource material.

#### *(b) The background and attitude of the students*

1. They regard it as history and they dislike history class!
2. They regard it just as boring as mathematics itself.
3. They do not have enough general knowledge of culture to appreciate it.

#### *(c) Assessment issues*

1. How can you set questions on it in a test or exam?
2. Is there any empirical evidence that students learn better when HM is made use of in the classroom?

Each of these objections addresses one or more of the four key issues mentioned in Section

1. Below we comment briefly on them in the light of these objections.

### **2.3.1 Which history is suitable, pertinent and relevant to Mathematics Education?**

This has been a permanent issue of debate among historians and mathematics educators with an interest in the HPM perspective. As early as 1984 at ICME 5, d’Ambrosio stressed the need to develop three separate histories of mathematics: history as taught in schools, history as developed through the creation of mathematics, and the history of that mathematics which is used in the street and the workplace. To deal with these differences he introduced the concept of *ethnomathematics* as compared to *learned mathematics* (Booker 1986).

In fact, implicit to the objections A(a1), A(a2), A(b1) is the idea that the term “history” is the same, whether used by historians, mathematicians, or teachers and mathematics educators. That this is not so lies at the heart of Grattan-Guinness’ early refutation of some of these arguments (Grattan-Guinness 1973; see also Kjeldsen 2011a, pp.1700-1701; 2011b, pp.166-167; Kjeldsen & Blomhøj 2012, §3, and references therein for a different recent approach). On the other hand, it is undeniable that quite often the historical development was complicated, followed a zig-zag path, led to dead ends, included notions, methods and problems that are no longer used in mathematics as we know and work with today, etc. (A(a2), A(b3)). Thus, its integration in ME, on the one hand is nontrivial, and on the other hand poses the question why it must be done at all. Therefore, integrating HM in the teaching and learning of mathematics, may force history “...to serve aims not only foreign to its own but even antithetical to them” (Fried 2011, p.13). In other words, the danger of either unacceptably simplifying or/and distorting history to serve education as still another of its tools is real by adopting what has been called a “Whig” (approach to) history, in which “...the present is the measure of the past. Hence, what one considers significant in history is precisely what leads to something deemed significant today” (Fried 2001, p.395).

In this connection, an important step was Grattan-Guinness’ distinction between what he called *History* and *Heritage* trying to clarify existing conflicts and tensions between a

mathematician's and a historian's approach to mathematical knowledge, and paying due attention to the relevance of HM to ME (Grattan-Guinness 2004a, b). In the context of the *HPM perspective*, this is a distinction close to similar ones between pairs of methodological approaches; *explicit & implicit* use of history, *direct & indirect* genetic approach, *forward & backward* heuristics (Tzanakis, Arcavi *et al* 2000, pp.209-210). Hence, this distinction is potentially of great relevance to ME (Rogers 2009, 2011; Tzanakis & Thomaidis 2012), serving, among other things, to contribute towards answering the recurrent question “*Why and which history is appropriate to be used for educational purposes?*” (Barbin 1997).

### 2.3.2 Which role can History of Mathematics play in Mathematics Education?

Perhaps, this is the question that has been discussed and analyzed most on the basis of both a priori theoretical and epistemological arguments and of empirical educational research. At least implicitly, such analyses try to refute some of the above objections, especially those concerning the barriers posed by the complexity of the historical development (A(a2), A(a3), A(b1)) and/or by students' predisposition to and general knowledge of both mathematics and history as taught subjects (objections (B(b1), B(b2) and B(b3), A(b2), respectively).

This question has been extensively discussed from several points of view, especially in relation to the appropriateness and pertinence of original historical sources in ME. In this context, HM can play three mutually complementary and supplementary roles (Barbin 1997; Jahnke *et al* 2000, §9.1; Furinghetti *et al* 2006, pp.1286-1287; Furinghetti 2012, §5; Jankvist 2013, §7):

A *replacement* role: Replacing mathematics as usually understood (a corpus of knowledge consisting of final results/finished and polished intellectual products; a set of techniques for solving problems given from outside; school units useful for exams etc.) by something different (to emphasize not only final results, but also mental processes that may lead to them; hence to perceive mathematics not only as a collection of well-defined and deductively organized results, but also as a vivid intellectual activity).

A *reorientation* role: Changing what is (supposed to be) familiar, to something unfamiliar; thus, challenging the learner's and teacher's conventional perception of mathematical knowledge as something that has always been existing in the form we know it, into the deeper awareness that mathematical knowledge was an invention, an evolving human intellectual activity.

A *cultural* role: Making possible to appreciate that the development of mathematics takes place in a specific scientific, technological or societal context at a given time and place; thus becoming aware of the place of mathematical knowledge as an integral part of human intellectual history in the development of society; hence, seeing mathematics from perspectives that lie outside its nowadays established boundaries as a discipline.

Considered from the point of view of the *objective* of integrating HM in ME, there are 5 main areas in which the *HPM perspective* could be valuable:

The learning of mathematics;

The development of views on the nature of mathematics and mathematical activity;

The didactical background of teachers and their pedagogical repertoire;

The affective predisposition towards mathematics; and

The appreciation of mathematics as a cultural-human endeavor.

These are analyzed in detail into more specific arguments in Tzanakis, Arcavi *et al*, 2000, §7.2, describing in this way the role of history in the educational process.

From the point of view of the *way* HM is accommodated into this perspective, a distinction was made by Jankvist (2009); namely, history serving as a *tool* for assisting the

actual learning and teaching of mathematics, and history serving as a *goal* in itself for the teaching and learning of the historical development of mathematics (see also Jankvist & Kjeldsen 2011). A similar distinction between *history for reflecting on the nature of mathematics as a socio-cultural process* and *history for constructing mathematical objects* was made by Furinghetti (2004; 2012, §5).

In this way, a finer and more insightful categorization of the possible roles of HM in ME resulted, reflecting the variety of their possible implementations in practice.

A small selection appears below (many more in section 3).

- Fauvel & van Maanen 2000: chs.7, 8 provide a variety of examples of possible classroom implementations, for several mathematical subjects; ch.9 gives examples of using original sources in the classroom and specific didactical strategies to do so.

- Katz & Tzanakis 2011, chs. 9, 10, 13, 14, 16, 19, and Sriraman 2012, chs. 2, 7, 14 provide particular examples, most of them emphasizing empirical results of actual implementations.

- Katz *et al* 2014: Rich on recent work in the *HPM domain*, including a sufficiently comprehensive old and recent bibliography in the editors' introduction and in its 12 papers. They concern theoretical issues on the history, philosophy and epistemology of mathematics, and on empirical investigations both in school and teacher education.

- Doctoral dissertations with considerable work on both the theoretical issues of the *HPM perspective* and on empirical investigation and evaluation of actual implementations: e.g. van Amerom 2002; Su 2005; Clark 2006; Jankvist 2009a; Glaubitz 2010.

### **2.3.3 To what extent the History of Mathematics has been integrated in Mathematics Education (curricula, textbooks, educational aids/resource material, teacher education)?**

Considerable work has been done over the last 15 years on understanding better and formulating more sharply the methodological issues raised by the integration of HM in ME, on producing appropriate educational aids of various types (B(a4)), and on designing and implementing teaching approaches to specific subjects and instructional levels in this context, with special emphasis on teacher education (B(a2), B(a3)).

According to the classification of the various approaches to integrate HM in teaching and learning mathematics given in Tzanakis, Arcavi *et al* 2000, there are three broad ways that may be combined (thus complementing each other), each one emphasizing a different aim:

*To provide direct historical information*, aiming to learn history;

*To implement a teaching approach inspired by history* (explicitly or implicitly), aiming to learn mathematics;

*To focus on mathematics as a discipline and the cultural and social context* in which it has been evolving, aiming to develop a deeper awareness of its evolutionary character, its epistemological characteristics, its relation to other disciplines and the influence exerted by factors both intrinsic and extrinsic to it.

From a methodological point of view, Jankvist 2009b classified the teaching & learning approaches in three categories:

*Illumination approaches*, in which teaching and learning is supplemented by historical information;

*Module approaches*, in the form of instructional units devoted to history, often based on specific cases;

*History-based approaches*, in which history shapes the sequence and the way of presentation, often without history appearing explicitly, but rather being integrated into teaching.

Approaches may vary in size and scope, according to the specific didactical aim, the mathematical subject matter, the level and orientation of the learners (A(b1), B(b3)), the

available didactical time (B(a1)), and external constraints (curriculum regulations, number of learners in a classroom etc.).

The crucial role of teachers' training for effectively following the *HPM perspective* has been stressed repeatedly (e.g. Barbin *et al* 2000, p.70; Furinghetti 2004, p.4; Barbin *et al* 2011b; Alpaslan *et al* 2014, pp.160-162). Similarly, the need for appropriate didactical resources is equally crucial (e.g. Tzanakis, Arcavi *et al* 2000, pp.212-213; Pengelley 2011, pp.3-4).

Though accommodating the *HPM perspective* in an essential way into the official national curricula does not seem to have attained wide applicability,<sup>3</sup> intensive efforts have been made to train teachers and explore changes in their attitude and/or teaching, and to design, produce and make available didactically appropriate resources. Some indicative examples (more in section 3):

**Teacher training:** Bruckheimer & Arcavi 2000; Liu 2003; Waldegg 2004; Arcavi & Isoda 2007; Clark 2011; Smestad 2011; Mosvold *et al* 2014; Povey 2014.

**Resource material & educational aids:** The need for didactical resources along the lines of the *HPM perspective* has been satisfied to a considerable extent in the last 15 years, so that such material is available nowadays in a variety of forms. Some examples:

- A wide spectrum of resource material can be found in [Convergence](#); e.g. (i) *HPM Newsletter*, No90/2015, pp.10-12 for a recent sample; (ii) Clark 2009 (detailed description of a teaching module).
- Katz & Michalowicz 2005: didactical source material in 11 mathematical modules.
- Siu 2007: a useful survey of the literature and available resources.
- Pengelley *et al* 2009: Didactical material for discrete mathematics based on original sources.
- Pengelley & Laubenbacher 2014: A website with many references to published work and material available online.
- Barnett *et al* (2014): Extensive information on teaching with historical sources and bibliography on its theoretical framework and available resource material.
- Books with material that can be used directly and/or inspire teaching; e.g. Stein 2010 (review in *HPM Newsletter* No77/2011, pp.8-9); Barbin 2015 (review in *HPM Newsletter* No 89/2015, pp.13-14).

### 2.3.4 How can this role be evaluated and assessed and to what extent it contributes to amend the teaching and learning of mathematics?

Evaluating the effectiveness of the *HPM perspective* on improving ME from the point of view of both teaching and learning mathematics is an issue clearly stressed in objections B(c1), B(c2). Those who oppose, or are reserved about the role of HM in ME rightly ask for sufficient empirical evidence about its effectiveness. Quite early it has become clear that this is a key issue (e.g. Siu & Tzanakis 2004, p.3; Jankvist 2007), and that any such evaluation is a complex process relying more on qualitative, than quantitative methodologies: to consider changes induced in teachers' own perception of mathematics; to examine how this may influence the way they teach mathematics; and to explore if and in which ways this affects students' perception and understanding of mathematics (Barbin *et al* 2000, particularly §§3.1, 3.2).

Additionally, any such evaluation goes together with actual classroom implementations, in school teaching and teacher pre- and in-service education. Therefore, many, if not all, works

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<sup>3</sup> One exception is Denmark (see Jankvist 2013, §3; Kjeldsen 2011b, §15.2; Niss & Højgaard 2011, ch.4). For a recent discussion and survey see Boyé *et al* 2011.

referring to such implementations necessarily address evaluation issues about the effectiveness of the approach considered in each case (e.g. those listed in §§2.3.2, 2.3.3).

This is an area of currently active research with no established results of universal acceptance because of several reasons:

(a) Such a complex process is not expected to lead to spectacular changes in a short time interval. Preconceptions, misconceptions, predispositions either of the teachers or the students are too stable to be easily and/or quickly modified. Therefore, one should expect to see such changes after a considerable time exposure to an approach adopting the *HPM perspective*; often this time is not available.

(b) There is strong dependence on the instructional level (primary, secondary, tertiary) and orientation of the students, teacher-students included (science or humanities; elementary or secondary school teachers etc.), as well as, on their entire previous educational path, which has determined their knowledge of, attitude towards, and preconceptions about mathematics.

(c) There is influence by external “technical” factors that may favor, impede, or even prevent the implementation of an approach based on the *HPM perspective*: the curriculum and the corresponding regulations; the number of students in the class (e.g. a small number facilitates group work and teacher’s effective supervision); the structure of the educational system (e.g. in a centralized system, teachers have less freedom, hence fewer possibilities to apply an innovative teaching approach not necessarily falling into the official curriculum regulations).

(d) Not all mathematical subjects are equally accessible or appropriate to be taught and/or learned in a historically motivated/driven context.

All this constitutes a complex network of factors interfering with each other, so that empirical findings of different research works are not easily comparable. Therefore, despite many thoughtfully designed and carefully applied empirical investigations, much work is still needed to evaluate the effectiveness of the role of HM in ME in an undisputable way.

### 3. A bibliographical survey in the *HPM domain* since 2000

This section provides a comprehensive bibliographical survey of work related to the *HPM perspective* since 2000, indicating the TSG 25 themes to which each item is related and relevant, except those collective works that practically touch upon all themes (cf. remark (a) and Appendix in §1.2). Within each subsection, items appear by publication year and for each year by authors’ alphabetical order. For those works included in Section 5 only author names and a note “see section 5” are given.

Throughout this study, journals, proceedings and conferences are abbreviated as follows:

#### ABBREVIATIONS

##### Journals

*BSHM*: BSHM Bulletin: Journal of the British Society for the History of Mathematics

*ESM*: Educational Studies in Mathematics

*FLM*: For the Learning of Mathematics

*Int. J. ME Sci. Tech.*: International Journal of Mathematical Education in Science and Technology

*Int. J. Math. Teach. Learn*: International Journal for Mathematics Teaching and Learning

*Int. J. Sci. ME*: International Journal of Science and Mathematics Education

*JMB*: Journal of Mathematical Behavior

*JRME*: Journal for Research in Mathematics Education

*MER*: Mathematics Education Review  
*MT*: The Mathematics Teacher  
*MTL*: Mathematical Thinking and Learning  
*MedJRME*: Mediterranean Journal for Research in Mathematics Education  
*NOMAD*: Nordic Studies in Mathematics Education  
*PRIMUS*: Problems, Resources, and Issues in Mathematics Undergraduate Studies  
*R-IREM*: Repères-IREM  
*RME*: Research in Mathematics Education  
*S&E*: Science & Education  
*TJME*: Taiwan Journal of Mathematics Education  
*TME*: The Mathematics Educator  
*ZDM*: ZDM - The International Journal of Mathematics Education

In Chinese:

*MB*: Mathematical Bulletin 数学通报  
*JME*: Journal of Mathematics Education 数学教育学报  
*CTMM*: Curriculum, Teaching Material and Method 课程·教材·教法  
*MTMS*: Mathematics Teaching in Middle Schools (Junior High School)/ (Senior High School) 中学数学教学参考  
*ERR*: Educational Researches and Reviews (High School Education and Instruction) 教育研究与评论  
*HSMM*: The High School Mathematics Monthly 中学数学月刊  
*CER*: Comparative Education Review 比较教育研究

### Conferences

*CERME*: Congress of European Research in Mathematics Education  
*ESU*: European Summer University on the History and Epistemology in Mathematics Education.  
*HPM XXXX*: The HPM Group Satellite meeting of ICME-Y<sup>4</sup>  
*ICME*: International Congress on Mathematical Education  
*ICTM*: International Conference on the Teaching of Mathematics (at the undergraduate level)

### Books

*Hand Int RME*: Handbook of International Research in Mathematics Education  
*ICMI Study Volume*: J. Fauvel & J. van Maanen (Eds.), *History in Mathematics Education, The ICMI Study*. Dordrecht: Kluwer, 2000.  
*Int. Hand. Res. Hi. Phil. Sci. Teach.*: International Handbook of Research in History, Philosophy and Science Teaching  
*TMMEM*: The Montana Mathematics Enthusiast Monographs

### Other

*ERME*: European Society for Research in Mathematics Education  
*HPM Group*: International Study Group on the Relations between the History and the Pedagogy of Mathematics  
*ICMI*: International Commission on Mathematical Instruction  
*MAA*: The Mathematical Association of America  
*NCTM*: National Council of Teachers of Mathematics (USA)

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<sup>4</sup> XXXX being the year that ICME-Y took place.

Proc.: Proceedings  
TSG: Topic Study Group  
WG: Working Group

### 3.1 Collective works

#### 3.1.1 *Collective volumes* in this area, with research papers, reviews of work etc.

##### 2000

Dorier, J.-L. (Ed) *On the teaching of Linear Algebra*. Dordrecht: Kluwer. T2, 5

This volume consists of two parts: On an epistemological analysis of the historical development of vector space theory; and on didactical issues addressed and actual implementations at the undergraduate level, to which this analysis is related.

Fauvel & van Maanen (see section 5).

Vol. 6 of the *New ICMI Study Series*: The outcome of a 4-year collectively realized international study of 62 scholars in 11 groups, each one authoring one chapter under the coordination of a convenor. A landmark in establishing the HPM domain.

Katz (see section 5).

26 chapters based on papers presented in the 1996 HPM Meeting: 7 on general issues of HM in ME; 5 on the teaching of a particular subject using the HM; 3 on teacher education; 11 on the HM.

