International Study Group on the Relations Between the HISTORY and PEDAGOGY of MATHEMATICS
An Affiliate of the International Commission on Mathematical Instruction

No. 75 November 2010

This and earlier issues of the Newsletter can be downloaded from our website
http://www.clab.edc.uoc.gr/hpm/

Professor R. C. Gupta receives Kenneth O. May Prize

Professor Radha Charan Gupta received the Kenneth O. May Prize for the History of Mathematics at the International Congress of Mathematicians (ICM) on 27 August 2010. Prof. Gupta was chosen for the 2009 prize jointly with Prof. Ivor Grattan-Guinness of UK by the International Commission for the History of Mathematics. Prof. Kim Plofker presented the award.

The Prize is named after the mathematician and historian Kenneth O. May, founder of the International Commission for the History of Mathematics and its journal *Historia Mathematica.* The prize, instituted in 1989, consists of a bronze medal and is given once in four years in appreciation of a mathematician's scholarly work in the history of mathematics.

Prof. Gupta's major contributions in the field include work on the history of development of trigonometry in India. He authored the chapter 'Historiography of Mathematics in India' in: Dauben and Scriba (eds.) *Writing the History of Mathematics: Its Historical Development*, Birkhauser, 2002. He was the President of the Association of Mathematics Teachers of India from 1994 until recently. He also founded the journal *Ganita Bharati* (meaning "Indian Mathematics") and has written several articles in the journal. He is also a frequent contributor to the HPM Newsletter (including this issue).

New Books

De grands défis mathématiques: d'Euclide à Condorcet

[On major challenges of mathematics: from Euclid to Condorcet]
Evelyne Barbin (ed.)
Paris: Vuibert; Adapt-Snes, 2010, 176pp,
ISBN 9782356560100;9782311000191

The output of IREM colleagues is admirable and enviable. This latest volume, under the editorship of Evelyne Barbin, provides nine incidents or problems that stimulated mathematical responses. The purpose of the

descriptions here is to provide original material that can be used in the classroom of upper secondary schools, that is, in those years immediately preceding university entrance. The claim is slightly misleading in that the article by Evelyne Barbin describes work with third year university students preparing to teach and the historical time frame of the articles extends to before Euclid and after Condorcet. The work is divided into four sections: measurement of magnitudes, representing magnitudes, probability, approximations to curves.

However entertaining and stimulating, I often feel that French studies linking history to pedagogy would not appeal to the Anglo-Saxon reader and I even wonder how much they might be taken up in the French classroom, save for the enthusiasts who have written these pieces. It is therefore encouraging to find, alongside suggested exercises based on historical texts, or at least inspired by them, some illustrations of work done by students themselves. Thus Dominique Tournès from Réunion has examples of students' constructions of solution curves of differential equations, using Euler's method, which they did following a guided reading of an extract of *Institutionum calculi integralis* (1768). From Patrick Guyot of Bourgogne we have an account of students working on the problem of inscribing a square inside a triangle which includes images of the students' initial naïve attempts. Indeed, it is clear that in almost all cases the writers are telling us of material they themselves have used.

But this collection of articles is more than a description along the lines of a simple 'these have worked for us'. Each piece comes with an introduction of the mathematical, historical and educational context and some offer further reflections and all have source bibliographies. There are some nice discoveries for me. Leibniz does not appear among the founders of probability calculus but Renaud Chorlay has used a letter from him describing the expected outcomes of a simple game of dice which is easy to read and has the pedagogic advantage that his assumptions are wrong. He fails to count both (3,2) and (2,3) as distinct ways of obtaining 5, something which students can easily correct. Gérard Harmon treats early approaches to Bayes' Theorem by Condorcet (1805) and then Lacroix (1816) both considering possible outcomes of taking black or white balls from an urn. I also enjoyed reading the background to modern digital type fonts and the use of Bézier curves which extended Loïc Le Corre's lesson on Dürer's geometric representation of fonts.

The educational context – defending the choice of material – makes us aware of French concerns that would not trouble the British secondary mathematics teacher, more accustomed to a pragmatic sloppiness of approach to teaching mathematics, particularly with regard to notation. *Vive la différence!*

Chris Weeks,
Great Britain

Tinhlelo

Interweaving Art and Mathematics: Colourful Basket Trays from the South of Mozambique
Paulus Gerdes
Lulu, 2010

"Tinhlelo" exist in each family of the vast Mozambique. We need to know that there are no factories for the production of the "tinhlelo." "Tinhlelo" knowledge is endogenous knowledge that repeats itself, recreates itself, and adorns itself in the multiplicity of its models, sizes, colours, and functions. No conventional school orients this learning. The families, the remarkable, curious and master artisans, are the ones who are creating and maintaining these products of eloquent usefulness in our families. The families recreate the art of this mathematical weaving!

(From the foreword)

Reports on new books are welcome.

Conference reports

6th European Summer University on the History and Epistemology in Mathematics Education
19-23 July 2010 Vienna

These Summer Universities started in Montpellier (France) in 1993 and from the beginning provided an occasion for researchers and teachers, not only to hear about new experiences and ideas and share their own, as in an ordinary international meeting, but above all to learn from each other, as in an open school inspired by cooperative learning. The main organizers were Evelyne Barbin, from University of Nantes (France); Manfred Kronfeller, from Vienna University of Technology (Austria); and Costantinos Tzanakis from University of Crete (Greece) and supported by the international Scientific Program Committee composed of 28 people from 18 different countries. There were 154 participants from 34 different countries, of whom about 75% made contributions (talks, workshops, posters). The conference was sponsored by the Austrian Federal Ministry of Science and Research, the Government of the City of Vienna, the Vienna Convention Bureau, Casio Europe and Texas Instruments.

This report is from a "teacher's point of view" and summarises the reactions of about 30 classroom teachers who attended the conference. I would particularly like to thank Alexandra Lux from BRG Purkersdorf (Austria) in helping to prepare the report.

