

International Study Group on the Relations Between the HISTORY and PEDAGOGY of MATHEMATICS An Affiliate of the International Commission on Mathematical Instruction

# No. 66 November 2007

This and earlier issues of the Newsletter can be downloaded from our website <u>http://www.clab.edc.uoc.gr/hpm/</u>

# A conversation between Judith Grabiner and Nikos Kastanis

**NK** Your paper *Is mathematical truth time-dependent?* [AMM, 81(1974), 354-365] is a milestone in the modern historiography of mathematics. What are the epistemological trends echoed in that paper?

**JG** In the nineteenth century Hermann Hankel, whom Carl Boyer quotes with apparent approval in his classic *History of Mathematics* (1967), wrote, "In most sciences one generation tears down what another has built, and what one has established another undoes. In mathematics alone each generation builds a new story to the old structure." This view of increasing linear progress was a

commonplace about the natural sciences also, until it was challenged for science, most notably by Thomas Kuhn in *The Structure of Scientific Revolutions* (1962). But there are numerous counterexamples in



the history of mathematics as well, examples when the rules of the game changed. One such revolution was the introduction of logical proof into ancient geometry. Another was the realization that there could be non-Euclidean geometries, showing that geometry is not the unique and necessary science of space. Examples of this new approach abound in the collection edited by Donald Gillies, *Revolutions in Mathematics* (1995).

The rigorization of the calculus associated with the names of Cauchy, Bolzano, and Weierstrass is another such example. This is the one addressed in my 1974 paper. Contrast Euler's successful discoveries by freewheelingly manipulating infinite series, even divergent ones, with Cauchy's "A divergent series has no sum" and Abel's "Can you imagine anything more horrible" than claims like  $1^n - 2^n + 3^n - 4^n + ... = 0$ . This too was a paradigm shift, a change in the agreed-on rules of the game.

Ideas like Kuhn's have had wide influence throughout the history, philosophy, and sociology of all the sciences. These ideas also call attention to the influences on science of the broader philosophical and social trends of the age - say, for instance, the professionalization of mathematics in the nineteenth century, or the need for mathematicians to teach. Scholars often speak of the contingency of history, focusing on the non-linear and non-predictable way events and ideas develop. This applies also to mathematics, which is a creative and social endeavor. Thus the work of mathematicians and mathematical communities resists rational reconstruction. That's why the history of mathematics is interesting.

**NK** During the last years a cognitive historiography of mathematics has developed, especially the aspect of conceptual changes. Your paper *The Changing Concept of Change: The Derivative from Fermat to Weierstrass* [Math. Mag., 56(1983), 195-205] has a relevant background. What is your opinion about this kind of historical understanding of mathematics?

JG Mathematical ideas often do not develop in the order they are later expounded. The axiomatic formulation and the fruitful definitions frequently come at the end of the development of a subject, not the beginning. This is especially striking in the case of the derivative, where properties of varying quantities when the quantities reach their maximum or minimum values were discovered and used by people like Fermat and Descartes, who did not see their calculations as examples of calculating rates of change. When Newton and Leibniz treated such problems as special cases of fluxions or of differential quotients, they opened the door for research into previously inconceivable topics like differential equations. Understanding and proving the properties of derivatives in terms of a sufficiently powerful and precise concept of limit came even later, and teasing out the distinction between pointwise and uniform convergence to a limit came later still. Beginning the study of rigorous calculus with the delta-epsilon definitions of limit and derivative turns this history on its head.

Students often have difficulties like those encountered by the mathematicians who invented key ideas. For example, "You mean 'can be made as close as you like' is the same as 'equals the limit' – how can that be?" is an excellent question. Thus Bishop Berkeley's criticisms of arguments about the limit of the ratio of two quantities that vanish - and the inadequacy of eighteenth-century attempts to answer him in his own terms - recur among very good students. Knowledge of the history of the relevant mathematical concepts can help the teacher in the classroom. And the history also lets those of us who learned a subject with ease appreciate which concepts are inherently hard; it's no accident that it

took a hundred and eighty years from the understanding of the derivative by Newton and Leibniz to the proofs of Weierstrass.

NK In 2000, in the course guidelines for your students in "Mathematics 10, Mathematics in Many Cultures," you started with the following: "General: All cultures have mathematics, though they may not have a class of people called 'mathematicians.' In this course, we will look together at the mathematical activities of a number of present-day cultures and of several historic cultures. We will concentrate on the mathematics, but will also try to gain some understanding of the cultural setting and to understand how culture and mathematics interact. Each student will do a project on a topic of his or her own choice, involving some mathematical subject in some cultural setting; more precise instructions will be forthcoming." Would you like to give a few examples of such historic projects?

**JG** This assignment has worked very well for me in "Mathematics in Many Cultures." I teach the course every year, and I go around showing my colleagues the list of student reports for weeks afterwards. Three such reports stand out in my mind.

First was a student who writes children's books. She said that a good children's book should have a character who has some sort of problem and who then thinks and works to solve it. Her project was a draft of such a book (I don't know whether she has tried to publish it) about the young son of a 15<sup>th</sup>century Inca quipu maker. The boy, after a series of mishaps, carries his father's completed quipu to the Inca ruler and then explains it. Her report drew on Marcia Ascher's discussion of number systems in the chapter on the quipu from Ascher's book *Ethnomathematics*, and on the illustrations of quipus in the manuscript known as the Inka Chronicle (El primer nueva corónica y buen gobierno, c. 1615) by Felipe Guamán Poma de Ayala.

The second was an art student, who reported on symmetries in Islamic art, concentrating (as did Maurits Escher) on those in the Alhambra in Spain. This student drew on Eva Wilson's *Islamic Designs for*  Artists and Craftspeople, and Keith Critchlow's Islamic Patterns. He remarked at the end that he was now using these geometric ideas in his own paintings.

The third student hadn't been looking for a report topic at all while she had searched for information about hairstyles for herself on the Internet. But she found the work of Gloria Gilmer on fractals and tessellations as illustrated by African-American hairstyles: www.math.buffalo.edu/mad/special/gilmergloria\_HAIRSTYLES.html.

She followed this up by reading Paulus Gerdes' book *Geometry from Africa*. Her report explained what fractals and tessellations are and was illustrated by pictures both from mathematical and hairstyling sources.

Each of these reports arose out of something important to the individual student. Doing the report got the students to understand and explain some new mathematics, and to place the mathematics in its cultural setting. And each report fascinated and instructed the entire class.

**NK** Do you have anything to add about these topics?

**JG** The history of mathematics has a lot to teach us. Mathematics is part of human culture. And mathematics answers questions of interest and importance to the society it's in. Mathematics progresses, not only by using and extending the prevailing ideas and techniques, but also by transforming and transcending them. Appreciating all of this can help teachers, attract students, and inspire researchers.

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### **Conference reports**

### ESU5 – 5<sup>th</sup> European Summer University on History and Epistemology in Mathematics Education, Prague, 19-24 July 2007

It is hoped that the Proceedings of the Prague Summer University will be available in early summer 2008. Meanwhile, here is a personal reflection on the summer university from Bob Burn. In the next issue, there will be an additional report by Costas Tzanakis.

The conference mounted a massive programme of 140 or so presentations. You can find them described, with abstracts, if you look at the website

#### http://www.pedf.cuni.cz/kmdm/esu5

and search through the section entitled "Final Programme". Even the most committed participant could not attend more than 20% of these presentations so choice had to be exercised, and that was welcome.

At the start of each day, in plenary, invited international scholars shared their insights and findings. These were always interesting, but not controversial, so they did not generate the issues with which the conference might have buzzed.