##### 2001

Barbin, É., Duval, R., Giorgiutti, I., Houdebine, J. & Laborde C. (Eds) *Produire et lire des textes de démonstrations*. Paris: Ellipses.

It addresses all problems encountered in learning proof, by following a variety of approaches: mathematical, historical, epistemological, didactical, linguistic, cognitive. Revealing the diversity of the points of view on proof and its teaching is the strong point of this book.

##### 2002

Boniface, J. (Ed) *Les constructions des nombres réels dans le mouvement de l'arithmétisation de l'analyse. Comprendre les mathématiques par les textes historiques*. Paris: Ellipses.

Cerquetti, F. & Rodriguez A. (Eds) *Faire des mathématiques avec des images et des manuscrits historiques*. Créteil: CRDP de Créteil. T2a, 4

An empirical study with high school students on rediscovering geometry using various educational aids; among others, excerpts from original sources (16-18th centuries)

##### 2003

Bekken & Mosvold (see section 5). T1, 4, 2a, 5

A continuation of Swetz *et al* 1995. Of its 27 chapters, 9 are related to the HPM perspective, the others concern either the HM or the history of ME.

##### 2005

Shell-Gellasch, A. & Jardine, D. (Eds) *From calculus to computers: using the last 200 years of mathematics history in the classroom*. MAA notes 68. Washington DC: MAA. T2, 1, 5

22 contributions focusing on 19<sup>th</sup> and 20<sup>th</sup> century mathematics, emphasizing recent history in the teaching of mathematics, computer science, and related disciplines; for details see

<http://www.maa.org/publications/books/from-calculus-to-computers> (accessed 18/2/2016)

## 2006

Thomaidis, Y., Kastanis, N. & Tzanakis, C. (Eds) *History and Mathematics Education*. Thessaloniki: Ziti Publications (in Greek). T4, 5, 1

Collective volume with 5 papers on the HPM perspective; 6 on the history of ME and 3 on historiographic approaches to ancient Greek mathematics (more in *HPM Newsletter*, No63/2006, pp.7-9).

## 2007

Barbin, E. & Bénard, D. (Eds) *Histoire et Enseignement des Mathématiques: Rigueurs, erreurs, raisonnements*. Lyon: Institut National de Recherche Pédagogique. T5, 4, 1

12 chapters on rigor, experiment and proof in geometry, reasoning between geometry and algebra, and multiplicity of points of view in analysis. Contributions are based on original texts and anchor on epistemological and didactical reflections.

Boero, P. (Ed) *Theorems in School. From History, Epistemology and Cognition to Classroom Practice*. Rotterdam: Sense Publishers. T5, 1

15 chapters in 4 sections exploring: the historical-epistemological nature of proof; the way national mathematics curricula and the underlying epistemological beliefs might influence students' perceptions of proof; the similarities and differences between argumentation and proof; the teaching of proof.

## 2008

Biegel, G., Reich, K., Sonar, T. (Eds.). *Historische Aspekte im Mathematikunterricht an Schule und Universität*. Göttingen: Termessos.

12 chapters about integrating history of mathematics in education. These proceedings of the conference "Historische Aspekte im Mathematikunterricht an Schule und Universität" (Historical perspectives in mathematics education at school and university) <https://www.degruyter.com/downloadpdf/j/dmvm.2004.12.issue-2/dmvm-2004-0036/dmvm-2004-0036.pdf> (accessed 19/6/2018) include topics from history of numbers, Adam Ries and Christoph Scheiner to elementary school and courses at the university level. For abstracts and contents see: <http://s-edition.de/sonarcontent/SonarContent01.htm> (accessed 19/6/2018)

Shell-Gellasch, A. (Ed) *Hands on History: A Resource for Teaching Mathematics*. MAA notes 72. Washington DC: MAA. T2b, 5, 6

16 chapters on historical examples appropriate for incorporating hands-on learning: From simple devices, to elaborate models of descriptive geometry, and detailed descriptions on how to build and use historical models in the high school or collegiate classroom; for details see <http://www.maa.org/press/books/hands-on-history-a-resource-for-teaching-mathematics> (accessed 18/2/2016)

Robson, E. & Stedall, J. (Eds) *The Oxford Handbook of the History of Mathematics*. Oxford: Oxford University Press. T6, 5

This is not just a 'history of mathematics'; it is unusual in that it is organized in 3 major themes: geographies and cultures; people and practices; interactions and interpretations that are sensitive to modern historiography and include social and cultural backgrounds and epistemologies.

## 2009

Greek Society of the Didactics of Mathematics (Eds). *The value of the history of mathematics in mathematics education*, Thessaloniki: Ziti Publications (in Greek). T2, 4, 1

9 contributions on one or more of the following issues: Whether and how crucial historical steps can be integrated into ME; whether and to what extent historical knowledge allows the prediction of students' difficulties; whether the integration of HM in ME is possible at all, given the constraints imposed by the official curriculum and the teachers' and students' established preconceptions.



## 2010

Barbin, E. (Ed) *Des grands défis mathématiques d'Euclide à Condorcet*. Paris: Vuibert. T2, 4, 5, 1

9 examples of introducing a historical perspective in ME, having as a starting point specific historical problems and organized in 4 parts: measuring magnitudes, representing magnitudes, calculating the probable, approaching a curve; thus illustrating the great domains of today's taught math: analysis, algebra, probability and geometry (for details see <http://culturemath.ens.fr/node/2582> (accessed 18/2/2016) or *HPM Newsletter No75/2010*, pp.1-2).

Hanna, G., Jahnke, N. & Pulte, H. (Eds) *Explanation and proof in mathematics: philosophical and educational perspectives*. New York: Springer. T1, 5

The 17 chapters assemble perspectives from ME, its history and philosophy to strengthen mutual awareness and share recent findings and advances in these interrelated fields. By a variety of examples, the authors explore the role of refutation in generating proofs, the varied links between experiment and deduction, the use of diagrammatic thinking in addition to pure logic, and the uses of proof in ME.

## 2011

Katz & Tzanakis (see section 5).

An all-embracing outcome of activities within the HPM Group during 2007-2009; to present an overview of the state of the art in this area after the appearance of the ICMI Study volume. 7 chapters on theoretical aspects of HM in ME; 10 with concrete implementations; 4 with particular focus on teacher education; 3 invited papers on the HM (indicative examples in section 5).

Shell-Gellasch, A. & Jardine, D. (Eds) *Mathematical Time Capsules: Historical Modules for the Mathematics Classroom*. MAA notes 77. Washington DC: MAA. T2b, 5

35 chapters offering teachers historical modules for immediate use in the math classroom (undergraduate or secondary math curricula). Each capsule presents at least one topic or a historical thread that can be used throughout a course and further references and resources on the chapter subject; for detail see <http://www.maa.org/press/ebooks/mathematical-time-capsules> (accessed 18/2/2016).

## 2012

Barbin, E. (Ed) *Les mathématiques éclairées par l'histoire: des arpenteurs aux ingénieurs*. Paris: Vuibert. T2, 4, 6

The 9 chapters provide examples of the influence or use of HM in the math classroom. Authors work with students from upper secondary school to tertiary, including teacher training. They show how their own reading and reflection has led to direct use of historical material in the classroom, either through the use of original texts or by devising tasks based on the methods or examples of their subjects (*HPM Newsletter No80/2012*, pp.12-13).

Chemla, K. (Ed) *The History of Mathematical Proof in Ancient Traditions*. Cambridge University Press. Cambridge, UK. T2b, 6

A number of important contributions from international scholars on the historiography, history and epistemology of mathematical proof, relevant for today's mathematics classroom.

Sriraman, B. (Ed.). *Crossroads in the History of Mathematics and Mathematics Education*, TMMEM vol.12. Charlotte NC: Information Age Publishing. T1, 2b, 5

15 chapters: 7 in the history and didactics of calculus and analysis, 4 in the history and didactics of geometry and number and 4 on HM in ME.

## 2013

Barbin, E. & Moyon, M. (Eds) *Les ouvrages de mathématiques dans l'histoire. Entre recherche, enseignement et culture*. Limoges: PULIM. T3b

For contents & summary, see <http://www.pulim.unilim.fr/index.php/notre-catalogue/fiche-detaillee?task=view&id=750> (accessed 18/2/2016).

## 2014

Matthews, M. (Ed) *Int. Hand. Res. Hi. Phil. Sci. Teach.* Dordrecht: Springer. T1, 4, 5, 3b

Of the 76 chapters, 7 are devoted to ME, 4 of which concern the HPM perspective.

## 2015

Barbin, É. & Cléro, J.-P. (Eds) *Les mathématiques et l'expérience: ce qu'en ont dit les philosophes et les mathématiciens.* Paris: Hermann. T5, 6

17 contributions on philosophical issues, pertinent to mathematics and its epistemological characteristics, on the basis of texts of various philosophers through the ages; for details see <http://les-livres-de-philosophie.blogspot.gr/2015/10/barbin-evelyne-clero-jean-pierre-dir.html> (accessed 18/2/2016).

Barbin, É. (Ed) *Les Constructions Mathématiques avec des Instruments et de Gestes (Mathematical Constructions: making and doing).* Paris: Ellipses-Editions. T2b

Any study of geometry involves both thinking and doing and it is through the doing (drawing, measuring, copying) that we develop a sense of what geometry is. School mathematics has today lost much of its geometry. It is in an attempt to recall this loss, and to report on the history of geometrical constructions, that provides the spur for this collection (see [HPM Newsletter No 89/2015](#), pp.13-14).

Rowe, D.E., & Horng, W.S. (Eds) *A Delicate Balance: Global Perspectives on Innovation and Tradition in the History of Mathematics: A Festschrift in Honor of Joseph W. Dauben.* Basel: Birkhäuser. T6

A collection of contributions on various historical issues, from the perspective and/or in the context of different cultures; for content & summary see <http://www.springer.com/gb/book/9783319120294> (accessed 18/2/2016).

## 2016

Larvor, B. (Ed.). *Mathematical Cultures: The London Meetings 2012-2014.* Berlin: Birkhäuser. T6, 1,

A collection of 23 contributions coming out of 3 meetings on exploring “mathematical cultures” from various perspectives, partly motivated by “a cultural turn in mathematics education research”; for content & summary see <http://www.springer.com/gp/book/9783319285801> (accessed 29/11/2016).

## 2018

Auvinet, J., Moussard, G., & Saint Raymond, X. (Eds.). *Circulation: Mathématiques, histoire, enseignement.* Presses universitaires de Limoges.

Barbin, É., Guichard, J.-P., Moyon, M., Guyot, P., Morice-Singh, C., Métin, F., Bühler, M., Tournès, D., Chorlay, R., Hamon, G. *Let history into the mathematics classroom.* Cham: Springer. T2b, 4, 6

English trans. from two French-language editions: É. Barbin (ed.) *De grands défis mathématiques d'Euclide a Condorcet* (2010), and É. Barbin (ed.) *Les mathématiques éclairées par l'histoire* (2012). A collection of 10 experiments which introduce historical perspectives into mathematics classrooms for 11 to 18-year-olds, suggesting that students should not only read ancient texts, but also should construct, draw and manipulate. The different chapters refer to ancient Greek, Indian, Chinese and Arabic mathematics as well as to contemporary mathematics, all having, as their starting points, historic problems—mathematical or not; for content & summary see <https://www.springer.com/la/book/9783319571492> ; see also [HPM Newsletter, No97/2018](#), pp.12-13 (accessed 15/5/2018).

Barbin, É., Bénard, D. & Moussard, G. (Eds.). *Les mathématiques et le réel: expérience, instruments, investigations.* Rennes: Presses Universitaires de Rennes.

This collection of fourteen essays is drawn from presentations made at an IREM conference of the same title, which took place in Le Mans in 2015. The authors show how there is a two-way relationship between

mathematics and the real world (see [HPM Newsletter, No98/2018](#), pp.10 (accessed 9/7/2018)).

Clark, K.M., Kjelden, T.H., Schorcht, S., & Tzanakis, C. (Eds) *Mathematics, Education and History: Towards a harmonious partnership*. Cham: Springer.

A collection of 17 chapters from 9 countries, originally presented in a shorter form at ICME-13, and an introductory chapter by its co-editors with an outline of recent developments in exploring the role of History and Epistemology in ME. It aims to serve as a valuable contribution in this domain, by making available reports on recent developments in this field, since 2000; for content & summary see <https://www.springer.com/gp/book/9783319739236> (accessed 15/5/2018)

Moyon, M. & Tournès D. (Eds) “*Passerelles*”: *Enseigner les mathématiques par leur histoire au cycle 3*. Commission Inter-IREM “épistémologie et histoire” Bouc-Bel-Air: ARPEME. [T4, 2](#)

This collective volume is based on the joint work of 9 French IREMs. It presents teaching sequences based on historical resources (original texts, drawings, or material artifacts), designed for and implemented in the French "cycle 3" (CM-1-CM-2-6e). A documented website accompanies the book to make all the necessary files accessible with a selected bibliography (<http://www.univ-irem.fr/spip.php?rubrique505>; accessed 17/5/2018).

Shell-Gellasch, A., & Jardine, D. (Eds) *The Courses of History: Ideas for Developing a History of Mathematics Course*, Washington, DC: MAA. [T1, 3b, 4, 5, 6](#)

## 2019

Chevalarias, N., Gandit, M., Morales, M. & Tournès, D. (2019). *Mathématiques récréatives. Éclairages historiques et épistémologiques*. Grenoble: UGA Éditions.

Collection of 10 chapters on mathematical games as recreational mathematics in historical perspective, structured along four main themes: Games of the society, or mirrors of a society?; Portraits of the authors of recreational problems in their era; Variations on combinatorics and algorithms; When recreational mathematics enter in the classroom.

(<https://laboutique.edpsciences.fr/produit/1062/9782759823192/Mathematiques%20recreatives>; accessed 21/2/2019).

### 3.1.2 Special issues of international journals of ME

## 2004

Siu & Tzanakis (see section 5). [T1, 2, 3a, 5](#)

A special double issue with 10 papers originally presented at ICME 10, TSG 17 “The role of the History of Mathematics in Mathematics Education” on epistemological issues; teacher education; didactical material; particular examples.

## 2007

Furinghetti, F., Radford, L. & Katz, V. (Eds) *The History of Mathematics in Mathematics Education: Theory and Practice*. *ESM* **66**(2), 107-271. [T1, 2a, 3b, 4](#)

A special issue with 10 papers, seeking to deepen the understanding of the pedagogical role HM may play in contemporary ME. Some provide examples of the use of the HM in school practice and teacher education; others address theoretical questions that have become crucial to understanding the profound intertwining of past and present, conceptual developments on spreading new epistemologies and theories of learning.

## 2010

Stedall, J. (Ed) Special Issue: The history of mathematics in the classroom. *BSHM* **25**(3), 131-179. [T2a](#)

4 articles on the HM in classroom practice, from teaching basic arithmetic in primary school to teaching statistics

to young adults.

## 2014

Clark, K. & Thoo, J.B. (Eds) *The Use of History of Mathematics to Enhance Undergraduate Mathematics Instruction*. *PRIMUS* **24**(8), 663-773. [T2, 4, 6](#)

8 papers focused on undergraduate math teaching and learning using HM as a vehicle: 4 on some aspect of including HM within coursework required of undergraduate students; 4 on authors' experiences with developing and using primary source material with undergraduate math students. Results from: classroom experiments; teaching HM courses for math, ME, or philosophy majors; ways of integrating original sources in the undergraduate classroom.

Katz *et al* (see section 5). [T1, 2a, 5](#)

Special issue with an introduction accompanied by an extensive bibliography and 12 papers directly related to the HPM perspective, divided into 4 sections: theoretical issues in the use of the HM in teaching; direct uses of the HM in the classroom; HM in teacher education; relations between the philosophy, the epistemology, the teaching and the sociology of mathematics.

Kourkoulos, M. & Tzanakis, C. (Eds) *History of Mathematics and Mathematics Education. Education Sciences*. Special Issue for 2014, 5-198. [T1, 2a, 3a](#)

Bilingual issue with 9 papers related to the *HPM perspective* (3 in English, 6 in Greek): 3 concern general ideas, conceptual frameworks and methodological schemes and 5 refer to specific issues with focus on classroom implementations (from elementary school to the university).

Lawrence, S. & O. Djokic (Eds.) Special Issue: History of Mathematics in Education and History of Mathematics Education – Mathematical Education Cultures. *Journal Teaching Innovations*, **27**(3) 9-126.

Special issue of a Serbian journal with 9 papers in English related to various aspects of the HPM perspective; among others, on teachers' implementation of the HM in the classroom, the use of didactical original sourced and the various traditions on the educational practices used in different countries and societies concerning the integration of the HM in MT.

## 2015

Karam, R. (Ed) Thematic issue: The Interplay of Physics and Mathematics: Historical, Philosophical and Pedagogical Considerations, *S&E* **24** (5-6), 487-805. [T1, 5, 6, 2a](#)

11 papers challenging the typical situation found in educational contexts: In physics education, math is a mere tool to describe and calculate; in ME, physics is only a possible context for applying mathematical concepts previously defined abstractly. Overcoming this dichotomy (which creates significant learning problems for the students) demands a systematic research effort in different fields, especially when aiming at informing educational practices by reflecting on historical, philosophical and sociological aspects of scientific knowledge.

## 2016

Nikolantonakis, K.: [The use of History in Mathematics Education](#), *Menon*, Thematic Issue **2**, 4-158. [T2a, T4, T5](#)

Thematic bilingual issue with 8 papers related to the *HPM perspective* (6 in English, 2 in French): Most of them refer to/are based on empirical studies (4 on elementary and/or junior high school and 4 on pre- and in-service teacher education) and are related to the use of original sources, either directly in the classroom, or/and as a source of inspiration and rethinking on basic issues in ME.