The main themes of this 6th ESU were:

- Theoretical and or conceptual frameworks for integrating history in mathematics education;
- History and epistemology implemented in mathematics education: Classroom experiments & teaching materials, considered from either the cognitive or/and affective point of view; survey of curricula and textbooks;
- Original sources in the classroom, and their educational effects;
- History and epistemology as tools for an interdisciplinary approach in the teaching and learning of mathematics and sciences;
- Cultures and Mathematics;
- Topics in the history of mathematics education.

Each theme had a plenary lecture, several workshops, oral presentations and posters, and some also had panel discussions. Since it was impossible to attend all the conferences and workshops, we shall only comment here on the ones we found most useful or interesting.

Of the plenaries, the most appreciated was that by Michael Glaubitz from the Albert Einstein Gymnasium in Hameln (Germany): *The Use of Original Sources in The classroom - Empirical Research Findings* (theme 3). He proposed the hermeneutic approach to the use of historical material claiming that reading an original source can be an especially rewarding enterprise which is capable of substantially deepening mathematical understanding, enriching classroom activities and developing learners' beliefs in mathematics. He set out, from an empirical point of view, feasible conceptual designs, necessary preparations and prospective effects when reading an original source in class using the example of quadratic equations. This was judged as "probably the most important lecture for actual teaching in class", and suggested further questions to be probed. We were really sorry

that the same author's workshop was cancelled.

Another successful plenary lecture was Michael Fried's: *History of Mathematics in Mathematics Education: Problems and Prospects* (theme 1). The speaker from the Ben Gurion University of the Negev (Israel), suggested that teaching maths using history has to involve a redefinition of our general goals for mathematics education more than the concrete ways in which we bring history of mathematics into the classroom. History as a goal means history as a new view point on the mathematical landscape in which a central idea is the presence of original texts in a form or another.

Also the two plenary lectures given by Marc Moyon, from the Centre d'Histoire des Sciences et d'Epistémologie de Lille1 & IREM de Lille; and Fulvia Furinghetti from Università di Genova (Italy) together with Livia Giacardi from Università di Torino (Italy) were judged very interesting. The first one, titled *Practical Geometries in Islamic Countries: the example of the division of plane figures* (theme 4) presented different geometric problems linked to various other subjects as among others, the practices of craftsmen, architect or jurists. The interest for the class practice is the wide range of procedures in which the whole mathematical knowledge seems to be involved. The second one, named *From Rome to Rome: Events, People, and Numbers during ICMI's First Century* (theme 6), was enjoyed because of his giving insights to some key ideas that led to the birth and to the growth of ICMI and to the changes in methodological approaches to problems that offered, especially to young teachers, a global perspective and a theoretical basis.

Our favourite panel discussion was the one chaired by Costantinos Tzanakis with contributions from Anne Boyé from Centre François Viète, Université de Nantes (France), Adriano Dematté from Università di Genova (Italy), and Ewa Lakoma from Military University of Technology Warsaw (Poland). The title was *The history of mathematics in school textbooks.* The close link with teachers' everyday practice together with the well constructed discussion appealed to us, even though some detected a lack of optimism in some panellists. For a future ESU it would be good to have a panel discussion on the use of history for students who find mathematics difficult. (Could history of maths help them to do better?)

The most interesting workshops we thought were: *Maps, Narratives and Orientations: The use of Concept Maps for exploring our Mathematical Heritage in the Classroom* (theme 2), by Leo Rogers from Department of Education, University of Oxford (UK); *Defining derivatives, integrals and continuity in secondary school: a phased*

approach inspired by history (theme 2), by Hilde Eggermont from Sint-Pieterscollege, Leuven (Belgium) together with Michel Roelens from Katholieke Hogeschool Limburg (Belgium); *Good Old arithmetic* (theme 3), by Frédéric Metin from IREM of Dijon, Université de Bourgogne (France); *Pedagogical and Mathematical Games throughout the Times: from Rithmomachia to Hex* (theme 3) by Jorge Nuno Silva from University of Lisbon (Portugal); *Digitising the past mathematics by the future mathematics* (theme 3), by Snezana Laurence from Bath Spa University, Culverhay Campus (UK); *Mathematisation of nature and new conceptions of curves in the years 1630* (theme 4), by Evelyne Barbin; *The mediaeval geometries: a way to use the history of mathematics in the classroom of mathematics* by Marc Moyon. The main reason why these were so popular was because they related directly to classroom work and/or offered useful material.

We have some critical comments and suggestions. There were rather too many short oral presentations. We noticed a really widespread range of quality, from rather poor, to quite useful. Since the Summer Universities declared aims are to provide a forum for presenting research in mathematics education and innovative teaching methods and to give the opportunity to mathematics teachers, educators and researchers to share their teaching ideas and classroom experience, it would be a good idea to reduce the number of oral presentations so as to make room for, say, "panel discussions workshops", coordinated by the theme of the discussion's proposer, in which each participant has the right to say his/her opinion. This would be an opportunity, especially for new comers, who often feel that they are the only ones not already known to everyone else, to be introduced to the researchers and teachers more interested in the same questions as them. It would help first-time teachers to identify and meet other teachers at the very beginning of the conference. Even though there should not be any real separation between researchers and teachers, a clearer indication of sessions of special interest to classroom teachers could help the absolute beginners to choose from such a widespread menu. Maybe, the workshops could be of two different types: the usual ones on one hand and some others more similar to the pre-service teachers trainee activities on the other. This would help young people or other new entries to participate in an active way and to make the experience more useful and nice.

To conclude, let me offer a suggestion about the arrangements. I think that the reason why in recent ESUs there have not been as many teachers participating is also a problem of sponsorship. The economic crisis has had the effect of drastically reducing the money available for these activities, especially for teachers (this is the case in Italy and must be the same for other European countries). So, even if personally I always was very happy when we stayed in a big city like Prague or, this year, Vienna (I really enjoyed the operas I've seen and listened to), it might be better for future meetings to take place in quieter and less expensive places. May be it also help if there were a cheaper registration fee for classroom teachers.