The bulk of the conference time was devoted to "workshops" which varied in style and presentation. At one extreme, a workshop was used for the reading of a research paper, which seemed to me to be an abuse of the term 'workshop'. Another style of workshop was the study of a primary source. This is of course a way of ensuring that the workshop is a genuine historical study, but the task of preparing the participants for working on the chosen primary source was not trivial, and the effort that this takes may not be commensurate with the mathematical pay-off. This was a thoroughly appropriate style for this conference, but it was not easy to see how it might be translated into a regular mathematics classroom. The high spot of the conference for me was the use of workshops for the conduct of what could have been regular lessons in high school maths. These were memorable, challenging and historically embedded. I remember a session on Leibniz' arithmetical triangle and another on the volume of a pyramid. It was the experience of a productive challenge for participants which made them so satisfactory. The engagement of participants demands priority over the imparting of information. It was massively impressive when some presenters of workshops offered their own simultaneous translation into French and English, when neither was their mother tongue. It was quite impossible to tell, from the published

abstracts, what the style of a workshop would be until one was in it. Perhaps the study of history is always like that!

After the workshops, short talks of 15 or 30 minutes were offered. The time constraint had clearly focussed the speakers, and I remember a convincing case being made for replacing the 'axiom, definition, theorem, proof' sequence of university mathematics with a genetic approach guided by the historical development of problems.

But there were things that do not appear on the timetable which made the conference even more worthwhile. There was always a chance, in the coffee breaks, to approach well-known researchers and ask one's own questions. And in the course of a guided tour of the city of Prague, a personal conversation raised the question of whether mathematics and history were to some extent at cross purposes in a regular maths class. Authentic history takes time and does not quite fit the mathematics, while useful history may have had the quirks of reality removed from it. Is it perhaps the teachers, above all, whose work is enriched by the insights of history?

The smooth running of the conference owed much to the local group organised by Nada Stehlikova with a very pleasant and competent group of undergraduates and technicians. To satisfy participants from 16 European nations, and 30 nations world wide, is no mean feat.

**Bob Burn** (Exeter, UK)

### Mathematics Education Through Lesson Study In The Asia-Pacific Region

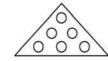
The conference took place from the 16<sup>th</sup> to 20<sup>th</sup> of August in Khon Kaen, a university city in the north-eastern part of Thailand, a province known as Isaan. It gathered a number of researchers from the APEC member countries, external observers and about 150 teachers and master students in mathematics education. Improving the quality of mathematics teaching is a big issue in Asia. The economic success of the four tigers, Japan, Singapore, South Korea and Taiwan, inspires the less developed countries to catch up. Mathematics is seen as leverage for raising the level of engineering and computer sciences and participating in the success of the four examplar countries. The participant from Singapore, Ban Yar Yeap, frankly stated that the prime reason for him engaging in research in mathematics education is the shameless fact that it contributes to the economic success of his country. Also the availability of comparative international assessment results such as PISA and TIMSS make the difference in performance of mathematics education between countries painfully clear. But even Singapore, ranking consistently at the top, seeks to improve its teaching methods. Singaporean students are excellent in procedural methods and symbolical representations. Still, Ban Yar Yeap sees the need to improve upon the problem-solving skills and conceptual understanding of students.

The conference is part of a joint project of research and human resource development of APEC with as main partners the Center for Research in Mathematics Education (CRME) of Khon Kaen University and the Center for **Research on Cooperation International** Development (CRIDED) of the University of Tsukuba, Japan. This year's theme is the development of mathematical thinking. The focus is on the observation and discussion of real-world class situations, called lesson study. The observation, recording and discussion of real-life lessons is a main tool in Japan to improve education. Several presentations discussed particular lessons with video, transcripts and even live classes. Having live lessons with a class of fifty Thai 5<sup>th</sup> grade students and a Japanese teacher before an audience of over 200 people is guite a feat. Translation of the Japanese to Thai and vice versa was done by project manager Maitree Inprasitha who is fluent in both. Translation to the English audience was provided through wireless headphones. Despite the obvious technical challenges and the cultural and language differences between teacher and students the lesson worked remarkably well. The Thai pupils responded wonderfully to the collaborative development of the tasks set by an experienced and

enthusiastic Japanese teacher. The lesson was consequently discussed by specialists as Alan Bishop and Kaye Stacy from Australia and David Tall from the UK. Cultural differences in such settings not only lead to communication problems, they also provide excellent opportunities for studying the sociocultural aspects of mathematics education. With additional contributions from Vietnam, Indonesia, Malaysia, Brunei, Hong Kong, the Philippines, South-Africa, Chile and Peru, the event was truly intercultural and thus exposed cultural-dependent factors and values in mathematics teaching.

Recorded lessons from Japanese schools were discussed in separate workshops. One lesson for 2<sup>nd</sup> grade pupils was rather interesting from the point of view of switching between representations and early symbolic reasoning. Mathematical thinking depends very much on representations. Many mathematical problems, once formulated within an adequate representational form, allow for an easy solution. The representation determines the operations that are possible on structures it represents. But students often find it difficult to formulate problems in the right representation. Typical for mathematical thinking is the switching between different representation forms. This lesson is an excellent example of teaching such skills at a voung age. Pupils were given an envelope of cards of triangular and square form with a number of dots on it, or candies, as they were told.

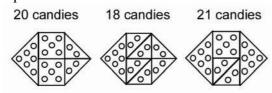


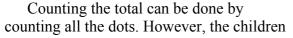


Then they were asked to assemble the cards in a predefined hexagonal shape and count the candies. Several configurations are possible and they can lead to a different total number of candies, as shown in three examples below:

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switched instead to a symbolic representation in which the total is expressed as the sum of dots on each of the cards, as 6+6+2+2+5=21in the third configuration. A symbolic representation is a more adequate representation to add several numbers together without making counting errors. Very interesting was that two different kinds of symbolic representations were used and that these are facilitated by the geometrical configuration. One student spontaneously suggest to make the sum by parentheses as in (2x6)+(2x2)+5=21. Undoubtedly this was inspired by the geometrical configuration of 2 large triangles, 2 small triangles and one square. This illustrated guite nicely how the two representations are reinforcing each other and contributed to the completion of the task. It was impressive to see how 7-year old Japanese pupils handled symbolic representations with ease.

The use of the history of mathematics for mathematics education was only marginally discussed in this conference. One example was the teaching of Pythagorean theorem as adding numbers in two dimensions. The lack of further efforts along this line is unfortunate as the multi-cultural setting allows for many opportunities. In general, there is a growing interest in the Asian countries to reassess and revalue their native mathematical heritages and put them to use for mathematics education. The Confucian-heritage countries have a rich tradition of ancient Chinese mathematical practice to draw from. In India there is a revival of Vedic arithmetical techniques as a basis for teaching the algorithms for operations on numbers. In Japan, traditional mathematics from the Edo period (1603-1868) was completely replaced by Western symbolic mathematics in a very short time span. Currently there is an increasing interest of going back to this wasan tradition for teaching elementary concepts of mathematics. This trend of drawing examples and techniques from the rich intellectual heritage of Asian cultures may be inspired by revaluating their traditions and possibly nationalistic tendencies. However, when we see mathematics as a product of socio-cultural

activities the use of culture-specific concepts and techniques makes sense.

The conference contributions can be downloaded from the Khon Kaen CRME website <u>http://www.crme.net/</u>. The lesson plan of the Japanese lesson discussed can be found at

http://www.criced.tsukuba.ac.jp/math/apec/ap ec2007/lesson\_plans/Takao\_Seiyama.pdf.

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#### XIXº Encontro do Seminário Nacional de História da Matemática,

at the Universidade de Aveiro in Portugal

The meeting took place in June, 2006, at the Departamento de Matemática of the Universidade de Aveiro, Portugal.