## 2022

Clark, K. M., Chorlay, R., & Tzanakis, C. (Eds), Exploring the significance of the History of Mathematics in Mathematics Education, *ZDM-Mathematics Education*, **54**(7), 1407-1670.

1 survey paper and 17 research papers on recent developments in the HPM domain, structured along five main themes: Theoretical and/or conceptual frameworks for integrating history in mathematics education; History and epistemology of mathematics in students' and teachers' mathematics education at all levels of instruction; Original historical sources and their educational effects; Surveys on the existing uses of history or epistemology in curricula, textbooks, and/or classrooms; History and epistemology of mathematics as a tool for an interdisciplinary approach in the teaching and learning of mathematics.

### **3.1.3 Proceedings of conferences and meetings (with reference to their accessibility via the Internet, wherever possible)**

#### **2000**

Horng & Lin (see section 5).

5 plenary lectures, 5 round tables, 4 panels, 35 oral presentations, 1 workshop with special emphasis on the effectiveness of the HPM perspective, cultural aspects and history of ME.

#### **2004**

Horng *et al* (see section 5).

Proceedings of a meeting preceding HPM 2004; see HPM Newsletter No55/2004, pp.12-14; No56/2004, p.16.

#### **2006**

Furinghetti *et al* (see section 5). [T4, 1, 2a](#)

Workshop on the original sources in ME. Research questions were identified which evolved from work in the past and are helpful in orienting future work. They reflect central issues related to the integration of original sources from the HM into ME, both in learning and teaching mathematics (see [HPM Newsletter, No62/2006](#), pp.7-10).

Furinghetti, F., Kaisjer, S. & Tzanakis, C. (Eds) [Proc. of HPM 2004 & ESU 4](#), Iraklion: University of Crete, Greece (accessed 18/2/2016).

Revised edition of the *HPM 2004 of ICME 10 & the 4th ESU* proceedings (Furinghetti *et al* 2004). 78 papers divided into 6 sections, corresponding to the 6 main themes of this joint meeting.

#### **2008**

Barbin *et al* (see section 5).

78 full papers & 42 abstracts, in 6 sections corresponding to the ESU 5 six main themes: 6 plenary lectures, 2 panel discussions, 19 workshops based on didactical & pedagogical material, 25 workshops based on historical & epistemological material, 44 oral presentations and 26 short communications.

Cantoral *et al* (see section 5).

70 papers on the 6 themes of the meeting, stemming from 4 plenary lectures, 5 workshops and 61 oral presentations.

#### **2010**

Furinghetti, F., Dorier, J.-L., Jankvist, U.T., van Maanen, J. & Tzanakis, C. (Eds) [WG 15: Theory and research on the role of history in mathematics education](#). In V. Durand-Guerrier, S. Soury-Lavergne, F. Arzarello (Eds), [Proc. of CERME 6](#) (pp.2677-2810). Lyon: Institut National De Recherche Pédagogique. [T2a, 4, 3b, 5](#) (accessed 18/2/16)

Group work structured along 8 main themes, leading to 14 papers included in this proceedings.

#### **2011**

Barbin *et al* 2011a (see section 5).

55 texts and 35 abstracts divided into 6 sections corresponding to the ESU 6 six main themes (see [HPM Newsletter No79/2012](#), pp.9-11).

Jankvist, U.T., Lawrence, S., Tzanakis, C. & van Maanen, J. (Eds) WG 12: History in Mathematics Education. In M. Pytlet, T. Rowland, E. Swoboda (Eds), [\*Proc. of CERME 7\*](#) (pp.1636-1779). Rzeszow, Poland: University of Rzeszow. [T1, 2a, 4](#) (accessed 18/2/2016).

Group work structured along 4 general topics, leading to 15 papers included in this proceedings. Topics: Research questions and relevance of research; use of HPM theory and ME theory; methods, data and analysis; validity, reliability and generality of research results.

## 2012

Barbin *et al* (sec. 5).

78 contributions on the 7 themes of the meeting, stemming from 7 plenary lectures, 7 workshops, 57 oral presentations, 5 posters & 2 exhibitions. Revised version in progress.

## 2013

Barbin, É. & Moyon, M. (Eds). *Les ouvrages de mathématiques dans l'histoire – Entre recherche, enseignement et culture*. Limoges: PULIM. [T2b, T3b](#) (<http://www.cfem.asso.fr/publications/barbin-moyon-ouvrages-mathematiques-histoire> accessed 21/2/2019)

Proceedings of the 19th “Colloque inter-IREM Épistémologie” with 22 papers examining in historical perspective the boundaries between mathematical books focused on research and those addressed to education and culture, and crossing them in order to explore their very existence, production, their authors’ visions, their readers’ expectations and reception.

Jankvist, U.T., Clark, K., Lawrence, S. & van Maanen, J. (Eds) WG 12: History in Mathematics Education. In B. Ubuz, Ç., Haser, & M.A. Mariotti (Eds) [\*Proc. of CERME 8\*](#) (pp.1945-2067). Ankara: Middle East Technical University. [T2a, 4, 5, 6](#) (accessed 18/2/2016).

Group work structured along 5 general themes leading to 13 papers and 3 posters included in this proceedings. Themes: interdisciplinarity; theoretical frameworks in history of ME; history in pre high school ME; history in high school ME; HM in teacher education and design.

## 2015

Barbin *et al* (see section 5). [T1, 2a, 2b, 4, 6](#)

53 texts and 34 abstracts corresponding to all types of activities during ESU 7, divided into 8 sections corresponding to its 7 themes and a poster session.

Jankvist, U. T., Lawrence, S., Clark, K., & Chorlay, R. WG 12: History in Mathematics Education. In K. Krainer & N. Vondrová (Eds) [\*Proc. of CERME 9\*](#) (pp.1778-1884). Prague: Charles University in Prague, Faculty of Education & ERME (accessed 4/4/2016).

Group work structured along 9 themes leading to 14 papers and 2 posters included in this proceedings, dealing with four areas of questions on history in ME-the student perspective; history in ME-the teacher perspective; history of ME-the mathematical education landscape; methodological reflections on history in/of ME.

## 2016

Radford, L., Furinghetti, F., Hausberger, T. (Eds.) (2016). *Proc. of the 2016 ICME Satellite Meeting – HPM 2016*, Montpellier: IREM de Montpellier.

84 contributions on the 7 themes of the meeting, stemming from 9 plenary lectures and presentations, 2 discussion groups, 2 panels, 15 workshops, 45 oral presentations, 9 posters & 2 exhibitions.

## 2017

Chorlay, R., Clark, K., Gosztonyi, K. & Lawrence, S. (Eds.). WG 12: History in Mathematics Education. In T. Dooley & G. Gueudet (Eds.), [\*Proceedings of CERME 10\*](#) (pp.1680–1815). Dublin: Dublin City University (accessed 18/2/2019)

Group work structured along 4 main themes, leading to 16 papers and 2 posters included in this proceedings concerning: the design and/or assessment of teaching/learning materials using the history of mathematics; surveys on the existing uses of history or epistemology in mathematics education; theoretical and methodological issues also in relation to other areas of mathematics education; and the history of mathematics education.

### 2019

Chorlay, R., Bernardes, A., Hamann, T. & Oler-Marcén, A. (Eds.). WG 12: History in Mathematics Education. In U. T. Jankvist, M. van den Heuvel-Panhuizen, M. Veldhuis (Eds.), [\*Proceedings of CERME 11\*](#) (pp.2089–2203). Utrecht: Freudenthal Group & Freudenthal Institute, Utrecht University, Netherlands and ERME (accessed 23/4/2023)

Barbin *et al* (see section 5)

### 2022

Chorlay, R., Hamann, T., Krüger, J. & Oler-Marcén, A. (Eds.). WG 12: History in Mathematics Education. In J. Hodgen, E. Geraniou, G. Bolondi & F. Ferretti (Eds.), [\*Proceedings of CERME 12\*](#) (pp.2010–2122). Bolzano: Free University of Bozen-Bolzano, Italy and ERME (accessed 23/4/2023)

**3.1.4 Resource material**, collectively produced, especially material available on the web.

### 2002

Bazin, M., Tamez, M. & Exploratorium Teacher Institute. *Mathematics and Science across Cultures. Activities and investigations from the Exploratorium*. New York: The New Press. [T2b, 6](#)

Addressed to grades 4-12 teachers, wishing to develop hands-on activities in classroom suggested by artefacts or instruments produced by different current and past civilisations.

### 2004

Barnett, J., Bezhanishvili, G., Leung, H., Lodder, J., Pengelley, D. & Ranjan, D. [\*Teaching discrete mathematics via primary historical sources\*](#) (accessed 18/2/2016). [T2a, 2b, 4](#)

Resource material on teaching projects via original sources, its rationale and extensive bibliography.

### 2005

IREM Basse-Normandie. *L'espérance du Hollandais ou le premier traité du calcul du hasard*. Paris: Ellipses. [T2b](#)

Annotated presentation of Huygens' book "De raticionis in ludo aleae" (the 1<sup>st</sup> treatise on probability calculus 1657) and the commentaries on it by its contemporaries and successors.

Katz & Michalowicz (see section 5); [T2b](#)

An electronically available resource based on work done by a larger group of people, with material on 11 mathematical modules (secondary education), useful in the classroom and/or for teachers to adapt it according to their needs and those of their classroom

### 2006

Demattè, A. & Furinghetti, F. *Fare matematica con i documenti storici: Una raccolta per la scuola secondaria di primo e secondo grado*. Trento: Editore Provincia Autonoma di Trento – IPRASE del Trentino. [T2b](#)

Includes selection of excerpts from original sources and aims to provide secondary school teachers with activities for their integration in the classroom; thus promoting alternative ways of teaching through text-based activities and

exercises in order to consolidate or/and introduce mathematical skills (one book for the teacher and one for the student; see [HPM Newsletter No64/2007](#), pp.6-8).

Groupe d'Histoire & d'Epistémologie des Mathématiques de l'IREM de Lyon. *Textes fondateurs du calcul infinitésimal*. Paris: Ellipses. T2b

Annotated texts of Newton and Leibniz on infinitesimal calculus ; for details see <http://www.decitre.fr/livres/si-le-nombre-m-etait-conte-9782729868222.html> (accessed 18/2/2016).

Swetz, F.J. *Shu Xue You Ji, (The History of Mathematics: A Study Guide)* Taipei: Transoxania Ltd (in Chinese). T2b

Based on an earlier teaching course on HM.

## 2007

Katz, V. (Ed) *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook*. Princeton: Princeton University Press. T2b, 6.

English translations of key mathematical texts from the 5 most important ancient and medieval non-Western mathematical cultures, putting them into full historical and mathematical context. A firsthand understanding and appreciation of these cultures' important contributions to world mathematics. An essential resource and indispensable guide for math teachers who want to use non-western mathematical ideas in the classroom.

Siu, M.K. [Some Useful References for Course MATH2001 \(Development of Mathematical Ideas\) Department of Mathematics, University of Hong Kong](#) (accessed 18/2/201). T2b

An extensive list of papers, collective volumes, individual books, special journal issues, websites on the HM and its relations to ME. References either in English or Chinese. A useful resource for those interested in the HPM perspective.

## 2009

Clark (see section 5). T2b, 4

Introduces undergraduate students preparing to teach mathematics to Stevin's pamphlet on decimal fractions and to encourage prospective math teachers to think about connections with pupils' initial learning of decimals' multiplication.

Pengelley *et al* (see section 5). T2b, 4

An extended presentation focusing on the pedagogy of historical projects, which offer excerpts from original sources, place the material in context, and provide direction to the subject matter.

Rogers, L. [Resources from Leo Rogers History and the Mathematics Curriculum](#) (accessed 18/2/2016). T2b

A website with a lot of information; useful websites; bibliography; suggestions/advices for HM in ME etc.

## 2012

Swetz, F.J. *Mathematical Expeditions: Exploring Word Problems across the Ages*. Baltimore, MD: Johns Hopkins University Press. T2b, 4, 6

Review at <http://www.maa.org/press/periodicals/convergence/review-of-emmathematical-expeditions-exploring-word-problems-across-the-agesem> (accessed 18/2/2016)

## 2013

Barnett, J., Bezhaniashvili, G., Leung, H., Lodder, J., Pengelley, D., Pivkina, I., Ranjan, D. & Zack, M. [Primary Historical Sources in the Classroom: Discrete Mathematics and Computer Science](#). *Convergence*, 10. DOI:10.4169/loci003984 T2b

Presentation of 16 separate curricular modules, each a project for students based on excerpts from primary historical sources, to provide context, motivation and direction for selected topics in discrete mathematics and computer science as an alternative form of instruction.



Siu, M.K. A reading list for teachers interested in history of mathematics.  
See [http://hkumath.hku.hk/~mks/Whynot%20Hsth\\_MKS\\_XZYS.pdf](http://hkumath.hku.hk/~mks/Whynot%20Hsth_MKS_XZYS.pdf) and  
<http://hkumath.hku.hk/~mks/HistMathTeachingReadingListChinese.pdf> (accessed 18/2/2016).  
T2b

An interesting list of books, collective volumes, and papers on the HM and on history in ME.

## 2014

Friedelmeyer, J-P. & Lubet, J-P. (Eds) *L'analyse algébrique – un épisode clé de l'histoire des mathématiques*. Paris: Ellipses. T2b

Original texts by Euler, Lagrange, Brisson, Servois and others, contextualized by appropriate comments and introductions.

Pengelley, D. & Laubenbacher, R. (see section 5). T2a, 2b, 4

A website with information and materials on using original historical sources in math teaching, including the authors' own experiences and materials, and those of others.

## 2015

Commission interIREM, Commission IREM de Basse-Normandie, Barbin, É. & Legoff, J-P. *Si le nombre m'était compté*. Paris: Ellipses. T2b

A useful resource for the generalization of the number concept beyond natural numbers

## 3.2 Individual books and papers

### 3.2.1 Books and Doctoral Dissertations

#### 2000

Kvasz, L. (2000). *Patterns of Change – Linguistic Innovations in the Development of Classical Mathematics*. Basel: Birkhäuser. T1, 6

#### 2001

Awosanya, A. (2001). *Using history in the teaching mathematics*. Dissertation. Florida State University, USA (accessed 30/11/16). T2a

Marshall, G.L. *Using history of mathematics to improve secondary students' attitudes towards mathematics*. Dissertation. Illinois State University, Bloomington–Normal, Illinois. T2a

#### 2002

Pizzamiglio, P. *Matematica e Storia. Per una didattica interdisciplinare*. Brescia: Ed. La Scuola. T5

van Amerom, B.A. (see section 5). T2, T1, T3a

#### 2003

Scimone, A. *Pupils' conceptions about a historical open question: Goldbach's conjecture. The improvement of mathematical education from a historical viewpoint*. Dissertation. University of Bratislava, Bratislava. T1, 2a

#### 2004

da Silva Souza, E. *A prática social do cálculo escrito na formação de professores: A história como possibilidade de pensar questões do presente*. Dissertation. UNICAMP, Campinas, Brazil. T2a

## 2004

Percival, I. [\*The use of cultures perspectives in the elementary school mathematics classroom.\*](#) Dissertation, Simon Fraser University, Canada. T2, 6

## 2005

Schubring, G. *Conflicts between Generalization, Rigor, and Intuition: Number Concepts Underlying the Development of Analysis in 17–19th Century France and Germany*, New York: Springer.

Su (see section 5). T2a

van Gulik-Gulikers, I. *Een studie naar de waarde en de toepassing van de geschiedenis van de meetkunde in het wiskundeonderwijs*. Dissertation. Rijksuniversiteit Groningen, The Netherlands. T2a, 3a, 5

## 2006

Bagni, G.T. *Linguaggio, Storia e Didattica della Matematica*. Bologna: Pitagora Editrice.

Clark (see section 5). T2a, 3a

Thomaidis, Y., Petrakis, Y., Touloumis, K. & Stafylidou M. *Language, History and Euclidean Geometry*. Thessaloniki: University of Macedonia (in Greek). T2a, 2b, 4, 5

## 2007

Goodwin, D.M. [\*Exploring the relationship between high school teachers' mathematics history knowledge and their images of mathematics.\*](#) Dissertation. University of Massachusetts, Lowell MA (accessed 18/2/2016). T3a, 2a

Knoebel, A., Laubenbacher, R., Lodder, J. & Pengelley, D. *Mathematical masterpieces – further chronicles by the explorers*. New York: Springer. T2b

## 2008

de Jesus Brito, A. *A Geometria de Euclides a Lobatschweski. Um estudo histórico-pedagógico*. Natal, RN, Brazil: Editora da UFRN. T2b

Fillooy, E., T. Rojano & L. Puig. *Educational Algebra. A Theoretical and Empirical Approach*. New York: Springer. T1, 5, 2a

Haile, T. [\*A Study on the Use of History in Middle School Mathematics: The Case of Connected Mathematics Curriculum.\*](#) Dissertation. Faculty of the Graduate School of the University of Texas, Austin (accessed 18/2/2016). T3a,b, T1

Stedall, J. *Mathematics Emerging: A Sourcebook 1540 – 1900*. Oxford: Oxford UP. T2b, 4

## 2009

Jankvist (2009a) (see section 5). T1, 2a

## 2010

Demattè, A. (2010). *Vedere la matematica. Noi, con la storia*. Trento: Editrice UNI Service. T2b

Stein (see section 5). T2b

Wardhaugh, B. *How to Read Historical Mathematics*. New Jersey: Princeton University Press. T4

Glaubitz (see section 5). T2a, T4

## 2011

Horton, L. B. (2011). [High school teachers' perception of the inclusion of history of mathematics in the classroom](#). Dissertation. University of Massachusetts Lowell. (accessed 30/11/16) T2a

Roth, W-M., Radford, L. (2011) *A cultural historical perspective on teaching and learning*. Rotterdam: Sense Publishers. T1, T2a

## 2012

Ostermann, A. & Wanner, G. *Geometry by its History*. Berlin: Springer. T2b, T4, T5

## 2013

Yıldız, C. (2013). *Analysis of history of mathematics usage of secondary school mathematics teachers on their lessons: Reflections on in-service training*. Dissertation. Karadeniz Technical University, Trabzon, Turkey. T2

## 2014

Katz, V.J & Hunger Parshall, K. *Taming the Unknown: A History of Algebra from Antiquity to the Early Twentieth Century*. Princeton, NJ: Princeton University Press.