Finally, we would all like to thank Manfred Kronfeller and the local organizing committee for the marvellous work they did

and for the kindness and the smiles they gave to all of us even if they sometimes were under stress. Thanks a lot to Evelyne Barbin and Costas Tzanachis too, for their giving the teachers the opportunity to contribute to the improvement of ESU's.

Caterina Vicentini
Istituto d'Arte "Max Fabiani", Gorizia (Italy)
Scienza under 18 isontina, Fogliano Redipuglia (Italy)

(Editor's note: Impressions on the ESU6 can also be read at the blog *Teacher Educator Bjørn* http://teachereducatorbjorn.blogspot.com/search?q=ESU from where the photos are taken.)

The editors welcome reports from conferences.

Work in progress

We encourage young researchers in fields related to *HPM* to send us a brief description of their work in progress or a brief description of their dissertation.

Have you read these?

Argün, Z., Arıkan, A., Bulut, S., & Sriraman, B. (2010). A brief history of mathematics education in Turkey: K-12 mathematics curricula. *ZDM, 42*(5), 429-441.

Bos, E.-J. (2010). Princess Elizabeth of Bohemia and Descartes' letters (1650-1665). *Historia Mathematica, 37*(3), 485-502.

Bressoud, David M. (2010). Historical Reflections on Teaching Trigonometry. *Mathematics Teacher, 104(2),* 106.

Catepillan, Ximena and Waclaw Szymanski, "Maya Calendar Conversions," *Loci* (July 2010).

Clark, K. M. (2010). Connecting local history, ancient history, and mathematics: the Eustis Elementary School pilot project. *BSHM Bulletin: Journal of the British Society for the History of Mathematics*, *25*(3), 132-143.

de Wreede, L. C. (2010). A dialogue on the use of arithmetic in geometry: Van Ceulen's and Snellius's Fundamenta Arithmetica et Geometrica. *Historia Mathematica, 37*(3), 376-402.

Descotes, D. (2010). An unknown mathematical manuscript by Blaise Pascal. *Historia Mathematica, 37*(3), 503-534.

Galuzzi, M. (2010). Newton's attempt to construct a unitary view of mathematics. *Historia Mathematica, 37*(3), 535-562.

Guan, Y. (2010). A new interpretation of Shen Kuo's *Ying Biao Yi*. *Archive for History of Exact Sciences, 64*(6), 707-719.

Hogendijk, J. P. (2010). The scholar and the fencing master: The exchanges between Joseph Justus Scaliger and Ludolph van Ceulen on the circle quadrature (1594-1596). *Historia Mathematica, 37*(3), 345-375.

Kourkoulos, M. & Tzanakis, C. (2010). History, and students' understanding of variance in statistics. *BSHM Bulletin: Journal of the British Society for the History of Mathematics*, *25*(3), 168-178.

Krüger, J. (2010). Lessons from the early seventeenth century for mathematics curriculum design. *BSHM Bulletin: Journal of the British Society for the History of Mathematics*, *25*(3), 144-161.

Mancosu, P., & Arana, A. (2010). Descartes and the cylindrical helix. *Mathematica, 37*(3), 403-427.

Maronne, S. (2010). Contexts, emergence and issues of Cartesian geometry: In honour of Henk Bos's 70th birthday. *Historia Mathematica, 37*(3), 341-344.

Maronne, S. (2010). The ovals in the Excerpta Mathematica and the origins of Descartes' method of normals. *Historia Mathematica, 37*(3), 460-484.

Musto, G. (2010). Mathematical timelines. *BSHM Bulletin: Journal of the British Society*

for the History of Mathematics, *25*(3), 162-167.

Papadopoulos, I. (2010). "Reinventing" Techniques For The Estimation Of The Area Of Irregular Plane Figures: From The Eighteenth Century To The Modern Classroom. *International Journal of Science and Mathematics Education, 8*(5), 869-890.

Pesic, P. (2010). Hearing the Irrational: Music and the Development of the Modern Concept of Number. *Isis, 101*(3), 501-530.

Rabouin, D. (2010). What Descartes knew of mathematics in 1628. *Historia Mathematica, 37*(3), 428-459.

Schwartz, Randy K., "Combining Strands of Many Colors: Episodes from Medieval Islam for the Mathematics Classroom," *Loci* (August 2010).

Wagner, R. (2010). The natures of numbers in and around Bombelli's *L'algebra*. *Archive for History of Exact Sciences, 64*(5), 485-523.

Have you been here?

In this section we bring links related to the scope of the HPM from around the world. Please send suggestions.

Societies and organisations

Commission on the History of Mathematics in Africa (including newsletter)
http://www.math.buffalo.edu/mad/AMU/amuchma_online.html

Association des Professeurs de Mathematiques de l'Enseignement Public [APMEP] History site:
http://www.apmep.asso.fr/BMhist.html

British Society for the History of Mathematics [BSHM]
http://www.bshm.org

HOMSIGMAA - History of Mathematics Special Interest Group of the MAA
http://www.maa.org/sigmaa/hom

HPM Americas
http://www.hpm-americas.org/

Italian Society of History of Mathematics
http://www.dm.unito.it/sism/indexeng.html

Association pour la Recherche en Didactique des Mathématiques:
http://www.ardm.asso.fr/

Commission Française pour l'Enseignement des Mathématiques:
http://www.cfem.asso.fr/

Instituts de Recherche sur l'Enseignement des Mathématiques (IREM):
http://www.univ-irem.fr/

Canadian Society for History and Philosophy of Mathematics
http://www.cshpm.org

Brazilian Society for History of Mathematics
http://www.sbhmat.com.br
***Nuncius* Newsletter**
http://brunelleschi.imss.fi.it/nuncius/inln.asp?c=5302

International History, Philosophy and Science Teaching Group
www.ihpst.org

Centre for the History of the Mathematical Sciences.
The Open University, UK
http://purematths.open.ac.uk/pmd_research/CHMS/index.html

Oxford Museum of the History of Science
www.mhs.ox.ac.uk/exhibits/
http://www.mhs.ox.ac.uk/measurer/text/title.htm
http://www.mhs.ox.ac.uk/geometry/title.htm
http://www.mhs.ox.ac.uk/scienceislam/

Topics and Resources

Maths is good for you!
http://www.mathsisgoodforyou.com/
With the heading 'History of mathematics for young mathematicians' Snezana Lawrence's very attractive website contains many suggestions for lesson starters and background information.