The oral contributions were the following: Saraiva, L.: História da Matemática - perspectivas sobre o seu desenvolvimento histórico enquanto disciplina científica em Portugal,

Domingues, F.C.: Os matemáticos de Coimbra e a História da Náutica em Portugal: a construção de uma disciplina científica,

Costa, C.: A História da Matemática no ensino antes de se constituir como disciplina científica: o caso de J. Vicente Gonçalves,

Vieira, R.: Anastácio da Cunha - algumas facetas menos conhecidas,

Silva, I.M.: Números complexos: História e Ensino,

Reich, K.: The Three Roots of Tensor Calculus, Malaquias, I.: A rede epistolar de João Jacinto de Magalhães,

D'Ambrósio, U: O Curso de História da

Matemática na Pontifícia Universidade de São Paulo, Wanner, G.: Geometry by Its History,

Malonek, H.: A abordagem geométrica no ensino da Teoria das Funções – uma perspectiva histórica 125 anos após as lições de Felix Klein,



Bebiano, N.: Matemática e Poesia, Reich, K.: A Lecture in History of Mathematics at Hamburg: *How the Arabic numerals came to Europe*.



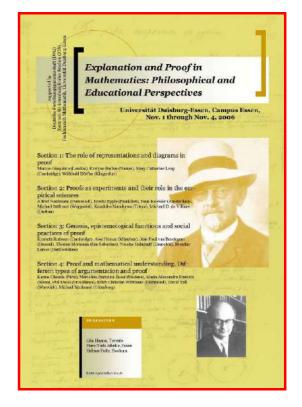
For more information (and photos), see: http://www2.mat.ua.pt/eventos/snhm19/resumos/RCom unicacoes.htm

### **Conference on Explanation & Proof**

Essen, Germany, November 1-4, 2006

Gila Hanna (Toronto), Hans Niels Jahnke (Essen) and Helmut Pulte (Bochum) organised a multi-disciplinary conference on Explanation and Proof in Mathematics involving mathematicians, philosophers and mathematics educators at the Essen campus of the University of Duisburg-Essen.

The poster of the Conference was:



The Program of the *Conference* organized in four Sections and three Round Tables. The presentations were the followings:

# Section 1: The role of representations and diagrams in proof

Marcus Giaquinto (University College London, London), A false dichotomy: algebraic vs geometric thinking in mathematics,

Mary Catherine Leng (Cambridge university), Mathematical Proof: An Algebraic Perspective,

Evelyne Barbin (Université de Nantes): *Proofs of the main proposition on geometrical proportion: from icons to symbols*,



Willibald Dörfler (Universität Klagenfurt): Verbal argumentation as talk about diagrams

# Section 2 : Proofs as experiments and their role in the empirical sciences

Alfred Nordmann (Technische Universität Darmstadt): *Proof as Experiment in Wittgenstein*,

Moritz Epple (Universität Frankfurt): Vague intuition vs. rigorous proof? Ways of argument in topology in late 19th and early 20<sup>th</sup> century,



Teun Koetsier (Vrije Universiteit Amsterdam), Motion and geometry in antiquity,

Michael Stöltzner (Universität Wuppertal): *The* principle of least action as a mathematical thought experiment,

Kazuhiko Nunokawa: *Explanations in* mathematical problem solving,

Michael D. de Villiers (University of Durban Westville): *Baking a mathematical pudding: what's the role of proof and experimentation?* 

# Section 3: Genesis, epistemological functions and social practices of Proof

Kenneth Ruthven (University of Cambridge): What needs explaining in classroom mathematics? What functions (h)as proof?,

Aiso Heinze (Universität München): On the acceptance of mathematical proofs: Observations about social processes in the mathematical community and possible implications for the mathematics classroom,

Jean Paul van Bendegem (Vrije Universiteit Brussel), What Turns an Argument into a Proof?,

Thomas Mormann (University of the Basque Country, San Sebastian): *Proof and Idealization in Mathematics*,

Nicolas Balacheff (Laboratoire Leibniz Grenoble): Bridging knowing and proving: the complexity of the epistemological genesis of mathematical proof,

Brendan Larvor (University of Hertfordshire de Havilland Campus): *What can Lakatos teach about teaching?*,



Phil Davis (Brown university): Why do I believe a theorem?

#### Section 4: Proof and mathematical understanding. Different types of argumentation and proof

Karine Chemla (CNRS Paris): Understanding, proving and the description of algorithms in the Book of mathematical procedures from China (ca 186 BCE),

Mariolina Bartolini-Bussi (Università di Modena): Contexts for Approaching at Validation: The Function of Artefacts of Ancient Technologies,



Maria Alessandra Mariotti (Università di Siena): Contexts for Approaching at Validation: The Function of Artefacts of Information Technologies, Michael Neubrand (Universität Oldenburg): Proving as Part of Mathematical Achievement: Concepts and Results from the PISA Study,

David Tall (University of Warwick): *The Cognitive Development of Different Types of Reasoning and Proof*,

Erich Christian Wittmann (University of Dortmund): *Operative Proofs* 

# Round Table 1: Proofs, diagrammatic thinking and empirical contexts Round Table 2: The cultural meaning of proof

**Round Table 3: Proof and explanation** 

The editors welcome reports from conferences.

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# Work in progress

We encourage young researchers in fields related to *HPM* to send us a brief description of their work in progress or a brief description of their dissertation.

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# Ph.D. thesis

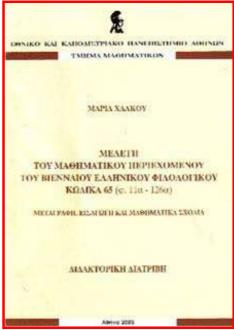
# *Transcription, Introduction and Mathematical comments (in Greek)*

Doctoral thesis of University of Athens (Dept. of Mathematics), Greece. Date of discussion: March 31th, 2004.

The Codex Vindobonensis phil. graecus 65 of the 15<sup>th</sup> century (f.11r- 126r) (Tractatus Mathematicus Vindobonensis Graecus, or TractMathVindGr), which according to my assessments was also aiming at the teaching of students at various levels of education, is made of paper and dates from the 15<sup>th</sup> century. The Code means that the pages (or the volumes) are not joined into a roll, but were simply placed between wooden frames or frames of some other material. It is considered to be a pioneering form of today's book. It appeared in the  $2^{nd}$  century and due to its ease in use, replaced the cylinder. At about the same time, the parchment (usually the hide of a cow or sheep), replaced the papyrus as writing material. The Code consists of

joined volumes and each volume of one (alternating) page number, folded in half.

The author and the origin of the TractMathVindGr are unknown. Augerius von Busbeck got the Code 65 when he was Ambassador to Emperor Ferdinand I in the court of Sultan Souleiman II (1555-1562). The pages 126v –140r contain a book of Arithmetic (Arithmetic includes logisticae and geodesia) with solved problems, which was published by H. Hunger and K. Vogel in 1963. The TractMathVindGr is the larger part of the Code 65 (f. 11r - 126r) and contains an anonymous arithmetic book with 240 chapters, of which the preface and the first two chapters were published by J. L. Heiberg in 1899. After the MS transcription I tried an interpretative approach of the methods of the Code's author, as well as comparison of these with the corresponding ones of today, which are taught in the secondary education. The methodologies of finding answers, even if in some cases they are not probably used in secondary education today, are analogous with those used today in corresponding problems.