## 2015

Shell-Gellasch, A. & Thoo, J. *Algebra in Context: Introductory Algebra from Origins to Applications*. Baltimore, MD: Johns Hopkins University Press. T2b

Guevara, I. [L'ús de contextos històrics a l'aula de matemàtiques de secundària: El cas concret de la visualització en la connexió geometria-àlgebra](#). Dissertation. Universitat de Barcelona. (accessed: 29/11/2016). T2a, 4

Guillemette, D. [L'histoire des mathématiques et la formation des enseignants du secondaire: sur l'expérience du dépaysement épistémologique des étudiants](#). Dissertation. Montréal: Université du Québec à Montréal (accessed 15/6/2016). T1, T2a, T4, T5

## 2018

Barbin, É., Moussard, G. & Bénard, D. *Les mathématiques et le réel. Expériences, Investigations, Instruments*. Rennes: PUR. T1, T5

de Varent, C. Pluralité des concepts liés aux unités de mesure. Liens entre histoire des sciences et didactique, le cas de l'aire du carré dans une sélection de textes anciens (Doctoral dissertation). University Paris 7 Diderot. <https://tel.archives-ouvertes.fr/tel-02411352>

Moyon, M. & Tournès, D. *Passerelles: Enseigner les mathématiques par leur histoire au cycle 3*. Bouc-Bel-Air: Arpeme. T1, T2, T6

Schorcht, S. *Typisierung mathemathikhistorischer Beispiele in deutschen Mathematikschulbüchern der Klassenstufe 1 bis 7*. Münster: WTM-Verlag. T2b, T3b

## 2019

Barbin, É. *Faire des mathématiques avec l'histoire au lycée*. Ellipses.

## 2020

Moustapha-Corrka, B. Rumo a uma postura problematizadora na formação de professores de matemática: Articulando práticas históricas e práticas de sala de aula. (Doctoral dissertation, Universidade Federal do Rio de Janeiro). <https://pemat.im.ufrj.br/>

### 3.2.2 Individual papers in Scientific Journals

#### 2000

- Arcavi, A., & Bruckheimer, M. Didactical uses of primary sources from the history of mathematics. *Themes in Education*, **1**(1), 55–74. T4, T2
- Barbin, É. Que faut-il enseigner? Pour qui? Pourquoi? Des réponses dans l'histoire des mathématiques, in *R-IREM*, **38**, 43-51. T2b
- Barbin, É. Construire la géométrie élémentaire. *R-IREM*, **40**, 5-9. T1, 5
- Barbin, É. Pourquoi démontrer? Les mathématiques grecques, *Cahiers de Science & Vie*, **55**, 42-49. T1
- Bkouche, R. Sur la notion de perspective historique dans l'enseignement d'une science. *R-IREM*, **39**, 35-59. T1
- Furinghetti, F. The history of mathematics as a coupling link between secondary and university teaching. *Int. J. ME Sci. Tech.*, **31**(1), 43–51. T1, 2
- Horng, W-S. Nam Byung Gil's comments on the "Tian Yuan Shu" versus "Jie Gen Fang": An HPM perspective. *Chinese Journal of Science Education*, **8**(3), 215-224 (in Chinese). T2b
- Krussel, L. [Using history to further the understanding of mathematical concepts](#). *PRIMUS*, **10**(3), 273-276. T2a
- Lawrence, H. S. A Visit from Pythagoras--Using Costumes in the Classroom. *MT*, **93**(8), 652-55. T2, 6
- Li, B.C. A survey on mathematics teachers' knowledge about the history of mathematics. *MB*, **39**(3), 39-40. T3a
- Matos, J. The historical development of the concept of angle (1). *TME*, **10**(2), 49-56. T2
- Testa, G. L'enseignement des coniques à travers une approche historique: comment saisir un texte? *R-IREM*, **41**, 105-119. T4, T2
- Tzanakis, C. & Thomaidis, Y. Integrating the close historical development of mathematics and physics in mathematics education: Some methodological and epistemological remarks. *FLM*, **20**(1), 44–55. T1, 5
- Wilson, P., & Chauvot, J. (2000). Who? How? What? A strategy for using history to teach mathematics. *MT*, **93**(8), 642-645. T2, 1

#### 2001

- Barbin, É. Qu'est-ce que faire de la géométrie? *R-IREM*, **43**, 59-83. T1, T5, T4, T6
- D'Ambrosio, U. What is ethnomathematics, and how can it help children in schools? *Teaching Children Mathematics*, **7**(6), 308., T6, T1
- Filep, L. The development, and the developing of, the concept of a fraction, *International Journal for Mathematics Teaching and Learning*. Available from <http://www.cimt.plymouth.ac.uk/journal/lffract.pdf> (accessed 18/2/201). T2b
- Fried (see section 5). T1
- Friedelmeyer, J.-P. Grandeurs et nombres: l'histoire édifiante d'un couple fécond. *R-IREM*, **44**, 5-31. T2a
- Furinghetti, F. & Somaglia, A. The method of analysis as a common thread in the history of algebra: reflections for teaching. *Themes in Education*, **2**(1), 3-14. T2a, 4, 1
- Jahnke, H.N. Cantor's cardinal and ordinal infinities: An epistemological and didactic view.

*ESM*, **48**, 175–197. [T1](#), [2](#)

Lit, C-K., Siu, M.K. & Wong, N.Y. The use of history in the teaching of mathematics: Theory, practice, and evaluation of effectiveness. *Educational Journal*, **29**(1), 17–31. [T2](#), [1](#)

Mayfield, B. A history of mathematics course as a senior seminar. *PRIMUS*, **11**(3), 245 - 257. [T2](#), [1](#)

Mendez, E.P. A history of mathematics dialogue in textbooks and classrooms, *MT* **94**(3), 170-173. [T2](#), [3b](#)

Percival, I. An artefactual approach to ancient arithmetic. *FLM*, **21**(3), 16-21. [T2a](#)

Povey, H., Elliott, S. & Lingard, D. The study of the history of mathematics and the development of an inclusive mathematics: Connections explored. *MER*, **14**, 8–18. [T2a](#), [1](#)

van Gulik-Gulikers, I. & Blom, K. ‘A historical angle’, a survey of recent literature on the use and value of history in geometrical education. *ESM*, **47**(2), 223–258. [T3](#)

## 2002

Boyé, A. & Comairas, M.-C. Moyenne, médiane, écart-type, quelques regards sur l’histoire pour éclairer l’enseignement des statistiques au lycée. *R-IREM* **48**, 27-40. [T2b](#), [5](#)

Fleener, M., Reeder, S., Young, E. & Reylands, A. History of mathematics: Building relationships for teaching and learning. *Action in Teacher Education*, **24**, 73-84. [T2a](#)

Miller, C. C. Teaching the history of mathematics. *PRIMUS*, **12**(4), 334 - 346. [T2a](#), [6](#)

Nooney, K.A. Critical question: Why can’t mathematics education and history of mathematics coexist? *TME*, **12**(1), 1–5. [T1](#)

Quinton, P. Activités mathématiques à propos de la mesure de la Terre. *R-IREM*, **49**, 73-92. [T2b](#), [a](#), [5](#)

## 2003

Friedelmeyer, J.-P. Euclide peut-il encore apprendre quelque chose au professeur de mathématiques d’aujourd’hui ? *R-IREM*, **53**, 23-42. [T2b](#), [4](#)

Furinghetti, F. & Paola, D. (2003). History as a crossroads of mathematical culture and educational needs in the classroom. *Mathematics in School*, **37**(1), 37-41. [T2b,a](#), [1](#)

Glaubitz, M. & Jahnke, H. N. Die Bestimmung des Umfangs der Erde als Thema einer mathemathikhistorischen Unterrichtsreihe, *Journal für Mathematik-Didaktik*, **24**(2), 71-95. [T4](#)

Glaubitz, M. & Jahnke, H. N. Texte lesen und verstehen. Eine Erwiderung. *Journal für Mathematik-Didaktik*, **24** (3/4), 252-260. [T4](#)

Guichard, J.-P. D’un problème de Diophante aux identités remarquables. *R-IREM*, **53**, 5-19. [T2b](#), [4](#), [6](#)

Jahnke, H. N. Numeri absurdi infra nihil. Die negativen Zahlen. *Mathematik lehren*, **121**, Dez. 2003, 21-22; 36-40. [T2b](#)

Liu (see section 5). [T1](#)

Massa, M. R. Aportacions de la història de la matemàtica a l’ ensenyament de la matemàtica. *Biaix*, **21**, 4-9. [T2](#)

Ransom, P. Drawing instruments – their history and classroom use. *BSHM*, **47**, 53-56 & **48**, 45-48. [T2b](#)

Streefland, L. Learning from history for teaching in the future. *ESM*, **54**, 37–62. [T1](#), [5](#), [2a](#)

Tournès, D. Du compas aux intégraphes: les instruments du calcul graphique.

*R-IREM*, **50**, 63-84. [T2b](#)

van Amerom, B. A. Focusing on informal strategies when linking arithmetic to early algebra.

*ESM*, **54**, 63–75. [T2a](#), [T4](#)

van Gulik-Gulikers, I. The seventeenth-century surveyor in class. *BSHM*, **47**, 56-63. [T2a](#), [4](#)

Wang, X.Q., & Ouyang, Y. Historical notice on HPM. *JME*, 12(3): 24-27. [T1](#)

## 2004

Arcavi A. Solving linear equations – why, how and when? *FLM* **24**(3), 24–28. [T2a](#), [1](#)

Barbin, É. L’outil technique comme théorème en acte, *Sciences et avenir*, **140**, 26-27. [T1](#)

Grattan-Guinness 2004a (see section 5). [T1](#)

Grattan-Guinness 2004b (see section 5). [T1](#)

Keiser, J.M. Struggles with developing the concept of angle: Comparing sixth-grade students’ discourse to the history of the angle. *MTL*, **6**(3), 285-307. [T2a](#), [T1](#)

Kidron, I. (2004). Polynomial approximation of functions: Historical perspective and new tools. *International Journal of Computers for Mathematical Learning*, **8**(3), 299–331.

Li, H.T. The new horizon of the curriculum reform: The history of mathematics entering the new mathematics curriculum. *CTMM*, **25**(9), 51-64. [T1](#)

Lopez-Real, F. Using the history of mathematics as a starting point for investigations: some examples on approximations. *Teaching Mathematics Applications*, **23**(3), 133-147. [T2b](#)

Mercier J.-P. Le problème des cinq carrés ou comment montrer l’intérêt des identités remarquables. *R-IREM*, **57**, 47-67. [T2b](#), [4](#), [6](#)

Merker, C. La méthode des indivisibles racontée lors d’un stage. *R-IREM*, **54**, 57-76. [T2b](#), [4](#)

Métin, F. Avez-vous lu Euclide, *R-IREM*, **55**, 101-103. [T2b](#)

Michalowicz, K. & McGee, R. [Using historical problems in the middle school](#), *Convergence*, **1** (accessed 18/2/2016). [T2b](#), [4](#)

Swetz, F.J. [Using problems from the history of mathematics](#), *Convergence*, **1** (accessed 18/2/2016). [T2b](#), [T6](#), [T4](#)

## 2005

Burn, B. The vice: Some historically inspired and proof-generated steps to limits of sequences. *ESM*, **60**, 269–295. [T2b](#)

Carroget, L. & Gairin C. Les Mathématiques sans ignorer nos anciens. *R-IREM*, **58**, 5-15. [T2a](#), [1](#)

Dubinsky, E., Weller, K., McDonald, M.A. & Brown, A. Some historical issues and paradoxes regarding the concept of infinity: An APOS-based analysis: Part 1, *ESM*, **58**(3), 335-359. [T1](#)

Durand-Guerrier, V. & Arsac, G. An epistemological and didactic study of a specific calculus reasoning rule, *ESM*, **60**(2), 149 – 172. [T2a](#), [5](#), [1](#)

Massa, M. R. Les equacions de 2n grau al llarg de la història. *Biaix*, **24**, 4-15. [T2](#)

Su, Y.W. Designing HPM work sheets. *Taiwan Journal of Mathematics Teachers*, **2**, 2-13 (in Chinese). [T2b](#)

Wang, X.Q. The Historical-genetic principle viewed from a test on the imaginary numbers and divergent series. *JME*, **14**(3), 30-33. [T3b](#)

## 2006

- Bakker A. & Gravemeijer, K P.E. An historical phenomenology of mean and median. *ESM*, **62**(2), 149 – 168. T1, 2a
- Barbin, É. Apports de l’histoire des mathématiques et de l’histoire des sciences dans l’enseignement, *Tréma*, **26**, 20-28. T1, 5
- Lawrence, S. [Maths is good fot you: web - based history of mathematics resources for young mathematicians and their teachers](#). *BSHM Bulletin* **21**, 90-96. T2a, b
- Leng, N. W. [Effects of an ancient chinese mathematics enrichment programme on secondary school students’ achievement in mathematics](#). *International Journal of Science and Mathematics Education*, **4**, 485-511. T2a, 6, 4
- Liu, P-H. & Niess, M.L. An exploratory study of college students’ views of mathematical thinking in a historical approach calculus course. *MTL*, **8**(4), 373-406/432. T2a
- Martínez-Torregrosa, J., López-Gay, R. & Gras-Martí, A. Mathematics in physics education: Scanning historical evolution of the differential to find a more appropriate model for teaching differential calculus in physics. *S&E*, **15**(5), 447–462. T5
- Savizi, B. Applicable problems in history of mathematics; practical examples for the classroom, *Philosophy of Mathematics Education Journal*, **19** (accessed 18/2/2016). Also in *Teaching Mathematics and its Applications*, **26** (2007), 51-54. T1, 2b
- Thiénard, J.-C. Les transformations en géométrie, introduction à une approche historique. *R-IREM*, **63**, 27-52. T2b, 4
- Tillema, E. Chinese algebra: Using historical problems to think about current curricula. *MT*, **99**(4), 238-245. T2, 6
- Wang, X.Q. & Zhang, X.M. Researches on HPM: Contents, methods and examples. *JME*, **15**(1), 16-18. T1

## 2007

- Barbin, É. L'arithmétisation des grandeurs. *R-IREM*, **68**, 5-20. T2b, 5
- Barbin, É. Les Récréations: des mathématiques à la marge, *Pour la science*, **30** 22-25. T3
- Barbin, É. Les avatars de la rigueur mathématique, *Pour la science*, **356**, 10-13. T1
- Carson, R. N. & Rowlands, S. Teaching the Conceptual Revolutions in Geometry, *S&E*, **16** (9-10), 921-954. T2b, 1
- Demattè, A. (2007). [Primary sources from the history of mathematics for secondary school students](#). *Acta Didactica Universitatis Comenianae Mathematics*, **7**, 47-66. T2b, 4, 1
- Farmaki V. & Paschos, Th. Employing genetic ‘moments’ in the history of mathematics in classroom activities. *ESM*, **66**(1), 83–106. T2a, 5
- Gérini ,C., & Verdier, N. Les Annales de Gergonne (1810-1832) et le Journal de Liouville (1836-1874): une mine de textes numérisés à exploiter dans notre enseignement. *R-IREM*, **67**, 55-68. T2b
- Guevara, I., Massa, M. R., & Romero, F. Enseñar matemáticas a través de su historia : algunos conceptos trigonométricos. *Epsilon*, **67**, 97-107. T2
- Hagerty, G.W., Smith, S. & Goodwin, D. [The unique effects of including history in college algebra](#). *Convergence*, **4**. DOI:10.4169/loci002530. T2, 4, 6
- Jankvist (see section 5). T3a
- Li, T. A. & Song, N. Q. Teaching of Analytic Geometry from the Historical Perspective. *JME*,

16(2): 90-94. [T1](#)

Meavilla, V. & Flores, A. History of mathematics and problem solving: a teaching suggestion. *Int. J. ME Sci. Tech.*, **38**(2), 253-259. [T2](#)

Ren, M. J., & Wang, X. Q. Senior high school students' understanding about the mathematical function. *JME*, **16**(4): 84-87. [T3b](#)

Thienard, J.-C. Redonner du sens aux mathématiques enseignées, *R-IREM*, **66**, 62-72. [T1](#), [5](#)

Thienard, J.-C. Introduction aux lois de probabilités continues: problèmes épistémologiques, *R-IREM*, **67**, 31-42. [T2b](#), [5](#), [4](#)

Yang, Y. H., Wei, J., & Song, N. Q. An analysis of historical materials in primary mathematics textbooks. *JME*, **16**(4): 80-83. [T3b](#)

## 2008

Bagni, G.T. A theorem and its different proofs: History, mathematics education and the semiotic-cultural perspective. *Canadian Journal of Science, Mathematics, and Technology Education*, **8**(3), 217-232. [T5](#), [1](#)

Bühler, M. & Michel-Pajus, A. Les démonstrations en arithmétique: à propos de quelques preuves historiques du petit théorème de Fermat. *R-IREM*, **71**, 23-39. [T2b](#), [4](#)

Clark, K.M. & Robson, E. Ancient accounting in the modern mathematics classroom. *BSHM* **23**(3), 129-142. [T2](#)

Doorman, M. & van Maanen, J. A historical perspective on teaching and learning calculus. *Australian Senior Mathematics Journal*, **22**(2), 4-14. [T2](#)

Fried, M.N. History of mathematics in mathematics education: A saussurean perspective. *TMMEM*, **5**(2), 185-198. [T1](#), [2](#)

Gérini, C. Variations pédagogiques sur un article de géométrie analytique d'Haton de la Goupillière, paru en 1872. *R-IREM*, **71**, 65-80. [T2b,a](#), [4](#)

Jahnke, H.N. & Richter, K. Geschichte der Mathematik. Vielfalt der Lebenswelten – Mut zu divergentem Denken. *Mathematik lehren*, **151**, Dez. 2008, 4-11. [T1](#)

Jamm, F. Le rêve de Ptolémée réalisé. *R-IREM*, **73**, 5-19. [T5](#), [2a](#), [b](#), [4](#)

Jankvist, U.T. A teaching module on the history of public-key cryptography and RSA. *BSHM*, **23**(3), 157-168. [T2](#)

Jankvist, U.T. Den matematikhistoriske dimension i undervisning—gymnasialt set (The dimension of the history of mathematics in teaching—the case of upper secondary level). *MONA*, **4**(1), 24-45 (in Danish). [T2](#), [1](#), [4](#)

Kaye, E. [The aims of and responses to a history of mathematics videoconferencing project for schools](#), *Proc. of the British Soc. for Res. into Learning Math.* **28**(3), 66-71.