A list of resources on history of mathematics related to school in the Norwegian language
http://eleviki.wikidot.com/ressurser-om-matematikkhistorie-pa-norsk

MATHS for EUROPE: The history of some aspects of mathematics like: history of mathematical persons, symbols, algorithms...
http://mathsforeurope.digibel.be/index.html
http://mathsforeurope.digibel.be/list.htm
http://mathsforeurope.digibel.be/olvp.htm
http://mathsforeurope.digibel.be/olvp2.htm
http://mathsforeurope.digibel.be/olvp3.htm

Ethnomathematics on the Web
http://www.rpi.edu/%7Eeglash/isgem.dir/links.htm

About Medieval Arabic Numbers
http://www.geocities.com/rmlyra/Numbers.html
http://www.geocities.com/rmlyra/arabic.html

Annotated Bibliography on Proof in Mathematics Education
http://fcis.oise.utoronto.ca/~ghanna/educationabstracts.html

BibM@th
http://www.bibmath.net/dico/index.php3?action=rub&quoi=0

Centro Virtual de Divulgación de las Matemáticas, esta siendo desarrollada por la Comisión de Divulgación de la *Real Sociedad Matemática Española (R.S.M.E.)*
http://www.divulgamat.net/index.asp

Digitization of the oldest extant manuscript of Euclid's *Elements*
http://librarieswithoutwalls.org/bookviewer/

History of Statistics
http://www.stat.ucla.edu/history/

Images of Lobachevsky's context
http://www.ksu.ru/eng/museum/page0.htm

Images of Mathematicians on Postage Stamps
http://members.tripod.com/jeff560/index.html

Photos of Mathematicians
http://www.math.uni-hamburg.de/home/grothkopf/fotos/math-ges/

Numdam-Digitization of ancient mathematics documents
http://www.numdam.org/en/ressnum.php

The Montana Mathematics Enthusiast (journal)
http://www.montanamath.org/TMME/

Loci: Convergence: an online magazine of the MAA providing resources to teach mathematics through its history
http://mathdl.maa.org/mathDL/46/

International Journal for Mathematics Teaching and Learning,
http://www.cimt.plymouth.ac.uk/journal/default.htm

Homepage of International Journal for the History of Mathematics Education
http://www.tc.edu/centers/ijhmt/index.asp?Id=Journal+Home

Documents for the History of the teaching of mathematics in Italy
http://www.dm.unito.it/mathesis/documents.html

Ethnomathematics Digital Library
http://www.ethnomath.org/

Some Japanese Mathematical Landscapes: The results of wandering in a beautiful country, with a mathematical eye, aided by a digital camera, by A. Arcavi
http://math.criced.tsukuba.ac.jp/museum/arcavi/arcavi_english/index.html

Wann-Sheng Horng's webpage
with HPM related materials in Chinese.
http://math.ntnu.edu.tw/~horng/

Fred Rickey's History of Mathematics Page
http://www.dean.usma.edu/math/people/rickey/hm/default.htm

Culture*MATH*. Ressources pour les enseignants de Mathématiques
www.dma.ens.fr/culturemath/actu/livres.htm

The French INRP (National Institute for Pedagogical Research) is developing a website on questions related to mathematics teaching: EducMath
http://educmath.inrp.fr

Geometrical books and instruments from 15th to 18th century
http://www.geometricum.com/

David Henderson' s Home Page [Educational and Historical Topics on Geometry]
http://www.math.cornell.edu/~dwh/

Homepage of Albrecht Heeffer
http://logica.ugent.be/albrecht/

Homepage of Jens Høyrup
http://www.akira.ruc.dk/~jensh/

***L'Enseignement Mathématique*, Archive**
http://retro.seals.ch/digbib/vollist?UID=ensmat-001

Homepage of Prof. Leo Corry
http://www.tau.ac.il/~corry/

Opera Mathematica of Christoph Clavius
http://mathematics.library.nd.edu/clavius/

Archimedes Project [Some famous mathematical books of the Renaissance period are available on line, i.e. Pacioli's *Summa*]
http://archimedes2.mpiwg-berlin.mpg.de/archimedes_templates

Simon Stevin's *De Meetdaet* [The Practice of Measuring]
http://www.math.leidenuniv.nl/~wiskonst/meetdaet/index.html
and The Principal Works of Simon Stevin
http://www.historyofscience.nl/works_detail.cfm?RecordId=2702

Mathematicians Gallery
http://www.math.uconn.edu/MathLinks/mathematicians_gallery.php?Rendition=printerfriendly

History of Mathematics
http://www.otterbein.edu/resources/library/libpages/subject/mathhis.htm

The Garden of Archimedes. A museum for Mathematics
http://web.math.unifi.it/archimede/archimede_NEW_inglese/

Mathematical instruments
http://brunelleschi.imss.fi.it/museum/esim.asp?c=500164
and
http://web.mat.bham.ac.uk/C.J.Sangwin/Sliderules/sliderules.html
and
http://www.mhs.ox.ac.uk/epact/catalogue.php?ENumber=52265

Homepage of Eleanor Robson
http://www.hps.cam.ac.uk/dept/robson.html

Flickr group for HPM related photos
http://www.flickr.com/groups/812621@N24/

Monuments on Mathematicians
http://www.w-volk.de/museum/exposi.htm

Video on the history of mathematics

http://www.youtube.com/watch?v=wo-6xLUVLTQ

International Journal for the History of mathematics Education, http://www.comap.com/historyjournal/

We would like to provide a more comprehensive list of websites containing resources useful to researchers and students (not necessarily in English). If there are any you use, or you know are useful for students or researchers, please send your recommendations to the editors.