The symbols, which are used in the Codex Vindobonensis phil. gr. 65 are the letters of the Greek alphabet but the calculations are carried out with the new decimal Hindu-Arabic system of numeration. Even though the author is not used to the new method, it should be emphasised that the use of letters and not numbers does not affect the result, since it concerns a system in which the arithmetical value of a letter depends upon its place. Thus, the author of this manuscript insisted on preservation of the old symbols, whilst other earlier scholars, such as Maximos Planudes (AD 1255-1305) in Byzantium and Fibonacci (born in 1170), who introduced the new arithmetical symbols in Western Europe, were familiar with the new symbolisation and the new arithmetical system.

It is highly likely that the contents of Codex 65 would have some particular meaning in relation to the teaching of mathematics in those days. The question therefore arises, concerning the relationship of Codex 65 with the manuscripts, namely the Arithmetic of Treviso and Pacioli's Suma. The Arithmetic of Treviso, which is anonymous, is considered to be the first commercial arithmetic of those times, and Pacioli's Suma was taught until the 16th century and was considered to be a mathematical encyclopaedia. Of course, this piece of work was not known for new discoveries in mathematics. However, it gives us information about the mathematical knowledge up to its time and is considered to have laid the foundations for the further development of algebra in the 16th century.

It is certain that many Latin scholars who knew ancient Greek read Greek manuscripts and were influenced by them. At the Mantua School in Italy (c. AD 1430), Vittorino da Feltre taught mathematics using Greek manuscripts. After his death his library came into the possession of Jacopo Cassiano who continued to teach mathematics at the same school.

Thus, in this case, in order to reach certain conclusions, a more detailed comparison between the contents of those Italian works and that of Codex 65 (f. 11r- 126r) is required in order to be able to give to the Tractatus Mathematicus Vindobonensis the title of the first Byzantine Mathematical Encyclopaedia.

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### **New Books**

# Amazing Traces of a Babylonian Origin in Greek Mathematics

Jöran Friberg World Scientific Publ., 2007

Jöran Friberg's book claims distinctive links between Babylonian and Ancient Greek mathematics. The following is from the publisher's blurb:

A sequel of Unexpected Links Between Egyptian and Babylonian Mathematics (World Scientific, 2005), this book is based on the author's intensive and ground breaking studies of the long history of Mesopotamian mathematics, from the late 4<sup>th</sup> to the 1<sup>st</sup> millennium BC. It is argued in the book that several of the most famous Greek mathematicians appear to have been familiar with aspects of Babylonian "metric algebra" – a convenient name for an elaborate combination of geometry, metrology, and quadratic equations that is known from both Babylonian and pre-Babylonian mathematical clay tables.

The book's use of 'metric algebra diagrams' in the Babylonian style, where the side lengths and areas of geometric figures are explicitly, indicated, instead of wholly abstract 'lettered diagrams' in the Greek style, is essential for an improved understanding of many interesting propositions and constructions in Greek mathematical works. The author's comparisons with Babylonian mathematics also led to new answers to some important open questions in the history of Greek mathematics.

**Contents:** *Elements* II and Babylonian Metric Algebra; *El* I.47 and the Old Babylonian Diagonal Rule; Lemma *El* X.28/29 Ia, Plimpton 322, and Babylonian igi-igi-bi Problems; Lemma *El* X.32/33 and an Old Babylonian Geometric Progression; *Elements* X and Babylonian Metric Algebras; *Elements* IV and Old Babylonian Figures Within Figures; *El* VI.30, XII.1-12, and Regular Polygons in Babylonian Mathematics; *El* XII.13-18 and Regular Polyhedrons in Babylonian Mathematics; *Elements* XII and Pyramids and Cones in Babylonian Mathematics; *El* I.43-44, *El* Vi.24-29, *Data* 57-59, 84-86, and Metric Algebra; Euclid's Lost Book *On Divisions* and Babylonian Striped Figures; Hippocrates' Lunes and Babylonian Figures with Curved Boundaries; Traces of Babylonian Metric Algebra in the *Arithmetica* of Diophantus; Heron's, Ptolemy's and Brahmagupta's Area and Diagonal Rules; Theon of Smyrna's Side and Diagonal Numbers and Ascending Infinite Chains of Birectangles; Greek and Babylonian Square Side Approximations; Theodorus of Cyrene's Irrationality Proof and Descending Infinite Chains of Birectangles; The Pseudo-Heronic *Geometrica*; A Chain of Trapezoids with Fixed Diagonals; A Catalog of Babylonian Geometric Figures.

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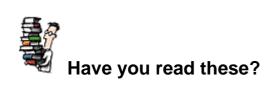
### Jacopo da Firenze's Tractatus algorismi and early Italian abbacus culture

Jens Høyrup

published in September 2007 by Birkhäuser.

This book deals with a Tractatus algorismi written in 1307 in Montpellier by a certain Jacopo da Firenze. This work is one of the earliest surviving "abbacus" treatises, which are textbooks for the vernacular school for merchant youth which sprung up in Northern Italy in the later thirteenth century. This one is by far more orderly than any of the extant predecessors and is also the first to contain a presentation of algebra.

The book contains an edition and an English translation of a manuscript from c. 1450 which is a meticulous copy of a meticulous copy of the original, together with a critical edition of an abridged version of which two manuscripts exist. In addition, the book features an extensive discussion of the contents of the treatise and its location within early abbacus culture, including evidence that this culture was transnational and probably older in the Catalan-Provençal area than in Italy.



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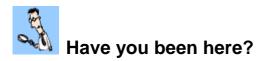
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New links in this issue

**Opera Mathematica of Christoph Clavius** <u>http://mathematics.library.nd.edu/clavius/</u>

Geometrical books and instruments from 15<sup>th</sup> to 18<sup>th</sup> century http://www.geometricum.com/

David Henderson's Home Page [Educational and Historical Topics on Geometry] http://www.math.cornell.edu/~dwh/

Archimedes Project [Some famous mathematical books of the Renaissance period are available on line, i.e. Pacioli's *Summa*] http://archimedes2.mpiwgberlin.mpg.de/archimedes\_templates

# Simon Stevin's *De Meetdaet* [The Practice of Measuring]

http://www.math.leidenuniv.nl/~wiskonst/meetdaet/ind ex.html

and The Principal Works of Simon Stevin http://www.historyofscience.nl/works\_detail.cfm?Recor dld=2702

#### **Mathematical instruments**

http://brunelleschi.imss.fi.it/museum/esim.asp?c=5001 64

#### and

http://web.mat.bham.ac.uk/C.J.Sangwin/Sliderules/slid erules.html

#### and

http://www.mhs.ox.ac.uk/epact/catalogue.php?ENumb er=52265



# Societies and organisations

African Mathematical Union: Commission on the History of Mathematics in Africa (including newsletter) http://www.math.buffalo.edu/mad/AMU/amuchma\_onl ine.html

Association des Professeurs de Mathematiques de l'Enseignement Public [APMEP] History site: http://www.apmep.asso.fr/BMhist.html

British Society for the History of Mathematics [BSHM] http://www.bshm.org

HOMSIGMAA - History of Mathematics Special Interest Group of the MAA http://www.maa.org/sigmaa/hom

HPM Americas http://www.hpm-americas.org/

Italian Society of History of Mathematics http://www.dm.unito.it/sism/indexeng.html Association pour la Recherche en Didactique des Mathématiques: <u>http://www.ardm.asso.fr/</u>

Commission Française pour l'Enseignement des Mathématiques: <u>http://www.cfem.asso.fr/</u>

Instituts de Recherche sur l'Enseignement des Mathématiques (IREM): <u>http://www.univ-irem.fr/</u>

Canadian Society for History and Philosophy of Mathematics http://www.cshpm.org

Brazilian Society for History of Mathematics http://www.sbhmat.com.br

*Nuncius* Newsletter http://brunelleschi.imss.fi.it/nuncius/inln.asp?c=5302

International History, Philosophy and Science Teaching Group www.ihpst.org

#### **Topics and Resources**

#### MATHS for EUROPE: The history of some aspects of mathematics like: history of mathematical persons, symbols, algorithms...