Zhao, Y. Y., & Zhang, X. M. Discussions about the Historical Parallelism: an Overview. *JME*, **17**(4), 53-56. [T1](#)

## 2009

Baki, A., & Güven, B. (2009). Khayyam with Cabri: experiences of pre - service mathematics teachers with Khayyam's solution of cubic equations in dynamic geometry environment. *Teaching mathematics and its Applications*, **28**, 1-9. [T2a](#), [6](#)

Charalambus, C., Panaoura, A. & Philippou, G. Using the history of mathematics to induce changes in preservice teachers' beliefs and attitudes: insights from evaluating a teacher education program. *ESM*, **71**(2), 161-180. [T2](#)



- Henry, M. Emergence de la probabilité et enseignement: définition classique, approche fréquentiste et modélisation. *R-IREM*, **74**, 76-89. [T2b, 5, 4](#)
- Guevara, I. & Massa, M. R. La historia de les matemàtiques en els nous curriculums de secundària. *Actes d' Historia de la Ciència i de la Tècnica*, **2**, 379-390. [T3b](#)
- Jankvist 2009b (see section 5). [T1](#)
- Jankvist, U.T. On empirical research in the field of using history in mathematics education. *Revista Latinoamericana de Investigación en Matemática Educativa*, **12**(1), 67-101. [T1, 2a](#)
- Jankvist, U.T. History of modern applied mathematics in mathematics education. *FLM*, **29**(1), 8-13. [T2](#)
- Kang, S.G. & Hu, G.H. Researches on the history of mathematics and mathematics education in primary and secondary schools in China: Analysis and reflections. *JME*, **18**(5), 65-68. [T3b](#)
- Kjeldsen, T.H. & Blomhøj, M. Integrating history and philosophy in mathematics education at university level through problem-oriented project work. *ZDM*, **41**, 87–103. [T1,2a,5](#)
- Lin, M.S. & Su, Y.W. Letting mathematics being interesting by history of mathematics. *The educator monthly*, **509**, 81-83 (in Chinese). [T2a](#)
- Liu, P-H. History as a platform for developing college students' epistemological beliefs of mathematics. *Int. J. Sci. ME*, **7**(3), 473–499. [T2, 1](#)
- Mayfield, B. & Kimberly, T. A [Locally Compact REU in the History of Mathematics: Involving Undergraduates in Research](#). *Convergence*, **6**. DOI: 10.4169/loci003263. [T2a](#)
- Nataraj, M. S.,& Thomas, M. O. J. (2009). [Developing understanding of number system structure from the history of mathematics](#). *Mathematics Education Research Journal*, **21**(2), 96 - 115. [T2a, 6](#)
- Shen, J.L. & Su, Y.W. Integrating history of mathematics into elementary teaching. *New Horizon Bimonthly for Teachers in Taipei*, **163**, 70-77 (in Chinese). [T2a](#)
- Zhang, X.M. & Wang X.Q. Integrating the History of Mathematics into Mathematics Teaching in Senior High Schools: An Action Research. *JME*, **18**(4), 89-92. [T2b](#)

## 2010

- Barbin, É. Epistémologie et histoire dans la formation mathématique. *R-IREM*, **80**, 74-86. [T1, 5](#)
- Bartolini Bussi, M.G., Taimina D. & Isoda, I. Concrete models and dynamic instruments as early technology tools in classrooms at the dawn of ICMI: from Felix Klein to present applications in mathematics classrooms in different parts of the world. *ZDM*, **42**(1), 19-31. [T2](#)
- Bellomo, C. & Wertheimer, C. A discussion and experiment on incorporating the history into the mathematics classroom. *Journal of College Teaching & Learning*, **7**(4), 19-24. [T2a](#)
- Bressoud, D. M. Historical reflections on teaching trigonometry. *MT*, **104**(2), 106-112. [T2b](#)
- Burns, B. A. [Pre-service teachers' exposure to using the history of mathematics to enhance their teaching of high school mathematics](#). *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal*, **4**, 1–9. [T2a](#)
- Clark, K. M. Connecting local history, ancient history, and mathematics: The Eustis Elementary School pilot project. *BSHM*, **25**(3), 132-143. [T2](#)
- da Conceição Ferreira Reis Fonseca, M. Adult education and ethnomathematics: appropriating results, methods, and principles. *ZDM*, **42**(3), 361-369. [T2, 6, 1](#)
- Haverhals, N. & Roscoe, M. [The history of mathematics as a pedagogical tool: Teaching the](#)

- [Integral of the secant via Mercator's projection](#). *The Montana Mathematics Enthusiast*, **7**(2), 339-368. T2a, T5
- Huntley, M. A. & Flores, A. A history of mathematics to develop prospective secondary mathematics teachers' knowledge for teaching. *PRIMUS: Problems, Resources and Issues in Mathematics Undergraduate Studies*, **20**(7), 603-616. T2a
- Jankvist, U.T. An empirical study of using history as a 'goal', *ESM* **74**, 53-74. T2a
- Karaduman, G. B. (2010). [A sample study for classroom teachers addressing the importance of utilizing history of math in math education](#). *Procedia-Social and Behavioral Sciences*, **2**(2), 2689-2693. T2a
- Lih, K.W. A remarkable Euler square before Euler *Mathematics Magazine* **83**, 163-167. T3b
- Lin, F.M. & Horng, W.S. *The Housekeeper and the Professor* by Yoko Ogawa. *Mathematical Intelligencer*, **32**(2), 75-76. T6
- Papadopoulos, I. "Reinventing" techniques for the estimation of the area of irregular plane figures: From the eighteenth century to the modern classroom. *Int. J. Sci. ME*, **8**(5), 869-890. T2
- Rowlands, S. A pilot study of a cultural-historical approach to teaching geometry. *S&E*, **19**(1), 55-73. T2a
- Tan, X. Z. Spiritual heritage and educational value of Greek mathematical culture. *JME*, **19**(1): 27-29. T3a
- Tatar, E., Tefvik I., Yasin S. & Levent A. Teaching square root numbers: from the perspective of Ibrahim Hakki of Erzurum, *Procedia – Social and Behavioural Sciences*, **2**(2), 1102-1106. T2b
- Xu, Z.T., Wang, X.Q. & Mei Q.X. The genetic approach to teaching and learning mathematics: from the history to the classroom. *JME*, **19**(1), 10-12. T1
- Yee, L. & Chapman, E. (2010). Using history to enhance student learning and attitudes in Singapore mathematics classrooms. *Education Research and Perspectives*, **37**(2), 110-132. T2
- Zhu, F. Q., & Xu, B. H. The modes of integrating the history of mathematics into teaching: an international perspective. *JME*, **19**(3), 22-25. T1

## 2011

- Gómez, B. (2011). *Marco preliminary para contextualizar la investigación en historia y educación matemática*. *Epsilon* **28**(1), 77, 9-22. T2b, 3b
- Guillemette, D. (2011). L'histoire dans l'enseignement des mathématiques: sur la méthodologie de recherche. *Petit x*, **86**(1), 5-26. T1
- Heffer, A. Historical objections against the number line. *S&E*, **20**(9), 863-880. T1
- Huntley, M. A. & Flores, A. Student-Made Games to Learn the History of Mathematics, *PRIMUS*, **21**(6), 567-576, DOI: 10.1080/10511970903521651. T2a,b
- Jankvist, U.T. Anchoring students' metaperspective discussion of history in mathematics, *JRME*, **42**(4), 346-385. T1, 2a
- Jankvist & Kjeldsen (see section 5). T1, 2a, 5
- Kaygın, B., Balçın, B., Yıldız, C., & Arslan, S. [The effect of teaching the subject of fibonacci numbers and golden ratio through the history of mathematics](#). *Procedia Social and Behavioral Sciences*, **15**, 961 - 965. T2a

- Klyve, D., Dtemkoski, L. & Tou, E. [Teaching and research using original sources from the Euler archive](#), *Convergence*, **8**. DOI:10.4169/loci003672. T2b
- Panagiotou, E. Using history to teach mathematics: The case of logarithms. *S&E*, **20**, 1–35. T2a
- Pu, S.P. & Wang, X.Q. Hans Freudenthal’s thought on HPM and its implications. *JME*, **20**(6), 20-24. T1
- Quale A. On the Role of Mathematics in Physics: A Constructivist Epistemic Perspective. *Science & Education* 20(7-8), 609–624. T5
- Shih, Y.K. & Su, Y.W. Integrating digital materials of history of mathematics into elementary teaching. *National Education*, **52**(3), 65-71. T2a
- Schubring (see section 5). T1
- Su, Y.W. An initial investigation into the design of materials of history of mathematics for elementary school education. *Research and Development in Science Education Quarterly*, **62**, 75-96 (in Chinese). T2b
- Wang, X.Q., Wang, M. & Zou, J. C. Teaching mathematics from the HPM perspective: the case of ellipse. *JME*, **20**(5), 20-23. T2b
- Yenilmez, K. Prospective mathematics teachers’ opinions about the history of mathematics course. *Pamukkale University Journal of Education*, **30**, 79 - 90.

## 2012

- Chan, Y.C. & Siu, M.K. Facing the change and meeting the challenge: Mathematics curriculum of Tongwen Guan in China in the second half of the nineteenth century, *ZDM*, **44**(4), 461-472. T6
- Chevalarias, N. & Minet, N. Des séances “Maths-Histoire” en classe de seconde. *R-IREM*, **86**, 5-25. T2a, b 4
- Clark, K.M. History of mathematics: illuminating understanding of school mathematics concepts for prospective mathematics teachers. *ESM*, **81**(1), 67–84. T2
- Fernandez, J. M. L. & Rodriguez, O. A. H. [Teaching the fundamental theorem of calculus: A historical reflection](#). *Convergence*, **9**. DOI:10.4169/loci003803. T2b
- Kjeldsen & Blomhøj (see section 5). T1, 2a, 5
- Li, G.Q. & Xu, L.H. Levels of mathematics teachers’ literacy in the history of mathematics based on the theory of SOLO. *JME*, **20**(1), 34-37. T1
- Liu, Z. D., Wang, Q. J., & Shao, R. Students’ motivation to learn mathematics: the historical perspective. *JME*, **20**(1), 30-33. T3b
- Özdemir, A. Ş., Göktepe, S., & Kepçeoğlu, İ. (2012). [Using mathematics history to strengthen geometric proof skills](#). *Procedia Social and Behavioral Sciences*, **46**, 1177-1181. T2a
- Panasuk, R. M., & Horton, L. B. Integrating history of mathematics into curriculum: what are the chances and constraints? *International Electronic Journal of Mathematics Education*, **7**(1), 3-20. T2, 1
- Pu, S.P. & Wang, X.Q. How to integrate the history of mathematics into mathematics textbooks: the cases of Chinese and French mathematics textbooks. *CTMM*, **32**(8), 63-68. T3b
- Pu, S.P. & Wang, X. Q. Junior high school students’ understanding about the letters: An empirical study of the historical parallelism. *JME*, **20**(3), 38-42. T3b

- Shih, Y.K., & Su, Y.W. Integrating digital materials of history of mathematics into elementary teaching. *National Education*, **523**(3), 65-71 (in Chinese). [T2a](#)
- Swetz, F.J. Similarity versus the “Out-in Complementary Principle” An example of a cultural faux pas in mathematical understanding. *Mathematics Magazine*, **85**(1), 3-11. [T6, 4](#)
- Wang, X.Q. The history of mathematics in French junior high school mathematics textbooks. *MB*, **51**(3), 16-20. [T3b](#)

### 2013

- Aydođdu, N., & Yüksel, İ. (2013). The relationship between prospective mathematics teachers’ beliefs and attitudes towards history of mathematics and their creativeness level. *Journal of Research in Education and Teaching*, **2**(4), 186-194.
- Baki, A. & Bütüner, S. Ö. (2013). The ways of using the history of mathematics in 6th, 7th and 8th grade mathematics textbooks. *Elementary Education Online*, **12**(3), 849-872. [T3b](#)
- Bernard, A. Résoudre un problème par l’algèbre sans en perdre le sens: sur les traces de Diophante d’Alexandrie. *R-IREM*, **92**, 59-74. [T2b, 4](#)
- Branson, W.B. [Solving the cubic with Cardano](#). *Convergence*, **10** (accessed 18/2/2016). [T2a, 2b, 4](#)
- Diamantopoulos, J. & Woodburn, C. [Maya geometry in the classroom](#). *Convergence*, **10**. (accessed 18/2/2016). [T2b, 6](#)
- Dubey, M. & Singh, B. Assessing the effect of implementing mathematics history with Algebra. *International Journal of Scientific and Research Publications*, **3** (8), 1-3. [T2a](#)
- Frejd, P. Old algebra textbooks: a resource for modern teaching. *BSHM*, **28** (1), 25-36. [T3b](#)
- Gazit, A. What do mathematics teachers and teacher trainees know about the history of mathematics? *Int. J. ME Sci. Tech* **44**(5), 501-512. [T2a](#)
- Goktepk, S. & Sukru, A. An example of using history of mathematics in classes. *European Journal of Science and Mathematics Education*, **1**(3), 125-136. [T2a](#)
- He, B.T. & Wang, X.Q. Senior high school students’ misconceptions of the tangent lines. *JME*, **21**(6), 45-48. [T3b](#)
- Jahnke, H.N. & Wambach, R. Understanding what a proof is: a classroom-based approach. *ZDM*, **45**(3), 469-482. [T2](#)
- Jankvist (see section 5). [T4, 5, 2](#)
- Kjeldsen, T.H. & Blomhøj, M. Developing students’ reflections about the function and status of mathematical modelling in different scientific practices: History as a provider of cases. *S&E*, **22**(9), 2157-2171. [T1, 2a, 5](#)
- Lawrence, S. Meno, his Paradox, and the Incommensurable Segments for Teachers. *Mathematics TODAY* (Inst. of Math. and its Appl.), April 2013, 84-85. [T2b](#)
- Liu, Z. & Yang, G.W. A comparative study on historical materials in junior high school textbooks of China and Japan. *HSMM*, **10**, 28-30. [T3b](#)
- Park, J., Güçler, B., & McCrory, R. (2013). Teaching prospective teachers about fractions: historical and pedagogical perspectives. *Educational Studies in Mathematics*, **82**(3), 455-479. [T2a](#)
- Moyon, M. & ERR. Diviser en multipliant les approches... Quand les mathématiques remontent aux sources, *R-IREM*, **93**, 47-77. [T2a, b 4, 6](#)
- Wu, J. & Huang, Q. Y. The understanding of mean, median and mode based on the history of

- mathematics. *MB*, **52**(11), 16-21. [T3b](#)
- Xie, Y.M. Historical materials in senior mathematics textbooks in Hong Kong: Characteristics and implications. *JME*, **21**(2), 67-70. [T3b](#)
- Wu, J. & Wang, X.Q. Integrating the history of mathematics into mathematics Teaching: Some foreign experience. *CER*, **8**, 78-82. [T1](#)
- Zhang, X. G. & Zhang, X. A review on using history in mathematics teaching and learning abroad. *JME*, **21**(4), 43-46. [T3b](#)