Notices

Volume of a sphere in Ancient China and India

Professor R. C. Bupta, Ph. D. (Hist. Of Math.)
Ganita Bharati Academy
R-20, Ras Bahar Colony, JHANSI-284003, India

The famous Greek mathematican Archimedes (died 212 BCE) had already found the correct formula

$$V_o = \left(\frac{\pi}{6}\right) d^3 = (4\pi/3)r^3 \qquad (1)$$

$$= 0.5236d^3 \text{ very nearly} \qquad (2)$$

for the volume of a sphere of diameter d (=2r). On the other hand, the popular ancient Chinese work *Jiu Zhang Suan Shu* (JZSS) which reached its present form in the first century CE, contains two numerical problems in which d is calculated by using the rule

$$d = \left(\frac{16v}{9}\right)^{1/3} \qquad (3)$$

This clearly implies the formula

$$v = \left(\frac{9}{16}\right) d^3 \qquad (4)$$

Suppose the sphere is inscribed in a right circular cylinder whose height is d and base is also of diameter d. Also let the cylinder be circumscribed by a cube of side d such that their bases and tops lie in the same planes. It was known that

$$\frac{\text{Vol.of the cylinder}}{\text{Vol.of the cube}} = \pi/4 \qquad (5)$$

Now let

$$\frac{\text{Vol.of the sphere}}{\text{Vol.of the cylinder}} = k \qquad (6)$$

From these two equations we see that

$$\text{Vol. of sphere} = (\pi k/4)d^3 \qquad (7)$$

The mathematically exact value of k is easily seen to be 2/3.

The *JZSS* uses the simple approximation $\pi = 3$ for calculating the area of a circle. Therefore a suggested use of the very high value $\pi = \frac{27}{8}$ for getting (4) from (1) is rejected as unlikely. In fact during the first century of CE and in the time of Zhang Heng (78-139 CE), the ratio $\frac{\pi}{4}$ in (5) was taken to be ¾ in China. At that time, it was thought that the ratio k also has the same value $\frac{\pi}{4}$ or ¾. Thus the early Chinese mathematicians were led, by (7), to the empirical rule

$$\text{Vol. of a sphere, } v = \left(\frac{\pi^2}{16}\right) d^3 \qquad (8)$$

$$\text{Or,} \qquad = (9/16)d^3 \qquad (9)$$

which is indeed the *JZSS* formula (4). This formula already greater than the true value of the volume but the calendrist Zhang Heng thought otherwise and made it even worse by taking

$$V_1 = (9/16)d^3 + (1/16)d^3 \qquad (10)$$

$$= (5/8)d^3 \qquad (11)$$

However, this, compared with (8), leads to $\pi = \sqrt{10}$ which is better than $\pi = 3$ and better even than another of Zhang Heng's values $\pi = \frac{92}{29}$.

A comment, attributed to Liu Hui (3rd cent. CE), on the *JZSS* sphere problem shows that (4) implies (6) with $k = \frac{\pi}{4}$. According to Li Chunfeng (602-670), Zu Geng (=Zu Xuan), son of Zu Chongzhi (429-500), had stated that Liu Hui (like Zhang Heng) took (6) with

$k = \frac{\pi}{4}$. There is some indication to show that Liu Hui considered this assumption to be wrong although he was unable to derive the correct rule.

In India, Āryabhaṭa (born 476 CE) in his work *Āryabhaṭīya* (II,7) gives the rule for the volume of sphere as

$$V_2 = A\sqrt{A} \text{ exactly} \qquad (12)$$

where A is the area of a central section of the sphere (i.e. $A = \pi r^2$). His commentator Bhāskara I (629 CE) quotes a rule (in Sanskrit) which gives a formula equivalent to (4). The rule was regarded empirical and so Āryabhaṭa attempted to find a better one.

Mahāvīra (about 850 CE) in his *Gaṇita-sāra Saṅgraha* (=GSS, VIII, 28) has also given the same rule and clearly stated it to be practical (i.e. not exact or accurate). The same rule also appears in the *Tiloyasāra* (gāthā 96) of Nemicandra (about 975 CE), and in the *Gaṇita-sāra* (V, 25) of Ṭhakkura Pherū (14th cent.) who gave it the form

$$d^3 \cdot \left(1 - \tfrac{1}{4}\right)\left(1 - \tfrac{1}{4}\right) \qquad (13)$$

A detailed discussion of all these Indian rules show that (4) should be interpreted as (8) with $\pi = 3$.

The modification of empirical rules by adjusting them to suitable values of π is an ancient practice. For example,

$$A = (c + h)h/2 \qquad (14)$$

for the area of a circular segment (of chord c and height h) was supposed to be based on $\pi = 3$ for which it gives exact value in the case of a semicircle. It was modified or adjusted to $\pi = \frac{22}{7}$ in the form

$$(c + h)\left(\tfrac{h}{2}\right) \cdot \left(\tfrac{22}{21}\right) \qquad (15)$$

In the *Mensurae* attributed to Heron (1st cent. CE). More interestingly, (14) wa adjusted to $\pi = \sqrt{10}$ (a popular Jaina value) in the form

$$(c + h)(h/2) \cdot \sqrt{\tfrac{10}{9}} \qquad (16)$$

In India by Śrīdhara (about 750 CE) in his *Triśatikā* (rule 47), by Pherū in his *Gaṇita-sāra* (III, 46) and by others. However, if a rule (to be modified) was presumed to involve the square of π or of circular circumference, the adjustment was done by the factor 10/9. Thus we find Mahāvīra giving his accurate rule for the volume of a sphere as (*GSS*, VIII, 28½)

$$V_3 = \frac{10v}{9} \qquad (17)$$

$$= 5r^3 \qquad (18)$$

which happens to be the same as V_1 in (11). For Mahāvīra $\pi = 3$ and (4) were rough or practical, and $\pi = \sqrt{10}$ was accurate. It may be pointed out that (18) can also be obtained from Āryabhaṭa's rule (12) by taking $A = 3r^2$, and using the usual Indian rule

$\sqrt{a^2 + x} = a + \frac{x}{(2a+1)}$, with a=1 and x=2.