http://mathsforeurope.digibel.be/index.html http://mathsforeurope.digibel.be/list.htm http://mathsforeurope.digibel.be/olvp.htm http://mathsforeurope.digibel.be/olvp2.htm http://mathsforeurope.digibel.be/olvp3.htm

#### Ethnomathematics on the Web

http://www.rpi.edu/%7Eeglash/isgem.dir/links.htm

#### **About Medieval Arabic Numbers**

http://www.geocities.com/rmlyra/Numbers.html http://www.geocities.com/rmlyra/arabic.html

# Annotated Bibliography on Proof in Mathematics Education

http://fcis.oise.utoronto.ca/~ghanna/educationabstracts. html

#### BibM@th

http://www.bibmath.net/dico/index.php3?action=rub& quoi=0

Centro Virtual de Divulgación de las Matemáticas, esta siendo desarrollada por la Comisión de Divulgación de la *Real Sociedad Matemática Española (R.S.M.E.)* http://www.divulgamat.net/index.asp

History of Statistics http://www.stat.ucla.edu/history/ Images of Lobachevsky's context http://www.ksu.ru/eng/museum/page0.htm

Images of Mathematicians on Postage Stamps <a href="http://members.tripod.com/jeff560/index.html">http://members.tripod.com/jeff560/index.html</a>

Photos of Mathematicians http://www.math.unihamburg.de/home/grothkopf/fotos/math-ges/

Numdam-Digitization of ancient mathematics documents http://www.numdam.org/en/ressnum.php

The Montana Mathematics Enthusiast (journal) http://www.montanamath.org/TMME/

Convergence: an online magazine of the MAA providing resources to teach mathematics through its history http://convergence.mathdl.org/

International Journal for Mathematics Teaching and Learning, http://www.cimt.plymouth.ac.uk/journal/default.htm

Homepage of International Journal for the History of Mathematics Education http://www.tc.edu/centers/ijhmt/index.asp?Id=Journal+ Home

# Documents for the History of the teaching of mathematics in Italy

http://www.dm.unito.it/mathesis/documents.html

# Ethnomathematics Digital Library <a href="http://www.ethnomath.org/">http://www.ethnomath.org/</a>

**Some Japanese Mathematical Landscapes**: The results of wandering in a beautiful country, with a mathematical eye, aided by a digital camera, by A. Arcavi

http://math.criced.tsukuba.ac.jp/museum/arcavi/arcavi\_english/index.html

Wann-Sheng Horng's webpage with HPM related materials in Chinese. http://math.ntnu.edu.tw/~horng/

#### Fred Rickey's History of Mathematics Page

http://www.dean.usma.edu/math/people/rickey/hm/defa ult.htm

**Culture***MATH*. Ressources pour les enseignants de Mathématiques www.dma.ens.fr/culturemath/actu/livres.htm

**The French INRP** (National Institute for Pedagogical Research) is developing a website on questions related to mathematics teaching: EducMath <u>http://educmath.inrp.fr</u>

#### Homepage of Albrecht Heeffer http://logica.ugent.be/albrecht/

Homepage of Jens Høyrup http://www.akira.ruc.dk/~jensh/

*L'Enseignement Mathématique*, Archive http://retro.seals.ch/digbib/vollist?UID=ensmat-001

Homepage of Prof. Leo Corry <a href="http://www.tau.ac.il/~corry/">http://www.tau.ac.il/~corry/</a>

We would like to provide a more comprehensive list of websites containing resources useful to researchers and students (not necessarily in English). If there are any you use, or you know are useful for students or researchers, please send your recommendations to the editors.

\* \* \*

# Notices

#### The Primitives: in reflection

One may wonder how the Flemish Primitives were able to paint with such precision. Contemporary artist David Hockney claims that from 1430 onwards painters used optical techniques: with the aid of a concave mirror a painter would have projected the scene onto a canvas.

In the interactive exhibition 'The Primitives: in reflection', a project of the Teacher Training College of the University College Antwerp one may test for oneself whether this is the case. Hockney's set up is recreated and the visitor can try for himself to project an image. Using mirrors and lenses the viewer gains an understanding of optics.

The experiments are accompanied by an exhibition of antiquarian books dealing with perspective, mirrors and lenses.

This interactive exhibition can be seen in the Nottebohm Room of the Antwerp City Library (Stadsbibliotheek Hendrik Conscienceplein) from November 9 to December 2 from 1 p.m. to 5 p.m. Entry is free.

#### Ad Meskens a.meskens@ha.be

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### *The Reorientation of the Russian Mathematical Mentality in the Age of Peter the Great*

Most histories of mathematics written in the West make no mention of mathematical activity in Russia until what Carl Boyer called the "Age of Euler". Indeed historians of Russian science such as V. Boss (Boss 1972, 5) have gone so far as to declare – falsely – that "mathematical learning, as opposed to rudimentary practice, was virtually nonexistent in Muscovy when the *Principia* [of Newton] was first published in 1687." Then we read that such figures as Nicolaus and Daniel Bernoulli, Euler, and Goldbach are actively working in the mathematical sciences at the Russian Imperial Academy of Sciences in St. Petersburg. How shall we explain this?

Until the end of the seventeenth century, Russia remained fairly isolated from the intellectual currents that shaped western Europe during the Renaissance, Reformation, and the seventeenth century. This is not to assert that Russia was immune from the influence of her neighbors on her western periphery who, like Poland, shared in the humanistic, religious, and scientific developments, or from farther-flung trading partners such as England. Thus, although Russian isolation from western Europe and its activities was far from complete, many of the English visitors to Russia in the sixteenth century still viewed her as a "rude and barbarous kingdom" (Berry & Crummey 1968). Besides merchants seeking to trade for Russian timber, furs, and other commodities, who visited Russia, Muscovite tsars had begun inviting western European specialists, especially those with military and engineering expertise, to Russia, starting from the sixteenth century. There had been alongside of these so-called "Germans", most of whom were actually British, a greater influx of Greek, Balkan Slavs, and western Russian intellectuals, especially from the Ukraine and Belarus'. They took part in helping to improve and translate Greek and Latin texts, especially religious tracts, with the aim of correcting the Old Church Slavonic texts. Pre-Petrine Russia certainly knew of the existence of works of foreign origin which contained

HPM Newsletter No. 66 November 2007, page 14 HPM webpage: <u>http://www.clab.edc.uoc.gr/hpm/</u> the most advanced theoretical mathematics of their day, and Russian historians of mathematics have studied a large number of medieval Russian mathematical manuscripts, not all of which are translations. As a result, we can comfortably conclude that the imported texts of the 15<sup>th</sup> century spurred a serious study of mathematics by 16<sup>th</sup> – and 17<sup>th</sup>-century Muscovites, but only among a small number, among whom were Prince Ivan (or Il'ya) Elizar'ev, reputed author of the *Sinodal'naya geometrya* (ca. 1625).