## 2014

- Alpaslan, M., Işıksal M., & Haser, Ç. Pre-service mathematics teachers' knowledge of history of mathematics and their attitudes and beliefs towards using history of mathematics in mathematics education. *Science & Education*, **23**(1), 159–183.
- Barnett *et al* (see section 5). [T4](#), [1](#), [2](#)
- Carter, J. [Writing in a history of mathematics capstone course](#). *Primus*, **24**(6), 544-556. [T4](#), [2a](#)
- Clark, K.M. & Thoo, J.B. Introduction to the special issue on the use of history of mathematics to enhance undergraduate mathematics instruction. *PRIMUS*, **24**(8), 663-668. [T1](#)
- Despeaux, S. E. [Collective research projects in history of mathematics classroom](#) *PRIMUS*, **24**(8), 684-697. [T2](#)
- Fenaroli, G., Furinghetti, F. & Somaglia, A. Rethinking mathematical concepts with the lens of the history of mathematics: an experiment with prospective secondary teachers. *S&E*, **23**, 185-203. [T2a](#), [1](#)
- Goodwin, D., Bowman, R., Wease, K., Keys, J., Fullwood, J. & Mowery, K. Exploring the relationship between teachers' images of mathematics and their mathematics history knowledge. *Philosophy of Mathematics Education Journal*. **28**, 1-15. [T3a](#), [2a](#)
- Heffer, A. How algebra spoiled recreational problems: A case study in the cross-cultural dissemination of mathematics. *Historia Mathematica*, **41**, 400–437.
- Jankvist, U. T. A historical teaching module on “the unreasonable effectiveness of mathematics”: Boolean algebra and Shannon circuits. *BSHM*, **29**(2), 120-133. [T2](#)
- Jankvist, U. T. The use of original sources and its potential relation to the recruitment problem. *FLM*, **34**(3), 8-13. [T4](#), [2a](#)
- Kjeldsen, T. H. & Petersen, P. H. Bridging History of the concept of a function with learning of mathematics: Students' meta-discursive rules, concept formation and historical awareness. *S&E*, **23**(1), 29-45. [T2a](#), [1](#), [4](#)
- Liu, P.-H. When Liu Hui meets archimedes: Students' epistemological and cultural interpretations of mathematics. *PRIMUS*, **24**(8), 710–721.
- Loats, J., White, D., & Rubino, C. [History of mathematics: three activities to use with undergraduate students and in service teachers](#). *PRIMUS*, **24**(8), 698-709. [T2b](#)
- Massa Esteve, M. R. Activities in the mathematics classroom; Tartaglia's *Nova Scientia* (1537). *Teaching Innovations*, **27**(3), 114-126. [T2a](#), [T4](#)
- Smestad, B., Jankvist, U.T. & Clark, K. Teachers' mathematical knowledge for teaching in relation to the inclusion of history of mathematics in teaching. *NOMAD*, **19**(3-4), 169-183. [T2a](#)
- Su, J. H. & Ying, J. M. Enhancing in-service mathematics teachers' professional knowledge

- with an HPM approach as observed in teachers' reflections. *TJME*, **1**(1), 79-97. **T2a**
- Su, Y. W., Huang, J. W., Chen, C. H. & Lin, M. Y. Enhancing teachers' professional development through HPM script design (in Chinese). *TJME*, **1**(2), 25-52. **T3a**
- Wu, J. & Wang, X. Q. A review on the practice of integrating the history of mathematics into mathematics teaching. *MB*, **53**(2), 13-16. **T2b**
- Wu, J. & Zhao, R. A survey on teachers' statistics knowledge for teaching from the perspective of HPM. *MB*, **53**(5), 15-18. **T3a**
- Ying, J.M. Mathematical canons in practice: The case of a nineteenth-century Korean scholar Nam Pyöng-Gil and his evaluation of two major algebraic methods used in East Asia. *East Asian Science, Technology and Society: An International Journal*, **8**(3), 347-362. **T3b**
- 2015**
- Bråting, K. & Pejlar J. On the relations between historical epistemology and students' conceptual developments in mathematics. *ESM*, **89**(2), 251-265. **T1**
- Bütüner, S. Ö. [Impact of Using History of Mathematics on Students Mathematics success: A Meta-Analysis Study](#). *Mevlana Int. J. of Educ. (MIJE)* **5**(2), 78-95. **T3a, 2a**
- Bütüner, S. Ö. [Impact of Using History of Mathematics on Students Mathematics Attitude: A Meta-Analysis Study](#). *European Journal of Science and Mathematics Education* **3**(4), 337-349. **T3a, 2a**
- Fried, M.N. & Jahnke, H.N. Otto Toeplitz's 1927 paper on the genetic method in the teaching of mathematics. *Science in Context*, **28**, 285-295. **T1**
- Fried, M.N. & Jahnke, H.N. (transl.) Otto Toeplitz, The problem of university courses on infinitesimal calculus and their demarcation from infinitesimal calculus in high schools. *Science in Context*, **28**, 297-310. **T1**
- Hašek, R., & Zahradník, J. Study of historical geometric problems by means of CAS and DGS. *International Journal for Technology in Mathematics Education*, **22**(2), 53–58.
- Jankvist, U.T. Changing students' images of "mathematics as a discipline". *JMB*, **38**, 41-56. **T2a**
- Jankvist, U.T., Mosvold, R., Fauskanger, J. & Jacobsen, A. Analyzing the use of history of mathematics through MKT. *Int. J. ME Sci. Tech.*, **46**(4), 495-507. **T1, 2**
- Kent, D., & Sherman, M. A GeoGebra rendition of one of Omar Khayyam's solutions for a cubic equation. *Convergence* <https://www.maa.org/press/periodicals/convergence/a-geogebra-rendition-of-one-of-omar-khayyams-solutions-for-a-cubic-equation>
- Kjeldsen T.H. & Lützen J. Interactions between mathematics and physics: The history of the concept of function – teaching with and about nature of mathematics. *S&E*, **24**(5-6), 543-559. **T1, 2a, 5**
- Lim, S. Y. & Chapman, E. Effects of using history as a tool to teach mathematics on students' attitudes, anxiety, motivation and achievement in grade 11 classrooms. *ESM*, **90**(2), 189-212. **T2a**
- Lombard, P. L'invention du zéro ou la revanche des bergers. *R-IREM*, **101**, 33-44. **T6**
- Pont, J.-C. À propos de l'introduction des nombres négatifs à l'école secondaire. *R-IREM* **101**, 69-86. **T2**
- Petrocchi, A. A new theoretical approach to sample problems and deductive reasoning in Sanskrit mathematical texts. *BSHM*, **30**(1), 2-19. **T6**

- Pu, S.P., & Wang, X.Q. Mathematics teachers' professional development promoted by HPM: researches and implications. *JME*, **23**(3), 76-80. **T4**
- Swetz, F.J. [Pantas' Cabinet of mathematical wonders: Images and the history of mathematics](#). *Convergence*, **12** (accessed 18/2/2016). **T2b, 4**
- Xenofontos, C. & Papadopoulos, C. E. [Opportunities of learning through the history of mathematics: the example of national textbooks in Cyprus and Greece](#). *Int. J. for Mathematics Teaching & Learning*, July 2015 (accessed 30/11/2016). **T3b**
- Xu, N.N., Kong, F.Z. & Liu, P.F. A Study on the presentation of history of mathematics in senior high school mathematics textbooks. *JME*, **23**(2), 61-65. **T3b**
- Ying, J.M., Huang, J.W. & Su, Y.W. An exploratory study on influences of a mathematical culture course on university students' mathematics beliefs – the case in a medical university. *TJME*, **2**(2), 1-24. **T2a**
- Zhong, P. & Wang, X.Q. The teaching of the concept of logarithm from the HPM perspective. *HSM*, **5**, 50-53. **T2a**

## 2016

- Barnett, J., Lodder, J. & Pengelley, D. Teaching and learning mathematics from primary historical sources. *PRIMUS*, **26**(1), 1-18. **T4, 1, 2**
- Barnett, J., Bezhanishvili, G., Lodder, J. & Pengelley, D. Teaching discrete mathematics entirely from primary historical sources. *PRIMUS*, **26**(7), 657-675. **T4, T2**
- Ju, M. K., Moon, J. E., & Song, R. J. History of mathematics in Korean mathematics textbooks: Implication for using ethnomathematics in culturally diverse school. *International Journal of Science and Mathematics Education*, **14**(7), 1321–1338.
- Kántor, T., & Tóth, A. Teaching of old historical mathematics problems with ICT tools. *Teaching Mathematics and Computer Science*, **14**(1), 13–24.
- Medrano, I. & Pino-Fan, L.R. . Estadios de comprensión de la noción matemática de límite finito desde el punto de vista histórico. *REDIMAT*, **5**(1), 287-323. doi: 10.4471/redimat.2016.1874. **T6, 5**

## 2017

- Barnett, J. H. (2017b). Generating Pythagorean triples: A mini-primary source project in number theory for mathematics majors, elementary teachers and others. *Convergence*. **14**, <https://www.maa.org/press/periodicals/convergence/generating-pythagorean-triples-a-mini-primary-source-project-in-number-theory-for-mathematics-majors>
- Guillemette, D. History of mathematics in secondary school teachers' training: towards a nonviolent mathematics education. *Educational Studies in Mathematics*, **96**(3), 349-365. **T2, T1**

## 2018

- d'Ambrosio, U. The program Ethnomathematics: Cognitive, anthropological, historic and socio-cultural bases. *PNA*, **12**(4), 229–247. **T6, T1**
- Fried, M. Ways of relating to the mathematics of the past. *Journal of Humanistic Mathematics*, **8**(1), 1-23. **T1**
- Wang, K., Wang, X.-q., Li, Y., & Rugh, M. S. A framework for integrating the history of mathematics into teaching in Shanghai. *Educational Studies in Mathematics*, **98**(2), 135–155.

Zengin, Y. Incorporating the dynamic mathematics software GeoGebra into a history of mathematics course. *International Journal of Mathematical Education in Science and Technology*, **49**(7), 1083–1098.

#### **2019**

Buchholtz, N., & Schorcht, S. (2019). Welche Überzeugungen haben Lehramtsstudierende zur Geschichte der Mathematik? – Ergebnisse der Studie ÜberLeGMA. *Mathematica Didactica*, **42**(1), 67–85.

Clark, K. M., Kjeldsen, T. H., Schorcht, S., & Tzanakis, C. History of mathematics in mathematics education—An overview. *Mathematica Didactica*, **41**(1), 1–26.

#### **2020**

Arnal-Bailera, A., & Oller-Marcin, A. M. Prospective secondary mathematics teachers read Clairaut: Professional knowledge and original sources. *Educational Studies in Mathematics*, **105**(2), 237–259.

Furinghetti, F. Rethinking history and epistemology in mathematics education. *International Journal of Mathematical Education in Science and Technology*, **51**(6), 967–994.

Jankvist, U. T., Clark, K. M., & Mosvold, R. Developing mathematical knowledge for teaching teachers: potentials of history of mathematics in teacher educator training. *Journal of Mathematics Teacher Education*, **23**(3), 311–332.

#### **2021**

Barnett, J. H., Can, C., & Clark, K. M. “He was poking holes ...”: A case study on figuring out metadiscursive rules through primary sources. *The Journal of Mathematical Behavior*, **61**, 100838

Barnett, J. H., Clark, K. M., Klyve, D., Lodder, J., Otero, D. E., Scoville, N., & White, D. A series of mini-projects from TRIUMPHS: TRansforming instruction in undergraduate mathematics via primary historical sources. *Convergence* (MAA online). <https://www.maa.org/press/periodicals/convergence/a-series-of-mini-projects-from-triumphs-transforming-instruction-in-undergraduate-mathematics-via>

Meadows, M., & Caniglia, J. That was then... This is now: Utilizing the history of mathematics and dynamic geometry software. *International Journal of Education in Mathematics, Science and Technology*, **9**(2), 198–212.

Moustapha-Corrka, B., Bernardes, A., Giraldo, V., Biza, I., & Nardi, E. Problematizing mathematics and its pedagogy through teacher engagement with history-focused and classroom situation-specific tasks. *The Journal of Mathematical Behaviour*, **61**, 1–10.

de Vittori, T., & Visentin, M. P. Supports historiques et enseignement des mathématiques: Sur le rôle de l’imagination. *Grand N*, **108**. IREM de Grenoble.

#### **2022**

Agterberg, D. A., Oostdam, R. J., & Janssen, F. J. J. M. From speck to story: relating history of mathematics to the cognitive demand level of tasks. *Educational Studies in Mathematics*, **110**, 49–64.

### **3.2.3 Individual papers in Collective Volumes**

#### **2000**

Dennis, D. [The role of historical studies in mathematics and science educational research](#). In D.



Lesh & A. Kelly (Eds.). *Research Design in Mathematics and Science Education* (pp.799-819). Mahwah, NJ: Lawrence Erlbaum (accessed 18/2/2016). [T1](#), [T3a](#), [T2](#)

Isoda, M., & Aoyama, K. (2000). The change of belief in mathematics via exploring historical text with technology in the case of undergraduates. In W.-C. Yang, S.-C. Chu, & J.-C. Chuan (Eds.), *Electronic Proceedings of the 5th Asian Technology Conference in Mathematics* (pp. 132–141).

## 2001

Kidron, I. Teaching Euler's algebraic methods in a calculus laboratory. In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), *Proceedings of the 12th ICMI Study Conference: The future of the teaching and learning of algebra* (vol. 2, pp. 368–375). University of Melbourne.

Radford, L. The historical origins of algebraic thinking. In R. Sutherland *et al* (Eds.), *Perspectives on School Algebra* (pp.13–36). Dordrecht: Kluwer. [T1](#)

Testa, G. Per un avvio alla ricerca “storica” in campo scientifico: studential lavoro. *L' insegnamento della matematica e delle scienze integrate*, **24B**, 39-68. [T2a](#)

## 2002

Furinghetti, F. & Radford, L. Historical conceptual developments and classroom learning in mathematics: from Phylogenesis and Ontogenesis Theory to classroom practice. In L. English (Ed.) *Hand Int RME* (pp.631–654). Mahwah (NJ): Lawrence Erlbaum Associates. [T1](#), [3a](#)

van Maanen, J. Research on history in mathematics education in the Netherlands: The ‘Reinvention Studies’. In F-L. Lin (Ed.), *Common Sense in Mathematics Education* (pp.191–201). National Taiwan Normal University, Taipei. [T3](#)

## 2003

Boyé, A. La musique au carrefour des mathématiques, des sciences et des arts. In E. Barbin (Ed) *La pluridisciplinarité dans les enseignements scientifiques, Histoire des sciences*. Caen: CRDP Basse-Normandie. [T5](#)

Jahnke, H. N. Texte von Studierenden zur Geschichte der Mathematik. In: L. Hefendehl - Hebecker & S. Hußmann (Eds.), *Mathematikdidaktik zwischen Fachorientierung und Empirie. Festschrift für Norbert Knoche*, Hildesheim: Franzbecker 2003, 105-116. [T4](#)

## 2004

Bagni, G., Furinghetti, F., & Spagnolo, F. [History and epistemology in mathematics education: 2000-2003](#). In L. Cannizzato, A. Pesci, O. Robutti (Eds.) *Italian Research in Mathematics Education: 2000-2003* (pp.207-221). Milano: Ghisetti & Corvi (accessed 18/2/2016). [T3a](#)

Puig, L. & Rojano, T. The history of algebra in mathematics education. In K. Stacey *et al* (Eds.), *The Future of the Teaching and Learning of Algebra. The 12th ICMI Study* (pp.189–223). Dordrecht: Kluwer. [T1](#)

## 2005

Grattan–Guinness I. History or heritage? An important distinction in mathematics and for mathematics education. In G. van Brummelen & M. Kinyon (Eds.), *Mathematics and the Historian's Craft* (pp.7–21). New York: Canadian Mathematical Society & Springer. [T1](#) (identical with Grattan–Guinness 2004b).