In China, the exact Archimedean formula (1) was proved by Zu Geng (6th cent. CE). He used it for both $\pi = 3$ and $\pi = \frac{22}{7}$ for convenience, although he knew even the far better value $\pi = \frac{355}{113}$ from his father. For the simple value $\pi = 3$, we have simply

$$V_4 = \frac{d^3}{2} \qquad (19)$$

If this is adjusted to a Jaina value $\pi = \frac{19}{6}$, we get

$$V_5 = \frac{d^3}{2} \cdot \frac{19}{18} \qquad (20)$$

which is found in the *Triśatikā* (rule 56) of Śrīdhara. The value $\pi = \frac{19}{6}$ is commonly derived from the usual Jaina value $\pi = \sqrt{10}$ (used in their sacred works) by using the approximation

$\sqrt{a^2 + x} = a + \frac{x}{(2a)}$, with a = 3 and x = 1.

The formula (20) is also found in the *Siddhānta Śekhara* (XIII, 46) of Śrīpati (11th cent.), and in the *Mahāsiddhānta* (XVI, 108) of Āryabhaṭa II whose date, following recent research, has been changed from the 10th to the 15th century.

The famous Indian mathematican Bhāskara II (12th cent.) not only gave the correct formulas for the surface and volume of a sphere but also supplied justifications. His *Līlāvatī* (rule 201) contains

$$S_0 = 4\pi r^2 \qquad (21)$$

$$V_0 = \frac{S_0 d}{6} = \left(\frac{\pi}{6}\right) d^3 \qquad (22)$$

The name of Archimedes, and his beautiful proof, is hardly ever met in ancient China or India. Was the ancient orient unaware of Greek achievements, or was it a case of older methods and practices being preferred even where new ones were known? The Jaina Pherū used the square form of the Jainisation adjustment factor 10/9 to improve the ancient formula (for surface of sphere)

$$S = \frac{C^2}{4} \qquad (23)$$

which can be derived from a rule of Mahāvīra (*GSS*, VII, 25); here C is the circumference of the great circle. Pherū also used the same factor for V_4 or $\frac{d^3}{2}$ which he perhaps picked up from (20). But in this case he was confused because here it is π which is involved and not π^2. Had he used the (linear) Jainisation factor $\sqrt{10/9}$, his result would have been far better.

References

1. L. Berggren et. al, *Pi: A Source Book*. Springer, New York, 2004.
2. R. C. Gupta, "Volume of a Sphere in Ancient Indian Mathematics", *Journal of the Asiatic Society*, XXX (1988), 128-140.
3. R. C. Gupta, "Techniques of Ancient Empirical Mathematics", *Indian Journal of History of Science* 45 (2010), 63-100.
4. J.-C. Martzloff, *A History of Chinese Mathematics*. Springer, Berlin, 1997.
5. D. B. Wagner, "Doubts Concerning the Attribution of Liu Hui's Commentary (on JZSS)", *Acta Orientalia*, 39 (1978), 199-212.
6. D. B. Wagner, "Liu Hui and Tsu Keng-chih on The Volume of a Sphere", *Chinese Science* III (1978), 59-79.

Announcements of events

CERME 7

February 9–13, 2011
Rzeszów, Poland
CERME is a Congress designed to foster a communicative spirit. It deliberately and distinctively moves away from research presentations by individuals towards collaborative group work. Its main feature is a number of thematic groups whose members will work together in a common research area. Researchers wishing to present a paper at the Congress should submit the paper to one of these groups. In 2011, Working Group 12 (WG 12) will be on "History in Mathematics Education", see directly below.
Website: http://www.cerme7.univ.rzeszow.pl/

CERME7: Working Group 12
History in mathematics education
Leaders
Uffe Thomas Jankvist (Denmark)
utj@imada.sdu.dk
Snezana Lawrence (UK), Jan van Maanen (The Netherlands), Constantinos Tzanakis (Greece).
Scope and Focus of WG12
The integration of history of mathematics in mathematics education has received increasing attention recently. This attention is reflected by the many publications both in journals and books and the increasing activities at international conferences and meetings. However, empirical research and coherent theoretical/conceptual frameworks within this area have been emerging relatively recently. The purpose of this CERME WG is to provide

a forum to approach mathematics education in historical context, dedicated *primarily* to theory and research on all aspects of the role, effect and efficacy of history in mathematics education.

Call for papers and poster proposals

WG12 in particular welcomes empirical and theoretical research papers, but to some degree also methodological and developmental papers, (10 pages max) and poster proposals (2 pages) related to one or more of the following issues – although any paper/poster of relevance to the overall focus of the group will be taken into consideration:

1. Theoretical, conceptual and/or methodological frameworks for including history in mathematics education;
2. Relationships between (frameworks for and empirical studies on) history in mathematics education and theories and frameworks in other parts of mathematics education;
3. The role of history of mathematics at primary, secondary, and tertiary level, both from the cognitive and affective points of view;
4. The role of history of mathematics in pre- and in-service teacher education, from cognitive, pedagogical, and/or affective points of view;
5. Possible parallelism between the historical development and the cognitive development of mathematical ideas;
6. Ways of integrating original sources in classrooms, and their educational effects, preferably with conclusions based on classroom experiments;
7. Surveys on the existing uses of history in curricula, textbooks, and/or classrooms in primary, secondary, and tertiary levels;
8. Design and/or assessment of teaching/learning materials on the history of mathematics;
9. Relevance of the history of mathematical practices in the research of mathematics education.

Papers and poster proposals should use the CERME7 WORD template, and conform to the guidelines at www.cerme7.univ.rzeszow.pl/?id=cerme-guidelines-for-authors. To submit it, you must email your paper as a WORD document to Uffe Th. Jankvist at utj@imada.sdu.dk, AND at the same time to the conference secretariat at s.cerme7@univ.rzeszow.pl

If possible please also send a pdf version *in addition* to the WORD document.