Among the Greeks who helped establish and taught school in Moscow were the Lichud brothers, Ioanniky [Ioann] (1663-1713) and Sophronius [Sofrony] (1652–1730), both of whom emigrated from Byzantinium to take their educations at Venice and Padua. From 1690 to 1691, they taught at the Hellenico-Slavonic Academy in Moscow, bringing with them the best that the Italian Renaissance humanists had to offer in the way of such subjects as grammar, rhetoric, logic, philosophy, and Aristotelian physics. The Hellenico-Slavonic Academy included Leontii Fillipovich Magnitskii (1669–1739), who graduated in 1701, and Michael Vasil'evich Lomonosov (1711–1756), both of whom went on to the newly founded Imperial Academy of Sciences in Saint Petersburg when it was founded in 1723. Magnitskii became the first native Russian instructor of mathematics at the Academy. Lomonosov, after a short time as a student at the Academy, was sent to the University of Marburg, where he studied mathematics and science with the Leibnizian philosopher Christian Wolff, and even while still a student at the Academy, was one of Euler's assistants. Beginning in 1745, he was refereeing mathematics textbooks for the publishing house of the new Academy. In January 1746, he even took on the task of refereeing Euler's article "Theorematum quorundam arithmeticorum demonstrationes" which appeared in volume ten of the Academy's Commentarii academiae scientiarum imperialis Petropolitanae (1747). Lomonosov became a polymath, making his name in chemistry, grammar, history, belleslettres, geography, meteorology, among other pursuits, and was the guiding force behind the

planning and founding of Moscow University, which grew out of the Hellenico-Slavonic Academy. Among his most noteworthy contributions in the application of mathematics to physical science was his *Elementa Chimiæ Mathematicæ* of 1741.

Prior to the start of the eighteenth century, there were a small number of mathematical works, circulating in manuscripts, primarily arithmetic and geometry tracts of a comparatively elementary character. The Synodal Geometry of Elizar'ev was a a massive, comprehensive and systematic treatise of theoretical geometry, and is thus difficult to summarize. Virtually all of the geometrical knowledge of its day is to be found there, including not only the classical knowledge of Pythagoras, Euclid, Archimedes, and Pappus of Alexandria, but also the very latest geometrical work in the textbooks of Petrus Ramus and of John Speidell. For the most part, however, arithmetic and geometry were the province of a handful of clerics and civil officials, geared respectively towards computing the dates of movable feasts, especially Easter, and for maintaining state budgets, computing taxes, cadastral rolls, and field measurements. The arithmetic textbook Kratkoe i poleznoe rukovedetsnie vo aritmetiku...[Concise and *Useful Guide to Arithmetic*] by Il'ya (Ivan) Fedorovich Kopievskii (also called Kopievich), which appeared in 1699 was among the first printed works to appear in Russian. Although it was printed in Amsterdam by the Tessing brothers under the direct auspicies of Peter the Great (r. 1682-1725), it still belonged in character to the western abacus books. The largest number of those Russians who, through the seventeenth century expressed their opinion about "western" learning and about mathematics and the sciences were hostile, in particular the leading clerics, who regarded these as akin to black magic, if not actually devilish. The first modern algebra text to appear in Russian was N. E. Murav'ev's Elementary Basics of Mathematics [Nachal'noe osnovanie *matematiki*]. It appeared in the same year that the Imperial Academy of Sciences, founded by Peter the Great under the inspiration of

Leibniz, officially opened its doors under the command of Peter's widow and successor, Catherine I (r. 1725–1727). Later editions of Murave'ev's text were prepared by the astronomer N. I. Popov (1720–1782).

It was the top of the political and social establishment that, even in the fifteenth century, saw the usefulness of the "western" sciences, and some members of the aristocracy as well as the tsar and members of his family began to engage in intellectual pursuits themselves. When Tsar Ivan IV established a printing press in Moscow ca. 1563, headed by Ivan Fedorov (ca. 1510– ca.1583), it was the civil clerks, fearing for their jobs, and their clerical allies, who drove away the printing establishment. For the political and military leadership, the advantages to western technological and industrial expertise was deemed crucial, and an increasing number of foreign specialists were invited to Russia especially by Tsarina Sofiya (r. 1682–1689), the half-sister of Peter the Great, and especially by Peter himself, who, during Sofiya's reign was exiled from the main court, spent his days with people from the foreign colony, mostly Dutch, British, and Germans, who taught him the rudiments of mathematics, navigation, and other useful sciences, and in the earliest years of his reign, he relied heavily for assistance of such men as the Scot Patrick Gordon (1635-1699), a general who made evident to Peter the value of engineers in the military.

Early on in his reign, Peter made a tour to western Europe and England, where he made contact, among others, with Leibniz and Edmund Halley. In Holland, he became acquainted with the work of the mathematicians Hans Gouda, Dirk Raven, and Hans Isbrandtsen Hoogzaat, which sparked his enthusiasm for the value of mathematics. A major result of this tour was the hiring of large numbers of foreign specialists of various expertise, including mathematicians. Among those hired was Henry (or Harry) Farquharson, called in Russia Andrei Danilovich Farkhvarson or Farvarson (1675-1739), who had taught mathematics and astronomy at the University of Aberdeen and was recommended by Halley and Jacob

Daniel Bruce (1670–1735), while John Colson was hired to teach Bruce mathematics. Farquaharson's task in Russia was to create and administer a Mathematics and Navigation School. It was under Farquharson's guidance that he and Tsar Peter wrote the mathematics curriculum for the new school. He was accompanied by Stephen Gwyn (1684–1720) and Richard Grice (1682?–1709), who were graduates of the England's Royal Mathematical School. The Mathematics and Navigation School's first native Russian professor, upon his graduation in 1701 from the Hellenico-Slavonic Academy, was Leontii Magnitskii. They arrived in Moscow in 1699.



#### Peter the Great (1672-1725)

Over the next decade, Peter became increasingly convinced by Leibniz that Russia needed to develop not only its own academic intellectual institutions, but its own academics and scholars, rather than relying almost exclusively upon others. Moreover, Peter began to realize that merely borrowing the technology and industry of the western Europeans would not of itself suffice to sustain comparable developments in Russia, even with the presence of foreign experts. Thus, a broad plan was conceived to create for Russia its own academic and scholarly institutions, which could one day compete on its own without depending upon foreign instructors and even foreign pupils for its viable continuance. Thus the Imperial Academy of Science was born, conceived by Peter and Leibniz, but officially opened by Catherine I, just after Peter's death. Between 1696 and 1716, Leibniz presented many memoranda and proposals on education in

Russia to Peter the Great, on the basis of which Peter founded the Imperial Academy of Sciences in a decree of 28 January 1724.

Thus, in 1725, the Moscow College of Mathematics and Navigation was removed to the newly-founded city of Saint Petersburg, and soon Nicholas (II) and Daniel Bernoulli and Leonhard Euler would join its staff. Lomonosov was one of its first students. Magnitskii became its first native faculty member, moving to the Academy along with the Mathematics and Navigation College.

Magnitskii's studies had included arithmetic texts and abacus books such as Jacob van der Scheuere, Arithmetica (1600); Johann Faulhaber, Arithmetischer Wegweyser...zur Rechenkunst (1614); Andreas Tacquet, Arithmeticæ Theoria et Praxis (1656); J. S. Farmancowicz (or Farmancovic), Arithmetica practica bipartita generalis numerorum et specialis (1669), and algebras such as Michael Stifel's Arithmetica integera (1544).



#### Magnitskii's Atithmetik

Magnitskii's *Arifmetika*, written in Church Slavonic, was a two-volume unprinted encyclopedic reader, published in Moscow which contained materials translated from Western European sources, on practical arithmetic, algebra, geometry, trigonometry, astronomy, geodesics, and navigation. It is a handbook of applied mathematics, rather than a systematic study. The work was divided into two books. The first book, called "Practical or active arithmetic" ("*Arifmetike praktika ili deyatel'noi*"), had a particular emphasis on military problems. The second book treated the other subjects. There were also supplementary sections dealing with arithmetic and geometric progressions, with decimal fractions, and with methods for extracting quadratic and cubic roots, including in particular a treatment of Viète's method. All together, forty pages were devoted to algebra. The book was both an encyclopedic work and a pedagogical text, combining traditional material from 17thcentury Russian manuscripts with the latest western European mathematical knowledge, all presented systematically, although no proofs were presented. Magnitskii himself made original contributions in the development of logarithmic tables and tables of trigonometric functions, although he never achieved a lasting reputation as a mathematician. The printed version of his book Arifmetika (1705), however, became the principal textbook for the next half-century.