## 2006

Fischer, W.L. Historical Topics as Indicators for the Existence of Fundamentals in Educational

Mathematics. In Leung, F.K.S., Graf, K.-D. & Lopez-Real, F.J. (Eds.), *Mathematics education in different cultural traditions. A comparative study of East Asia and the West. The 13<sup>th</sup> ICMI Study* (pp. 95-110), New York: Springer. T6

## 2007

D'Ambrosio, U. Mathematical marginalisation and meritocracy: Inequity in an English classroom. In S. Barath (Ed.), *Peace, Social Justice and Ethnomathematics*, TMMEM vol.1 (pp.25-34). Charlotte NC: Information Age Publishing. T6

Heeffer, A. [Learning concepts through the history of mathematics](#), In K. François, K. & J.P. van Bendegem, (Eds.). *Philosophical Dimensions in Mathematics Education* (pp. 83–103). New York: Springer (accessed 18/2/2016). T5, 1

Pinxten, R. & François, K. [Ethnomathematics in practice](#), In K. François, K. & J.P. van Bendegem, (Eds.). *Philosophical dimensions in mathematics education* (pp. 213-227). New York: Springer (accessed 18/2/2016). T6, 1

## 2008

Clark, K.M. Heeding the call: History of mathematics and the preparation of secondary mathematics teachers. In F. Arbaugh, & P.M. Taylor (Eds.), *Inquiry into mathematics teacher education: Association of Mathematics Teacher Educators (AMTE)* (pp. 85-95). San Diego, CA: Association of Mathematics Teacher Educators. T2

Furinghetti, F. & Radford, L. [Contrasts and oblique connections between historical conceptual developments and classroom learning in mathematics](#). In L. English (Ed.), *Hand Int RME*, 2nd Edition (pp.626–655). New York: Routledge, Taylor and Francis (accessed 18/2/2016). T1, T3a

## 2009

Jankvist, U.T. A teaching module on the early history of error correcting codes. In I. Witzke (ed.), *18th Novembertagung on the History, Philosophy and Didactics of Mathematics* (pp.153–163). Berlin: Logos Verlag. T2

Swetz, F.J. Culture and the Development of Mathematics: An Historical Perspective. In Brain Greer et al (Eds.), *Culturally Responsive Mathematics Education* (pp.11-41). New York: Routledge. T6

Swetz, F.J. Word problems: Footprints from the history of mathematics, In Lieven Verschaffel et al (Eds.) *Words and Worlds: Modelling Verbal Descriptions of Situations* (pp. 73-91). Rotterdam: Sense Publishers. T2b, 4

## 2010

Costabile, F. & Serpe, A. Archimedes in secondary schools: A teaching proposal for the math curriculum. In S. A. Paipetis & M. Ceccarelli (Eds.). *The genius of Archimedes: 23 centuries of influence on mathematics, science and engineering*, History of Mechanism and Machine Science, vol.11 (pp.479–491). Netherlands: Springer. T2

## 2011

Jahnke, H.N. The conjoint origin of proof and theoretical physics. In M. Pitici (Ed.), *The Best Writing on Mathematics 2011* (pp.236 – 256). Princeton: Princeton University Press. T2

Goddijn A. Algebra from Ahmes to applet. In P. Drijvers (Ed.), *Secondary Algebra Education. Revisiting Topics and Themes and Exploring the Unknown*, (pp.27–68). Rotterdam: Sense Publishers. T2, 5

## 2012

Furinghetti (see section 5). [T3a](#), [T1](#)

Siu, M.K. Proof in the Western and Eastern traditions: Implications for mathematics education. In G. Hanna, M. de Villiers (Eds.) *Proof and Proving in Mathematics Education: The 19th ICMI Study* (pp.431–440). New York: Springer. [T1](#), [6](#), [5](#)

## 2013

Barbin (see section 5). [T3a](#), [b](#), [1](#), [4](#)

Wang, X.Q. Research on HPM: An overview. In J. S. Bao & B. Y. Xu (Eds.), *Guide to the Research to Mathematical Education II* (pp. 403-422). China: Jiangsu Education Publishing House. [T1](#), [2](#), [3](#), [4](#)

## 2014

Barbin & Tzanakis (see section 5). [T3a](#)

Clark, K.M. History of mathematics in mathematics teacher education. In M.R. Matthews (Ed.) *Int. Hand. Res. Hi. Phil. Sci. Teach.*, Volume 1, (pp.755–791). Dordrecht: Springer. [T1](#), [2](#)

Fried, M.N. History of mathematics and mathematics education. In M.R. Matthews (Ed.) *Int. Hand. Res. Hi. Phil. Sci. Teach.*, Volume 1, (pp.669–703). Dordrecht: Springer. [T1](#)

Jahnke, H. N. History in Mathematics Education. A hermeneutic approach. In: M. Fried & T. Dreyfus (Eds.), *Mathematics & Mathematics Education: Searching for Common Ground* (pp.75-88). Dordrecht: Springer. [T1](#), [4](#)

Jankvist, U.T. On the use of primary sources in the teaching and learning of mathematics. In M.R. Matthews (Ed.) *Int. Hand. Res. Hi. Phil. Sci. Teach.*, vol. 1, (pp.873–908). Dordrecht: Springer. [T4](#)

Kjeldsen T.H. & Carter, J. The role of history and philosophy in university mathematics education. In M.R. Matthews (Ed.) *Int. Hand. Res. Hi. Phil. Sci. Teach.*, vol. 1, (pp.837–871). Dordrecht: Springer. [T1](#)

Radford, L. Bernard, A., Fried, M. N., Furinghetti, F., Sinclair, N. Reflections on History of Mathematics: History of mathematics and mathematics education. In M. N. Fried & T. Dreyfus (Eds.). *Mathematics & Mathematics Education: Searching for Common Ground* (pp. 89-109). Dordrecht: Springer. [T1](#), [2](#) (a collective work consisting of individual subchapters by each co-author)<sup>5</sup>

Rogers, L. History of mathematics in and for the curriculum. In H. Mendick & D. Leslie (Eds.), *Debates in Mathematics Education* (pp.106-122). London: Routledge. [T3](#)

## 2015

Jankvist, U. T. History, application, and philosophy of mathematics in mathematics education: Accessing and assessing students' overview and judgment. In S. J. Cho (Ed.) *Selected regular lectures from the 12th International Congress on Mathematical Education* (pp.383-404). Cham: Springer

Siu, M. K. “*Zhi yì xíng nán* (knowing is easy and doing is difficult)” or vice versa? -A Chinese mathematician's observation on HPM (History and Pedagogy of Mathematics) activities. In

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<sup>5</sup>A. Bernard: History within math and science teaching: A historical issue; M.N. Fried: Mathematicians, historians of mathematics, mathematics teachers, and mathematics education researchers: The tense but ineluctable relations of four communities; F. Furinghetti: History in the mathematics classroom; N. Sinclair: Aesthetic considerations; L. Radford: Three provocative questions and one remark.

B. Sriraman, J.F. Cai, K. Lee, L. Fan, Y. Shimuzu, C. Lim, K. Subramanian (Eds.), *The First Sourcebook on Asian Research in Mathematics Education* (pp.27-48). Charlotte NC: Information Age Publishing. [T3a, 6](#)

### 2016

Chorlay, R., & de Hosson, C. (2016). History of science, epistemology and mathematics education research. In B. R. Hodgson, A. Kuzniak, & J.-B. Lagrange (Eds.), *The didactics of mathematics: Approaches and issues*, (pp. 155–189). Springer.

### 2018

Fried, M. N. History of mathematics, mathematics education, and the liberal arts. In G. Kaiser, H. Forgasz, M. Graven, A. Kuzniak, E. Simmt, & B. Xu (Eds.), *Invited lectures from ICME-13* (pp. 85–100). Springer.

Kjeldsen, T. H. Creating inquiry-reflective learning environments in mathematics through history and original sources. In J. Auvinet, G. Moussard & X. Saint Raymond (Eds.), *Circulation: mathématiques, histoire, enseignement* (pp. 13–29). PULIM Presses universitaires de Limoges.

### 2019

Clark, K. M. History and pedagogy of mathematics in mathematics education: History of the field, the potential of current examples, and directions for the future. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European society for Research in Mathematics Education* (pp. 29–55). ERME.

Kjeldsen, T. H. A multiple perspective approach to history of mathematics: Mathematical programming and Rashevsky's early development of mathematical biology in the twentieth century. In G. Schubring (Ed.), *Interfaces between mathematical practices and mathematical education* (pp. 143–167). Springer.

Moyon, M. Recréations mathématiques et algorithmique dans le *Liber abaci* de Fibonacci (XIII<sup>e</sup> siècle). In N. Chevalarias, M. Gandit, M. Morales, & D. Tournes (Eds.), *Mathématiques récréatives : éclairages historiques et épistémologiques* (pp.225– 252). UGA Editions-EDP Sciences.

Pejlare, J., & Bråting, K. Writing the history of mathematics: Interpretations of the mathematics of the past and its relation to the mathematics of today. In B. Sriraman (Ed.), *Handbook of the mathematics of the arts and sciences* (pp. 1–25). Springer.

### 2020

Barbin, É., Guillemette, D., & Tzanakis, C. (2020). History of mathematics and education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (2nd ed.) (pp.333–342). Springer.

### 2021

Barnett, J. H., Can, C., & Clark, K. M. (2021a). Learning mathematics from primary sources: Metadiscursive rules, exogenous growth and transgressive acts. In B. Pieronkiewicz (Ed.), *Different perspectives on transgressions in mathematics and its education* (pp. 293–310). Wydawnictwo Naukowe Uniwersytetu Pedagogicznego.

Thomsen, M., & Jankvist, U. T. Mathematical thinking in the interplay between historical original sources and GeoGebra. In U. T. Jankvist, A. Clark-Wilson, H.-G. Weigand, R. Elicer, & M. Thomsen (Eds.). *ICTMT-15 Book of accepted contributions: 15th*

*International Conference on Technology in Mathematics Teaching – Making and Strengthening “Connections and Connectivity” (C&C) for Teaching Mathematics with Technology* (pp. 283– 291). Danish School of Education

### 3.2.4 Individual papers in Proceedings of Conferences

#### 2000

Fried, M.N. Some difficulties in incorporating history of mathematics in teaching-training. In A. Ahmed, J.M. Kraemer and H. Williams (Eds.) *Cultural Diversity in Mathematics (Education): CIEAEM 51* (pp.73-78). Chichester: Horwood Publishing Ltd. [T1](#)

#### 2002

Kronfellner, M. A genetic approach to axiomatics. In I. Vakalis, D. Hughes-Hallett, Ch. Kourouniotis, D. Quinney, & C. Tzanakis (Eds.), *Proc. of the 2nd ICTM* (e-book). New York: J. Wiley & Sons Inc. [T2b](#)

Safuanov, I.S. Design of genetic teaching of algebra at universities. In I. Vakalis, D. Hughes-Hallett, Ch. Kourouniotis, D. Quinney, & C. Tzanakis (Eds.), *Proc. of the 2nd ICTM* (e-book). New York: J. Wiley & Sons Inc. [T2b](#)

#### 2003

Puig, L. (2003) Historia de las ideas algebraicas: componentes y preguntas de investigación desde el punto de vista de la matemática educativa. In: E. Castro, P. Flores, T. Ortega, L. Rico, A. Vallecillos (Eds.). *Investigación en Educación Matemática. Proc. VII Simposio de la SEIEM* (pp. 97-108). Granada: Univ. de Granada. [T1](#)

#### 2004

Jahnke, H. N. Historical sources in the mathematics classroom: ideas and experiences. In: H. Fujita, Y. Hashimoto, B.R. Hodgson, P.Y. Lee, S. Lerman & T. Sawada (Eds.). *Proc. of the 9<sup>th</sup> ICME* (pp.136-138). Dordrecht: Kluwer Academic Publishers. [T4](#)

#### 2005

Mason, R., & Janzen Roth, E. [Thinking like Archimedes: An instructional design experiment](#). In *Proc. of the 8<sup>th</sup> International Congress of the History, Philosophy, Sociology and Science Teaching*, Leeds, UK (accessed 10/10/ 2015); abstract in <http://www.mern.ca/abstract.asp?id=23> (accessed 18/2/ 2016)

#### 2006

Charbonneau, L. Histoire des mathématiques et les nouveaux programmes au Québec: un défi de taille. In N. Bednarz & C. Mary (Eds.), *Actes du Colloque EMF2006* (pp. 11–21). Sherbrooke: Éditions du CRP et Faculté d'éducation, Université de Sherbrooke. [T3b](#)

El Idrissi, A. (2006). L'histoire des mathématiques dans les manuels scolaires. In N. Bednarz & C. Mary (Eds.), *Actes du Colloque EMF2006* (in CD-Rom). Sherbrooke: Éditions du CRP et Faculté d'Éducation, Université de Sherbrooke. [T3b](#)

Fried, M.N. & Bernard, A. History of mathematics in the curriculum and the possibility of a new Paideia. In F. Spagnolo, B. Di Paola (Eds.), *Proc. of CIEAEM 57: Changes in Society: A Challenge for Mathematics Education*, (pp.217-220). Piazza Armerina, Sicily. [T1, 3](#)

Guevara, I; Massa, M.R.; Romero, F. Textos históricos para la enseñanza de las matemáticas. In: J.A. Pérez-Bustamante, et al. (Eds.), *Actas del IX Congreso de la Sociedad Española de*

*Historia de las Ciencias y de las Técnicas* (pp. 1301-1304) .Cádiz: SEHCYT.

Heeffer, A. [The methodological relevance of the history of mathematics for mathematics education](#). In G. Dhompongsa, F.M. Bhatti & Q.C. Kitson (Eds.) *Proc. of the International Conference on 21st Century Information Technology in Mathematics Education* (pp. 267-276). Chang Mai, Thailand [T1](#) (accessed 18/2/2016).

Rowlands, S. & Carson, R. [Proof, reason, abstraction and leaps: A cultural-historical approach to teaching geometry](#) In D. Hewitt (Ed.), *Proc. of the British Society for Research into Learning Mathematics* **26**(1), 71-76 (accessed 18/2/2016). [T1](#), [2b](#)

Schubring (see section 5). [T1](#)

### **2009**

Montelle, C. & Clark, K.M. Using history to enrich undergraduate mathematics: Beyond the anecdote. In D. Wessels (Ed.), *Seventh Southern Right Delta Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics* (pp.185-195). Stellenbosch, South Africa: DELTA. [T1](#), [2](#)

### **2010**

Kjeldsen, T.H. History in mathematics education - why bother? Interdisciplinarity, mathematical competence and the learning of mathematics. In B. Sriraman & V. Freiman (Eds.) *Interdisciplinarity for the 21st Century: Proc. of the 3rd International Symposium on Mathematics and its Connections to Arts and Sciences* (pp.17-48). *TMMEM*, vol. 11. Charlotte, NC: Information Age Publishing. [T1](#), [2a](#), [5](#)

### **2011**

Boyé, A., Moussard, G. L'enseignement des vecteurs au XX<sup>e</sup> siècle: diversité des héritages mathématiques et circulation entre disciplines. In IREM Basse-Normandie, (Eds.) *Circulation, Transmission, Héritage* (pp.219-238). Caen: IREM de Basse-Normandie. [T2b](#)

### **2012**

Barbin, É. L'histoire des mathématiques dans la formation: une perspective historique (1975–2010). In J.-L. Dorier & S. Coutat (Eds.) *Actes du congrès de l'Espace Mathématique Francophone 2012* (pp. 546–554). Genève: EMF. [T1](#), [T2](#)

Rogers, L. Practical mathematics in 16<sup>th</sup> century England: Social-Economic contexts and emerging ideologies in the new Common Wealth. In Bjarnadóttir, K., Furinghetti, F., Matos, J. & Schubring, G. (Eds.), *“Dig where you stand” 2. Proc. of the conference on the History of Mathematics Education* (pp.421- 444). Universidade Nova: Lisbon. [T6](#)

### **2013**

Barbin, É. L'histoire des mathématiques dans la formation: une perspective historique (1975–2010). In J.-L. Dorier (Ed.) *Actes du Colloque Espace Mathématique Francophone* (pp.546-554). University of Genève: Genève. [T2b](#)

Smestad, B. Uses of history of mathematics in school (pupils aged 6–13). In Cho, S. J. (Ed.) *The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and attitudinal challenges* (pp.601-603). New York: Springer. [T2a](#)

### **2017**

Tzanakis, C., & Wang, X., Clark, K., Kjeldsen, T.H., Schorcht, S. *Topic Study Group 25: The role of history of mathematics in mathematics education*. In G. Kaiser (Ed.), *Proc. of the 13th ICME*. (pp. 491-495). Cham: Springer.

2018

Radford L (2018) A cultural-historical approach to teaching and learning: the theory of objectification. In F-J Hsieh (Ed.), *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education*, vol. 1 (pp. 137-147). Taipei: EARCOME. T1, T6

#### 4. Concluding remarks

The *HPM perspective* described in §1.1 emerged gradually over the last decades as a perception of mathematics worth exploring, thanks to research and teaching work done worldwide, thus establishing the *HPM domain* as a valuable research area in the context of ME. Launching the ICMI Study volume in 2000 was a decisive step in this direction. This highly collective work motivated, stimulated, oriented, encouraged and supported research in this area, to a large extent realized in the context of the *HPM Group* and the main activities related to it. At that time central issues were (and still are):

- To put emphasis on *pre- and in-service teacher education* as a necessary prerequisite for the *HPM perspective* to be possible at all.
- To design, produce, make available and disseminate a variety of *didactical source material* in the form of anthologies of original sources, annotated bibliography, description of teaching sequences/modules to serve as a source of inspiration and/or as generic examples for classroom implementation, educational aids of various types, appropriate websites etc.
- To perform systematically, carefully designed and applied *empirical research* in order to examine in detail and *evaluate* convincingly the *effectiveness* of the *HPM perspective* on improving the teaching and learning of mathematics, as well as students and teachers' awareness of mathematics as a discipline and their disposition towards it.
- To acquire a deeper understanding of theoretical ideas put forward in the HPM domain and to carefully develop them into coherent *theoretical frameworks and methodological schemes* that will serve as a foundation for further research and applications in this area.

In the last 10 to 15 years much work has been done on these issues and more is still in progress. In this survey

- An attempt was made to provide enough evidence - mainly based on the literature - that HM is relevant to ME in several ways and may have a multifaceted influence on improving the teaching and learning of and about mathematics. More specifically:
- An outline of the development of the HPM domain has been given;
- The key issues in this domain have been formulated and briefly discussed; and
- A sufficiently comprehensive survey of the existing literature has been included.

We hope that the present survey will serve both as a working document and as a motivation for all those who desire information about the *HPM perspective* and to explore further the possibilities offered for supporting and improving ME.