Reviews and Decisions

Each paper will be peer-reviewed by at least two persons from among those who submit papers to this Working Group and the leaders. Please expect to be asked to review up to three papers yourself between 15th September and 22nd October 2010. It may be necessary for you to revise your paper before final acceptance. Please reserve some time to do this in the second half of November. The group leaders will decide about the acceptance of posters.

Important dates

15th September 2010: Deadline for submission of papers.
1st October 2010: Deadline for submission of poster proposals
22nd October 2010: Deadline for reviewers to submit their reviews.
1st December 2010: Deadline for revisions to papers

Colloque « L'enseignement des mathématiques, des mathématiques du quotidien à la théorie en l'honneur de Nicolas Rouche »

March 16-18, 2011

Mons, Belgium (March 16th), Lille, France (March 17th-18th)

This symposium in honor of Nicolas Rouche will be in French, so the information is also given in French:

Quels qu'ils soient et quels que soient les systèmes éducatifs dont ils sont des maillons, les enseignants de mathématiques sont obligés à un moment ou un autre de s'interroger sur le sens de leur enseignement en terme éducatif et politique, en terme de rapport à la réalité (celle des problèmes pratiques qui se posent à toute société). Cette réalité s'invite aussi par le biais des étonnements ou questions, venant de

jeunes enfants comme d'étudiants en thèse ; elle s'hybride à la symbolisation, elle se transforme dans des réseaux de techniques et de théorisations. S'agit-il d'aménager la construction, avec les meilleurs procédés, d'une science déductive déjà faite ou s'agit-il d'enseigner à penser mathématiquement ? A chaque niveau de rigueur et d'exigence, comment la pensée mathématique s'ancre-t-elle dans les perceptions, les actions, les mouvements ? Quel part peut y prendre le jeu, l'expérimentation ? Comment se construisent définitions et concepts dans leurs rapports à l'intuition, aux problèmes et aux démonstrations ? L'histoire des mathématiques et l'histoire de l'enseignement informent sur toutes ces questions, quelles ressources offrent-elles aux enseignants ?

De l'école élémentaire à l'université, chercheur-e-s et/ou enseignant-e-s apporteront des éléments pour travailler ces questions dans tous les champs de la discipline (algèbre, géométrie, analyse…) par des conférences plénières, des exposés ou des ateliers de 1h30.

Conférences invitées : Erich Wiitmann, Rudolf Bkouche, Jean Mawhin, Christine Decoq, Christiane Hauchart, Evelyne Barbin, Thérèse Gilbert, Luc Sinègre.

Ce colloque est organisé à l'initiative de la Régionale Pays-Bas, (Groupe d'Enseignement Mathématique de Louvain la Neuve, Groupe de Leuven, Freudenthal Institute et IREM de Lille), du CREM (Centre de Recherche sur l'Enseignement des Mathématiques, Nivelles), et des deux Commissions Inter-IREM "Histoire et Epistémologie" et "Géométrie".

Informations et appels à contributions à partir du 1er décembre 2010 :

http://irem.univ-lille1.fr/

11th International Conference of The Mathematics Education into the 21st Century Project: Turning Dreams into Reality: Transformations and Paradigm Shifts in Mathematics Education

September 10–16, 2011

Rhodes University, Grahamstown, South Africa

The Mathematics Education into the 21st Century Project has just completed its tenth successful international conference in Dresden, Germany, following conferences in Egypt, Jordan, Poland, Australia, Sicily, Czech Republic, Malaysia and the USA. Our project was founded in 1986 and is dedicated to the planning, writing and disseminating of innovative ideas and materials in Mathematics, Statistics, Science and Computer Education. The next conference is planned for September 10–16, 2011 in Grahamstown, South Africa. The chairman of the Local Organising Committee is Professor Marc Schafer of Rhodes University. The conference will open with an evening welcome reception on Sunday, Sep 10th and will close with lunch on Saturday, Sep 16th.

The title of the conference is "***Turning Dreams into Reality: Transformations and Paradigm Shifts in Mathematics Education***". Paper proposals are now invited on all innovative aspects of mathematics, statistics, science and computer education. Our conferences are renowned for their friendly and productive working atmosphere. They are attended by innovative teachers and mathematics educators from all over the world, 44 countries were represented at our last conference for example.

Plenary speakers: Prof. Dr. Ludwig Paditz from Germany and Professor Ubiratan D'Ambrosio from Brazil.

There will be an additional full social programme for accompanying persons.

For ALL further conference details please email Alan Rogerson, Chairman of the International Programme Committee, at alan@rogerson.pol.pl

ICME 12

July 8–15, 2012
Seoul, South Korea
http://www.icme12.org/
First Announcement now available from
http://www.icme12.org/eng/announ_first_welcome.html

HPM 2012

July 16–20, 2012
Daejeon, South Korea

Photo from a meeting of some of the people responsible for the HPM 2012 (from left to right): Sunwook Hwang (chair of the Local Organising Committee (LOC), president of KSME), Jinho Kim (secretary of LOC), Evelyne Barbin (HPM AdB), Pamela Chae (Daejeon Convention Center), Sung Sook Kim (vice-president of KSME and member of LOC), Masami Isoda (HPM AdB), Chang Kyoon Park (president of KSHM and member of LOC), Sangki Choi (vice-chair of LOC).

ESU7

2014
To be announced...

A note from the Editors

The Newsletter of HPM is primarily a tool for passing on information about forthcoming events, recent activities and publications, and current work and research in the broad field of history and pedagogy of mathematics. The Newsletter also publishes brief articles which they think may be of interest. Contributions from readers are welcome on the understanding that they may be shortened and edited to suit the compass of this publication.

Distributors:

If you wish to be a distributor in a new or unstaffed area please contact the editor.