In 1705, Magnitskii's student Vasilii Onofrievich (or Anofrievich) Kipriyanov (d. 1728) prepared Novyi sposob Arifmetiki Feoriki ili zritel'nyi, Sochinen voprosam radi udobneishego or New Method of Arithmetic..., sometimes referred to under the title *Matematika*, a workbook and problem book for Magnitskii's text which was edited by Magnitskii (1705). In 1703, Magnitskii, Farquharson, and Gywn published their frequently reprinted Tables of Logarithms, and of Sines, Tangents, Secants..., the Tablitsy Logarifmov, i sinusov, tangensov, sekansov..., much of it based on Magnitskii's work. It went through five editions, the fifth appearing in 1791



The book Geometry published in Moscow, March 17, 1708

The lectures for a one-year course on arithmetic and geometry, taught at the Kiev-Mogila Academy by Feofan Prokopovich (Ukrainian Prokopovych, Polish Theophanes Prokopowicz; 1681–1736) form another significant work of this period. The lectures were given in Latin. The lecture course was the "Duo primi et uberrimi rerum matematicarum fontes: Arithmetica et Geometrica... " (1707-08), and only fragments of the transcript survive in manuscript. These lectures are nevertheless considered by some to be far superior mathematically to Magnitskii's Arifmetika insofar as they represent the first mathematics course in Russia to be presented in a "scientific" way, that is, presumably, in a logical presentation. The contemporary Danish scholar Peder van Haven, after meeting Prokopovich, wrote (van Haven 1744, 19) that he was also most impressed by Prokopovich's "deep knowledge of mathematics and extraordinary fondness for that art." Moreover, Prokopovich's mathematical lectures were on a par with typical lectures in arithmetic and geometry of the Renaissance era, but more detailed than most.

The first Russian translation of Euclid's Elements, the Evklidovy elementy, however, was begun only in 1719, edited by Farguharson and translated by Ivan Satarov, and published in 1739; it included as well, though without its being indicated, a Russian translation of relevant parts of the Latin text by Andreas Tacquet of Elementa Geometriæ plane et solidea. Quibus accedunt selecta ex Archimede theoremata (1654), as Nachal ploskoi i prostranstvennoi geometrii, s prilozheniem izbrannykh teorem Arkhimeda. Shortly thereafter, Farguharson and Satarov collaborated on a translation of Tacquet's The Archimedean Theorems of André Taquet... (1654) and George Peter Domcke's *Abridgement...* (*Arkhimedovy teoremy* Andreem Takkvet i Georgiem Petrom Domkino sokrashchennye), which appeared in 1739, just after Farguharson's death. This latter work is essentially a new edition of the Evklidovy elementy. The history of these two works is convoluted and controversial at best.

The Evklidovy elementy advertises itself as a Russian translation of a condensation by Newton of Euclid's *Elements*, but both works are in fact more than mere translations of Tacquet's condensation of Euclid; rather they are translations of Tacquet's condensation along with George Peter Domcke's discussion of the mathematics of Newton's Principia along with Farguharson's additional discussion. A "cleansed" translation of Euclid's *Elements*, free that is of Farquharson's "Newtonian" insertions, was prepared by Nikolai Gavrilovich Kurganov (1722–1796), a former student of Bruce's from the School of Navigation who became a professor at the Academy of Sciences in 1774, who also compiled a number of other mathematical textbooks. Kurganov's Elements of the Geometry of Euclid, the Elementy geometrii po Evklidov, appeared in 1769.

The anonymous *Geometry* — *Practice* with Figures (Geometriya — Praktika s *figurami*); was the first modern geometry textbook originating in Russia, and it included trigonometric and logarithmic tables. It has been conjectured that the Geometry — *Practice with Figures* was written either by Farguharson between 1730 and 1739, or in Saint-Petersburg in 1714 by Bruce. It was not, however, the first modern geometry textbook to appear in the Russian language: that distinction belongs to a 1708 translation of Anthony Ernst Burckhardt von Pürkenstein's Ertzherzogliche Handgriffe desz Zirckels und *Lineals* 1686), translated into Russian as the Slavic Geometry of the Art of Earth Measurement (Geometriya slavenski zemlemerie...).

In 1731, Vasilii Evdokimovich Adodurov (1709–1778 or 1780), a graduate of the University of St. Petersburg, attached to the Imperial Academy of Sciences, was appointed an adjunct member of the Imperial Academy, the first native Russian to receive such an appointment. From 1733 to 1741, he was deputy head of the academy's Department of Higher Mathematics. He is perhaps best known for his translation into Russian of the first part of Euler's German *Introduction to Arithmetic for Gymnasium Use*, the

# *Rukovodstvo k arifmetike dlya upotrebleniya gimnazii....* (1740).

It is fair to suggest that there already existed in pre-Petrine and early Petrine Russia a budding mathematical tradition upon which to base Russia's "age of Euler". From this point forward, that is, beginning with the first years of the Russian "age of Euler", the history of mathematics in Russia is better known in the West, and Russian achievements in mathematics, under the leadership of Euler and his colleagues and students, took an international place.

If one were to summarize the situation in Russia in the pre-Petrine era, one would be forced to admit that mathematical knowledge was limited, both in scope — depth and breadth — and in the number of those versed in the subject, and haphazard in what was studied and known. Under the urging of Peter and his advisors, who recognized that the technological and scientific advances that were required in order for Russia to maintain an equilibrium with the advances of in technology and practical arts of western Europe, it was necessary for Russia to sustain a systematic scientific and educational equilibrium with western Europe as well, including in the areas of mathematics, one of the goals of Peter's educational reforms was to train Russians to attain the level of their imported teachers. With few exceptions, this did not happen overnight, or within one or two generations, and, at the elementary level, the "cipher schools" that Peter decreed were a monumental failure. But within several generations, the higher echelons of the educational system eventually bore their fruit, in the likes, for example, of the geometer, astronomer, and pedagogue Nikolai Lobachevskii, whose work takes us, however, chronologically far beyond our discussion of the mathematics of the age of Peter the Great. The years from 1698 to 1725 provide the preparatory grounds for Russia's solid entry into the international arena of mathematics.

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### First Announcement HPM 2008

History and Pedagogy of Mathematics The HPM Satellite Meeting of ICME 11 14-18 July 2008, Mexico City

#### 1. Aim and focus

The HPM 2008 is the seventh quadrennial meeting of the International Study Group on the Relations between the History and Pedagogy of Mathematics (the HPM Group), affiliated to ICMI. It is a satellite meeting of the corresponding ICME (International Congress on Mathematical Education) and is scheduled close to ICME. These quadrennial meetings are a major activity of HPM, to bring together those who are interested in the relation between the history of mathematics and mathematics education such as:

• Researchers in mathematics education, and its relation to the history of mathematics;

• Mathematics teachers at all levels who are eager to get insights on how the history of mathematics may be integrated into teaching and help students to learn mathematics;

• Historians of mathematics, who wish to talk about their research;

• Mathematicians, who want to learn about new possibilities to teach their discipline;

• All those with an interest in the history of mathematics and pedagogy.