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## 5. Bibliography

- Alpaslan, M., Işıksal M. & Haser, Ç. (2014). Pre-service mathematics teachers' knowledge of history of mathematics and their attitudes and beliefs towards using history of mathematics in mathematics education. *S&E*, **23**(1), 159-183.
- Arcavi, A. & Isoda, M. (2007). Learning to listen: From historical sources to classroom practice, *ESM*, **66**(2), 111-129.
- Barbin, É. (1997). Histoire des Mathématiques: Pourquoi? Comment? *Bulletin de l'Association Mathématique du Québec*, **37**(1), 20-25.
- Barbin, É. (2013). History and Pedagogy of Mathematics in France. In V. L. Hansen & J. Gray (Eds.), *History of Mathematics*, in *Encyclopedia of Life Support Systems* (e-book). Oxford, UK: EOLSS Publishers.
- Barbin, É. (Ed.) (2015). *Les Constructions Mathématiques dans l'histoire: avec des Instruments et de Gestes (Mathematical Constructions: making and doing)*. Paris: Ellipses-Editions.
- Barbin, E. *et al* (2000). Integrating history: research perspectives. In J. Fauvel & J. van Maanen (Eds.), *ICMI Study Volume* (pp. 63-90). Dordrecht: Kluwer.
- Barbin, É., Stehlikova, N. & Tzanakis, C. (Eds) (2008). *History and Epistemology in Mathematics Education: Proc. of the 5<sup>th</sup> ESU*, Prague: Vydavatelský servis, Plzeň.
- Barbin, E., Kronfellner, M. & Tzanakis, C. (Eds) (2011a). *History and Epistemology in Mathematics Education: Proc. of the 6<sup>th</sup> ESU*, Wien: Holzhausen Verlag.
- Barbin, É., Furinghetti, F, Lawrence, S. & Smestad, B. (2011b). The role of the history and epistemology of Mathematics in pre-service teachers training. In E. Barbin, M. Kronfellner & C. Tzanakis (Eds), *Proc. of the 6<sup>th</sup> ESU* (pp.27-46). Wien: Holzhausen Verlag.
- Barbin, É., Hwang, S. & Tzanakis, C., (Eds.) (2012). *Proc. of HPM 2012 – The HPM Satellite Meeting of ICME-12*. Daejeon, South Korea: Korean Society of Mathematical Education & Korean Society for History of Mathematics.
- Barbin, É., Tzanakis, C. (2014). History of mathematics and education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp.255-260). New York: Springer.
- Barbin, É., Jankvist, U., Kjeldsen, T.H. (Eds.) (2015). [\*History and Epistemology in Mathematics Education: Proc. of the 7<sup>th</sup> ESU\*](#). Copenhagen: Danish School of Education, Aarhus University (accessed 18/2/2016).
- Barbin, É., Jankvist, U. T. , Kjeldsen, T. H., Smestad, B., Tzanakis, C. (Eds.) (2019). [\*Proceedings of the 8<sup>th</sup> European Summer University on History and Epistemology in Mathematics Education\*](#), Oslo: Oslo Metropolitan University.
- Barnett J. Heine, Lodder, J. & Pengelley, D. (2014). The pedagogy of primary historical sources in mathematics: Classroom practice meets theoretical frameworks, *S&E*, **23**(1), 7-27.
- Bekken, O. & Mosvold, R. (Eds.) (2003). *Study the Masters*. Nationellt Centrum för Matematikutbildning NCM, Göteborg: Göteborgs Univesitet.
- Booker, G. (1986). Topic Area: Relationship between the History and Pedagogy of Mathematics. In M. Carss (Ed.), *Proc. of the 5<sup>th</sup> ICME* (pp.256-260). Boston: Birkhäuser.
- Boyé, A., Demattè A., Lakoma, E. & Tzanakis, C. (2011). The history of mathematics in school textbooks. In E. Barbin, M. Kronfellner & C. Tzanakis (Eds) (2011). *Proc. of the 6<sup>th</sup> ESU* (pp.153-163). Wien: Holzhausen Verlag.
- Bruckheimer, M. & Arcavi, A. (2000). Mathematics and its history: An educational partnership. In Katz, V. (Ed.) (2000). *Using History to Teach Mathematics. An International Perspective*. MAA notes 51 (pp.135-146). Washington DC: MAA.
- Calinger, R. (Ed.) (1996). *Vita Mathematica*, MAA Notes 40, Washington DC: MAA.



- Cantoral, R., Fasanelli, F., Garciadiego, A., Stein, R. & Tzanakis, C. (Eds.) *Proc. of HPM 2008: The HPM Satellite Meeting of ICME 11*. Mexico City: Centro de Investigación y de Estudios Avanzados del IPN.
- Clark, K. (2006). [\*Investigating teachers' experiences with the history of logarithms: A collection of five case studies\*](#). Dissertation. Faculty of the Graduate School of the University of Maryland, College Park, USA (accessed 18/2/2016).
- Clark, K. (2009). [\*'In these numbers we use no fractions': A classroom module on Stevin's decimal fractions\*](#). *Convergence* **6**. DOI:10.4169/loci003333.
- Clark, K. (2011). Reflections and revision: Evolving conceptions of a *Using History* course. In Katz, V. & Tzanakis, C. (Eds.) (2011). *Recent Developments on Introducing a Historical Dimension in Mathematics Education* (pp.211-220). MAA notes 78. Washington DC: MAA.
- Fasanelli, F. & Fauvel, J. (2006). History of the International Study Group on the relations between the history and pedagogy of mathematics: the first 25 years 1976–2000. In Furinghetti, F., Kaisjer, S. & Tzanakis, C. (Eds). *Proc.of HPM 2004 & ESU 4* (pp.x-xxviii), Iraklion: University of Crete, Greece.
- Fauvel, J. & van Maanen, J. (1997). The role of the history of mathematics in the teaching and learning of mathematics: Discussion document for an ICMI Study (1997-2000). *ESM*, **34**, 255-259.
- Fauvel, J.G. & van Maanen, J. (Eds.) (2000) *History in Mathematics Education: The ICMI Study*, New ICMI Study Series, vol.6, Dordrecht: Kluwer.<sup>6</sup>
- Fried, M. (2001). Can mathematics education and history of mathematics coexist? *S&E*, **10**, 391–408.
- Fried, M. (2011). History of mathematics in mathematics education: problems and prospects. In E. Barbin, M. Kronfellner & C. Tzanakis (Eds) (2011). *Proc. of the 6<sup>th</sup> ESU* (pp.13-26). Wien: Holzhausen Verlag.
- Furinghetti, F. (2004). History and mathematics Education: A look around the world with particular reference to Italy. *MedJRME*, **3** (1-2), 1-20.
- Furinghetti, F. (2012). [\*History and epistemology in mathematics education\*](#). In V. L. Hansen & J. Gray (Eds.), *History of Mathematics*, in *Encyclopedia of Life Support Systems* (e-book). Oxford, UK: EOLSS Publishers (accessed 18/2/2016).
- Furinghetti F., Kaisjer S. & Vretblad A. (Eds.) (2004). *Proc. of the HPM Satellite Meeting of ICME 10 & the 4<sup>th</sup> ESU*. Uppsala: Uppsala University.
- Furinghetti, F., Jahnke, H.N. & van Maanen, J. (Eds.) (2006). *Report No22/2006 on the Mini-Workshop on Studying Original Sources in Mathematics Education*. Oberwolfach: Mathematisches Forschungsinstitut Oberwolfach.
- Glaubitz, M.R. (2010). [\*Mathematikgeschichte lesen und verstehen: eine theoretische und empirische Vergleichstudie\*](#). Dissertation. Universität Duisburg-Essen, Duisburg, Essen (accessed 18/2/2016).
- Grattan-Guinness, I. (1973). Not from nowhere. History and philosophy behind mathematical education. *Int. J. ME Sci. Tech.*, **4**, 421-453.
- Grattan-Guinness, I. (2004a). The mathematics of the past: distinguishing its history from our heritage. *Historia Mathematica*, **31**, 163-185.
- Grattan-Guinness, I. (2004b). History or Heritage? An important distinction in mathematics for mathematics education. *American Mathematical Monthly*, **111**(1), 1-12.
- Horng, W-S. & Lin, F-L. (Eds.) (2000). *Proc. of the HPM Conference History in Mathematics*

---

<sup>6</sup> Abbreviated as *ICMI Study Volume* throughout this study.

- Education: Challenges for a new millennium - A satellite meeting of ICME-9*. Taipei: National Taiwan Normal University.
- Horng W-H., Lin Y-C., Ning T-C. & Tso T-Y. (Eds.) (2004). *Proceedings of Asia-Pacific HPM 2004 Conference: History, culture and mathematics education in the new technology era*. Taichung, Taiwan: National Taichung Teachers College, Department of Mathematics Education.
- International Study Group on the Relations between the History and the Pedagogy of Mathematics (1978). *Historia Mathematica*, **5**(1), 76. doi:10.1016/0315-0860(78)90135-0.
- Jankvist, U.T. (2007). Empirical research in the field of using history in mathematics education: Review of empirical studies in HPM2004 & ESU4. *NOMAD* **12**(3), 83–105.
- Jankvist, U.T. (2009a). *Using history as a “goal” in mathematics education*. Dissertation. Department of Science, Systems and Models, IMFUFA, Roskilde University, Denmark (accessed 18/2/2016).
- Jankvist, U.T. (2009b). A categorization of the “whys” and “hows” of using history in mathematics education. *ESM*, **71**(3), 235–261.
- Jankvist, U.T. (2013). History, applications, and philosophy in mathematics education: HAPh—A use of primary sources. *S&E*, **22**, 635-656.
- Jankvist, U.T. & Kjeldsen, T.H. (2011). New avenues for history in mathematics education: mathematical competencies and anchoring. *S&E*, **20**, 831-862.
- Jankvist, U.T., Lawrence, S., Tzanakis, C. & van Maanen, J. (Eds.) (2011). In M. Pytlet, T. Rowland, E. Swoboda (Eds.), *Proc. of CERME 7* (pp.1636-1779). Rzeszow, Poland: University of Rzeszow.
- Jahnke, H. N., et al. (2000). The use of original sources in the mathematics classroom. In: J. Fauvel & J. van Maanen (eds.), *ICMI Study Volume* (pp.291-328). Dordrecht: Kluwer Academic Publishers.
- Katz, V. (Ed.) (2000). *Using History to Teach Mathematics. An International Perspective*. MAA notes 51. Washington DC: MAA.
- Katz, V. & Michalowics, K.D. (Eds.) (2005). *Historical Modules for the Teaching and Learning of Mathematics* (e-book). Washington DC: MAA.
- Katz, V. & Tzanakis, C. (Eds.) (2011). *Recent developments on introducing a historical dimension in mathematics education*. MAA notes 78. Washington DC: MAA.
- Katz, V., Jankvist, U.T., Fried, M.N. & Rowlands, S. (Eds.) (2014). Thematic issue: History, Philosophy and Mathematics Education. *Science & Education*, **23**(1), 1-250.
- Kjeldsen, T.H. (2011a). Uses of history in mathematics education: development of learning strategies and historical awareness. In M. Pytlet, T. Rowland, E. Swoboda (Eds.), *Proc. of CERME 7* (pp.1700-1709). Rzeszow, Poland: University of Rzeszow.
- Kjeldsen, T.H. (2011b). History in a competence-based of the practice of mathematics in a competence based mathematics education: A means for the learning of differential equations. In V. Katz & C. Tzanakis (Eds.): *Recent Developments on Introducing a Historical Dimension in Mathematics Education*, pp.165-173. MAA notes 78. Washington: MAA.
- Kjeldsen T.H. & Blomhøj, M. (2012). Beyond motivation: history as a method for learning meta-discursive rules in mathematics. *ESM*, **80**(3), 327–349;
- Lagarto, M. J., Vieira, A. & Veloso, E. (Eds.) (1996). *História e Educação Matemática: Proc. of Deuxième Université d'Été Européenne sur l'Histoire et Épistémologie dans l'Éducation Mathématique and the ICME-8 Satellite Meeting of HPM*. Braga: Portuguese Association of the Teachers of Mathematics & Department of Mathematics, University of Minho. Available online [Vol. I](#), [Vol. II](#) (accessed 18/2/2016).

- Lalande F., Jaboeuf F. & Nouazé Y. (Eds.) (1995). *Actes de la première Université d'Été Européenne sur l'Histoire et Épistémologie dans l'Éducation Mathématique*. Montpellier: IREM de Montpellier, Université Montpellier II.
- Liu, P-H. (2003). Do teachers need to incorporate the history of mathematics in their teaching? *MT*, **96**(6), 416-421.
- Mosvold, R., Jakobsen, A. & Jankvist, U.T. (2014). How mathematical knowledge for teaching may profit from the study of history of mathematics, *S&E*, **23**(1), 47-60.
- NCTM, 1969, *Historical topics for the mathematics classroom*, Reston VA: NCTM (31<sup>st</sup> NCTM Yearbook, reprinted 1989).
- Niss, M. & Højgaard, T. (Eds.) (2011). [\*Competencies and mathematical learning-ideas and inspiration for the development of mathematics teaching and learning in Denmark\*](#). Roskilde: Roskilde University (accessed 18/2/2016).
- Pengelly, D. (2011). Teaching with primary historical sources: should it go mainstream? Can it? In V. Katz & C. Tzanakis (eds.): *Recent Developments on Introducing a Historical Dimension in Mathematics Education*, pp.1-8. MAA notes 78. Washington: MAA.
- Pengelly, D., Lodder, J., Barnett, J., Bezhanishvili, G., Ranjan, D. & Leung, H. (2009). [\*Historical projects in discrete mathematics and computer science\*](#). In B. Hopkins (Ed.), *Resources for Teaching Discrete Mathematics* (pp.163-276). MAA notes 74. Washington, DC: MAA (accessed 18/2/2016).
- Pengelly, D. & Laubenbacher, R. [\*Teaching with Original Historical Sources in Mathematics, a resource web site\*](#) (accessed 18/2/2016).
- Povey, H. (2014). 'Walking in a foreign and unknown landscape': Studying the history of mathematics in initial teacher education. *S&E*, **23** (1), 143-157.
- Radelet de Grave, P. & Brichard, R. (Eds.) (2001). *Proc. of the 3<sup>rd</sup> ESU*. Leuven & Louvain-la-Neuve: Université Catholique de Louvain.
- Radford, L. *et al.* (2000). Historical formation and student understanding of mathematics. In J. Fauvel & J. van Maanen (eds.), *ICMI Study Volume* (pp. 143-170). Dordrecht: Kluwer Academic Publishers.
- Radford, L., Furinghetti, F., & Hausberger, T. (Eds) (2016). *Proc. of the 2016 ICME Satellite Meeting of HPM*. Montpellier: IREM de Montpellier.
- Rogers, L. (2009). History, heritage and the UK mathematics classroom. In V. Durand-Guerrier, S. Soury-Lavergne, F. Arzarello (Eds.). *Proc. of CERME 6* (pp.2781-2790). Lyon: Institut National De Recherche Pédagogique.
- Rogers, L. (2011). Mapping our heritage to the curriculum: historical and pedagogical strategies for the professional development of teachers. In V. Katz & C. Tzanakis (Eds.) *Recent Developments on Introducing a Historical Dimension in Mathematics Education*. MAA notes 78 (pp.221-230). Washington DC: MAA.
- Schubring, G. (2006). Ontogeny and phylogeny. Categories for cognitive development. In F. Furinghetti, S. Kaisjer & C. Tzanakis (Eds.), *Proceedings of HPM 2004 & ESU 4* (pp.329–339). Iraklion: University of Crete.
- Schubring, G. (2011). Conceptions for relating the evolution of mathematical concepts to mathematics learning – epistemology, history, and semiotics interactive. *ESM*, **77**(1), 79–104.
- Siu, M-K. (2006). No, I don't use history of mathematics in my classroom. Why? In F. Furinghetti, S. Kaisjer & C. Tzanakis (Eds.) *Proceedings of HPM 2004 & ESU 4* (pp.268-277). Iraklion: University of Crete, Greece.
- Siu, M-K. & Tzanakis, C. (Eds.) (2004). The role of the History of Mathematics in Mathematics Education. *Med JRME*, **3**(1-2), 1-166; special double issue.

- Smestad, B. (2011). History of mathematics for primary school teacher education, or: Can you do something, even if you can't do much?. In V. Katz & C. Tzanakis (Eds.) (2011). *Recent Developments on Introducing a Historical Dimension in Mathematics Education* (pp.201-210). MAA notes 78. Washington DC: MAA.
- Stein, R., (2010). *Math for Teachers: An exploratory approach*. Dubuque, IA: Kendall Hunt Publishing Co.
- Su, Y-W. (2005). *Mathematics teachers' professional development: Integrating history of mathematics into teaching*. Dissertation. Department of Mathematics. Taipei: National Taiwan Normal University (in chinese).
- Swetz, F., Fauvel, J., Bekken, O., Johansson, B. & Katz, V. (Eds.) (1995). *Learn from the masters!* Washington DC: MAA.
- Thomaidis, Y., & Tzanakis, C. (2007). The notion of historical “parallelism” revisited: Historical evolution and students’ conception of the order relation on the number line. *ESM* **66**, 165–183.
- Tzanakis, C., Arcavi A., *et al.* (2000). Integrating history of mathematics in the classroom: an analytic survey. In J. Fauvel & J. van Maanen (Eds.), *ICMI Study Volume* (pp. 201–240). Dordrecht: Kluwer.
- Tzanakis, C & Thomaidis, Y. (2012). Classifying the arguments & methodological schemes for integrating history in mathematics education. In B. Sriraman (Ed.) *Crossroads in the History of Mathematics and Mathematics Education* (pp.247-294). Charlotte NC: Information Age Publishing.
- van Amerom, B.A. (2002). [\*Reinvention of early algebra. Developmental research on the transition from arithmetic to algebra.\*](#) Dissertation. Utrecht: CD-β Press, Centre for Science and Mathematics Education. Freudenthal Institute, University of Utrecht (accessed 18/2/2016).
- Waldegg, G. (2004). Problem solving, collaborative learning and history of mathematics: Experiences in training in-service teachers. *Med JRME*, **3** (1-2), 63-71.