Area	Name and address	Email address
Argentina	Juan E. Nápoles Valdés, Lamadrid 549, (3400) Corrientes, ARGENTINA	napoles4369@gmail.com
Australia	G. FitzSimons, Faculty of Education, P.O.Box 6, Monash University, 3800 Victoria, AUSTRALIA	gail.fitzsimons@education.monash.edu.au
Austria	Manfred Kronfellner, Institute of Discrete Mathematics and Geometry, Vienna University of Technology, Wiedner Haupstr. 8-10, A-1040 Wien, AUSTRIA	m.kronfellner@tuwien.ac.at
Belgium and The Netherlands	Sylvia Eerhart, Freudenthal Instituut, Aïdadreef 12, 3561 GE Utrecht, THE NETHERLANDS	S.Eerhart@fi.uu.nl
Canada	Thomas Archibald, Mathematics Department, Acadia University, Wolfville, NS B0P1X0, CANADA	Tom.Archibald@acadiau.ca
China	Ma Li, Linkoping University, ITN, SE - 601 74 Norrkoping, SWEDEN	ma_li@mac.com
Eastern Europe		
France	Evelyne Barbin, Centre François Viète, Faculté des sciences et des techniques, 2 Chemin de la Houssinière, BP 92208, 44322 Nantes cedex, FRANCE	evelyne.barbin@wanadoo.fr
Germany	Gert Schubring, Inst. f. Didaktik der Math., Universitaet Bielefeld, Postfach 100 131, D-33501, Bielefeld, GERMANY	gert.schubring@uni-bielefeld.de
Iran	Mohammad Bagheri, P.O.Box 13145-1785, Tehran, IRAN	sut5@sina.sharif.edu
Israel	Ted Eisenberg, Mathematics Department, Ben Gurion University of the Negev, Beer-Sheva 84105, ISRAEL	eisen@math.bgu.ac.il eisenbt@barak-online.net
Italy	Giorgio T. Bagni †, Department of Mathematics and Computer Science, University of Udine, Polo Rizzi, via delle Scienze 206, I-33100 Udine, ITALY and Marta Menghini, Dipartimento di Matematica (Universita` La Sapienza), Piazzale A. Moro 5, 00185 Roma ITALY	marta.menghini@uniroma1.it
Japan	Osamu Kota, 3-8-3 Kajiwara, Kamakura Kanagawa-ken, 247-0063 JAPAN	kota@asa.email.ne.jp
Malaysia	Mohamed Mohini, Department of Science and Mathematical Education, Universiti Teknologi Malaysia, 81310 Johor, MALAYSIA	mohini@fp.utm.my
Mexico	Alejandro R. Garciadiego, Caravaggio 24, Col. Nonoalco Mixcoac Del. Benito Juárez 03700 México, D. F. México	gardan@servidor.unam.mx
Morocco	Abdellah El Idrissi, E.N.S. B.P: 2400 Marrakech, C.P: 40 000, MOROCCO	a_elidrissi@hotmail.com
New Zealand	Bill Barton, Mathematics Education Unit, Dept of Mathematics and Statistics University of Auckland, Private Bag 92-019, Auckland, NEW ZEALAND	b.barton@auckland.ac.nz
Other East Asia	Gloria Benigno, Department of Education, Culture and Sports, Region X, Division of Misamis Occidental, Oroquieta City, PHILLIPINES	glorya4444@yahoo.com
Russia	Vasilii Mikhailovich Busev	vbusev@yandex.ru
Scandinavia	Sten Kaijser, Department of Mathematics, P.O. Box 480, SE- 751 06 Uppsala, SWEDEN	sten@math.uu.se
South America	Marcos Vieira Teixeira , Departamento de Matemática , IGCE - UNESP, Postal 178 13 500 - 230 Rio Claro, SP BRAZIL	marti@rc.unesp.br
South Asia	Prof. R. C. Gupta, Ganita Bharati Academy, R-20, Ras Bahar Colony, Jhansi-284003, U.P. INDIA	

South East Europe	Nikos Kastanis, Department of Mathematics, Aristotle University of Thessaloniki, Thessaloniki 54006, GREECE	nioka@auth.gr
Southern Africa	Paulus Gerdes, Mozambican Ethnomaths Research Centre, C.P. 915, Maputo, MOZAMBIQUE	paulus.gerdes@gmail.com
Spain and Portugal	Carlos Correia de Sá, Departamento de Matemática Pura; Faculdade de Ciências da Universidade do Porto; Rua do Campo Alegre, 687 P - 4169 - 007 Porto, PORTUGAL	csa@fc.up.pt
Taiwan	Wann-sheng Horng, Math dept NTNU, 88 Sec.4, Tingchou Rd., Taipei, TAIWAN	horng@math.ntnu.edu.tw
Turkey	Funda Gonulates, Bagazici Universitesi, Egitim Fakultesi, Bebek- Istanbul, TURKEY	oprukcuf@boun.edu.tr
United Kingdom	Snezana Lawrence, Bath Spa University, Bath, UK	snezana@mathsisgoodforyou.com
United States of America	David L. Roberts, Prince George 's Community College, Largo, Maryland, USA	robertsdl@aol.com

Table of contents

The views expressed in this Newsletter may not necessarily be those of the HPM Advisory Board.
Please pass on news of the existence of this newsletter to any interested parties.
This and previous newsletters can be downloaded from our website:
http://www.clab.edc.uoc.gr/hpm/

Items for the Newsletter should be sent to the editors, preferably by email (see addresses below).
The Newsletter appears three times a year with the following deadlines for next year.

No.	Deadline for material	Sent to distributors
76	12 February 2011	1 March 2011
77	12 June 2011	1 July 2011
78	12 October 2011	1 November 2011

The Newsletter is the communication of the International Study Group on the Relations between the History and Pedagogy of Mathematics, an affiliate of the International Commission on Mathematical Instruction.
The Newsletter is free of charge, available upon request from the distributor for your area, and may be reproduced with acknowledgement.

Editors:
Chris Weeks, chrs.weeks@gmail.com (Downeycroft, Virginstow, Beaworthy, GB - EX21 5EA, Great Britain)
Bjørn Smestad, bjorn.smestad@lui.hio.no (Faculty of Education and International Studies, Oslo University College, Postbox 4 St. Olavs plass, N-0130 Oslo, Norway)