#### 2. Main themes

The HPM 2008 is a place where mathematicians, educators, historians, researchers and students, can make presentations and participate in discussions. The programme and activities are structured around the following main themes:

1. Integrating the History of Mathematics in Mathematics Education.

2. Topics in the History of Mathematics Education.

3. Mathematics and its relation to science, technology and the arts: historical issues and educational implications.

4. Cultures and Mathematics.

5. Historical, philosophical andepistemological issues in Mathematics Education.6. Mathematics from the Americas.

#### 3. Activities during HPM2008

During HPM 2008 there will be

• one-hour plenary lectures on each of the five main themes

- two one-hour panel discussions.
- parallel sessions of 25-minute oral

presentations, followed by 5-minute discussions. • poster exhibitions with discussion sessions

exhibitions of books and other didactical material

A limited number of 1-hour workshops may be included in the program upon special request to be further considered by the Organizers. In such cases, participants are actively participating in studying a specific subject and having a follow-up discussion. The role of the workshop organizer is to prepare, present and distribute the historical, or didactical material, which motivates and orients the exchange of ideas and the discussion among the participants. Participants read and work on the basis of this material (e.g. original historical texts, didactical material, students' worksheets etc).

#### 4. Time and place

HPM 2008 will be held from Monday 14 July to Friday 18 July 2008 in Mexico City

Sessions will be held on Monday, Tuesday, Thursday and Friday with a mathematical tour on Wednesday.

#### 5. Official Languages

The official languages are English and Spanish. More specifically:

• All plenary talks and panel discussions will be in English with simultaneous translation if possible.

• Oral presentations will be in either English or Spanish with one set or transparencies in Spanish and one set in English utilizing either two projectors and screens or two power point computers.

### 6. Submission of proposals

ABSTRACTS

30 November 2007: deadline for submitting Abstracts of proposals for all types of activities.

31 December 2007: Notification of acceptance or not of the submitted proposals.

*Important*: Please, use the Application Form and send it in electronic form to Constantinos TZANAKIS, Chair of HPM 2008, email: <u>tzanakis@edc.uoc.gr</u>. Postal address: Department of Education, University of Crete, Rethymnon 74100, Crete, Greece.

The members of the Scientific Program Committee (SPC) will review the submitted abstracts. At this stage, acceptance of a proposal means that the proposed activity will be included in the HPM 2008 Scientific Programme. However, this does not imply that a full text based on this activity will automatically be included in the HPM 2008 Proceedings, which are going to be available on the spot.

Full texts will be further reviewed by members of the SPC at the usual international standards. For more details, see Proceedings. FULL TEXTS

15 February 2008: deadline for submitting full texts of proposals for all types of activities.

31 March 2008: Notification of acceptance or not of the submitted proposals.

# 7. The (international) Scientific Program Committee (SPC)

#### Main Organizers

• Ricardo Cantoral, Departamento de Matemática Educativa, Centro de Investigación y de Estudios Avanzados del IPN, MEXICO.

• Florence Fasanelli, American Association for the Advancement of Science, USA.

• Alejandro Garciadiego, Departamento de Matemáticas, Facultad de Ciencias, UNAM. MEXICO

• Robert Stein, California State University, San Bernandino, USA.

• Constantinos Tzanakis, Chair of the HPM Study Group, Department of Education University of Crete, GREECE.

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• Evelyne Barbin, IREM-Centre François Viète, Université de Nantes, France

• Carlos Correia de Sà, Departamento de Matemática Pura da Faculdade de Ciências da Universidade do Porto, Portugal

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• Chris Weeks, Downeycroft, Virginstow Beaworthy, UK

• Greicy Winicki Landman, Department of Mathematics and Statistics, California State Polytechnic University, USA

#### The Local Organizing Committee (LOC)

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• Alejandro Garciadiego, co-chair, Departamento de Matemáticas, Facultad de Ciencias. Universidad Nacional Autónoma de México

• Rosa Marva Farfán, Dirección de Educación, Ciencia y Tecnología del Instituto de Ciencia y Tecnología del DF

• Gisela Montiel, Centro de Investigación en Ciencia Aplicada y Tecnología Avanzada del IPN.

• Gabriela Buendía, Cimate de la Universidad Autónoma de Chiapas

• Rodrigo Cambray Nuñez, Universidad Pedagógica Nacional

• Edmundo Palacios, Universidad Iberoamericana,

#### 8. The web site

Making known the HPM 2008 in various countries is a major task to be realized by the SPC. To this end, a web site is available at <u>http://www.red-cimates.org.mx/HPM2008.htm</u>. This is going to be a very efficient tool to make known the HPM 2008 worldwide, to allow for online registration etc.

#### 9. Proceedings

Publishing the Proceedings of HPM 2008 is also a major task, and will be available on the spot.

Each submitted full text for an oral presentation, or workshop will be reviewed by members of the SPC at the usual international standards.

More details on the size of the texts, the format guidelines will be announced in due course from the HPM 2008 and HPM websites <u>http://www.red-cimates.org.mx/HPM2008.htm</u> and <u>http://www.clab.edc.uoc.gr/hpm/</u> respectively.

10. Contact

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\* \* \*

# Announcements of events

### The First Century of the International Commission on Mathematical Instruction (1908–2008): Reflecting and Shaping the World of Mathematics Education

#### March 5-8, 2008

Accademia dei Lincei and Istituto dell'Enciclopedia Italiana, Rome, Italy



This symposium in Rome will celebrate the centennial of the International Commission on Mathematical Instruction (ICMI). Starting from a historical analysis of principal themes regarding the activities of the ICMI (reforms in the teaching of the sciences, teacher training, relations with mathematicians and with research, and so on), discussions will focus on identifying future directions of research in mathematics education and possible actions to be taken to improve the level of scientific culture in various countries.

The program includes plenary sessions, invited short talks, and working groups. More information can be found at the symposium website:

http://www.unige.ch/math/EnsMath/Rome200 8

#### 5th International Colloquium on the Didactics of Mathematics April 17-19, 2008

Department of Education, University of Crete, Rethymnon, Crete, Greece http://www.edc.uoc.gr/5colloquium

Call for papers

Deadline for *Abstract submission*: 10 September, 2007. Abstracts should not exceed 500 words (approximately, one A4 page). Deadline for *Full Text submission*: 10 November, 2007. Full texts will be reviewed by the members of the International Scientific Committee.

*Notification of acceptance*: by 10 January, 2008.

# ICME-11

July 6-13, 2008 Monterrey, Mexico http://www.icme11.org.mx/icme11/

Topic Study Group (TSG) 23: The role of history of mathematics in mathematics education: http://tsg.icme11.org/tsg/show/24

# HPM 2008

History and Pedagogy of Mathematics The HPM Satellite Meeting of ICME 11 July 14-18, 2008 Mexico City, Mexico See First Announcement p. 19

# Models in Developing Mathematics

**Education** (10th International Conference of The Mathematics Education into the 21st Century Project) **September 12-18, 2008** Dresden, Germany For further information contact arogerson@inetia.pl

# Models in Developing Mathematics Education September 11–17, 2009

Dresden, Germany

\* \* \*

# A note from the Editors

The Newsletter of HPM is primarily a tool for passing on information about forthcoming events, recent activities and publications, and current work and research in the broad field of history and pedagogy of mathematics. The Newsletter also publishes brief articles which they think may be of interest. Contributions from readers are welcome on the understanding that they may be shortened and edited to suit the compass of this publication.

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Items for the Newsletter should be sent to the editors, preferably by email (see addresses below).

The Newsletter appears three times a year with the following deadlines for next year.

No.	Deadline for material	Sent to distributors	
67	12 February 2008	1 March 2008	
68	7 June 2008	1 July 2008	
69	12 October 2008	1 November 2008	

The Newsletter is the communication of the International Study Group on the Relations between the History and Pedagogy of Mathematics, an affiliate of the International Commission on Mathematical Instruction.

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