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In Memoriam
Karen Dee Michalowicz
(see page 21)

Ivor Grattan-Guinness talks to Chris Weeks

After you had studied mathematics at Oxford, you decided to begin studies in the history of mathematics. Could you say a few words about what motivated you to do this?

The reason why I became interested in the history of mathematics in the first place was as a direct effect of my own experience as an undergraduate in mathematics. As was usual in a mathematics course – this was round about 1960 – there was no heuristics given or very little, and I don't think any historical remarks were made whatsoever: and I vaguely felt that something was missing from the educational experience here. This is one of the reasons why I have always been interested in the connections between the history of mathematics and education.

When I eventually came to decide to do something about this, I realised the question was more than just a mathematics question and learning more mathematics wouldn't help me at all. I needed to do something about philosophy and history of mathematics. I didn't really know what I was looking for, but by great good fortune I was recommended to consider the London School of Economics (LSE) where a new masters course was opening up (this was about June 1964). So I went along to the History of Philosophy and Science Department and enrolled there for a masters degree in Karl Popper's department.

What aspect of Karl Popper's thinking do you think had the most influence on you?

Listening to Popper lecturing was just exactly what I wanted to do – hearing him say that science was guesswork, and I realised that mathematics was guesswork as well. He also stressed that the problems in philosophy come from outside philosophy; and that theories must be distinguished from theories about theories (here he was very influenced by Alfred Tarski). He didn't speak very much about maths, but I could reorient my questions with this Popperian background – in fact I saw them partly as an *application* of Popper to history – history was a kind of meta-theory. Then all sorts of connections came to light.

The other influence of the masters course was that it as well as a compulsory part called the Popperian brainwash, there were various options for the other part, and one of these was mathematical logic and the foundations and the philosophy of mathematics. So I took it because I could see it used a lot of set theory which I had already studied at Oxford; so that was how I came to learn mathematical logic in the first place. While I was studying logic and set theory in this part of the course, I was applying the other part to it, by looking at logic historically and philosophically in various sorts of ways.

So this is the background to my work – it has a very strong Popperian background. In the 1990s I did a general history of mathematics at the invitation of Fontana Press,[1] and while I was writing the book Popper died (in 1994): so I dedicated the book to his memory, partly because he had died while I was writing the book, but mainly because Popperian philosophy is applied to it all over the place. I do not mention this, and I doubt that people notice it.

There were also some very important negative lessons I learned from Popper. One of them was what he calls 'essentialism' – don't get so bogged down and fussy about the meanings of words, because you're just going to get into vicious circles and infinite regress and so on. This is a particularly interesting matter in mathematics where definitions do play a role and have a place in the more formalised sides of the subject (analysis and set theory and logic, for example), yet the thrust of the point still applies; so I rarely use the word 'concept' in my writings, because of its essentialist connotations.

How did you decide on a topic of study for your doctorate research?

Any topic would have done, but I decided to pick one which I liked and in which education played an important role, so I could study the history of maths, maths education and history of maths education, all at the same time. I chose the development of the calculus and mathematical analysis in the early nineteenth century, and this became my PhD and my first book: 'The development of the foundations of mathematical analysis from Euler to Riemann'. One of the reasons why I picked the calculus came from the educational experience at Oxford. In the second year, one morning we had the analysis course where Wilfrid Ferrar used to write all his textbooks up on the blackboard, proving things about uniform and continuous functions, and convergent series, and so on, all using ε, δ tricks. Then, as it happens straight afterwards, George Temple gave us a course in hydrodynamics, and he was using *a completely different kind of calculus* from the one we had from Ferrar. Ferrar's was, as I now understand it, a Cauchy-Weierstrass type of analysis, based on limits, with a deliberate avoidance of infinitesimals; Temple's was the classic applied mathematician's Leibniz-Euler calculus, so when he wrote down dy/dx, he literally meant dy divided by dx. And I realised at the time, that while this was the same subject that Ferrar did (calculus), it was being handled in a completely different way. When Ferrar wrote down dy/dx there were no real dy's or dx's.

This particular detail had stuck in my mind and was one of the reasons why I took up the calculus and analysis as my thesis topic; and my career has circled out from that centre. There was Cauchy on the pure side and Joseph Fourier on the applied side, so while I did my PhD I also started working on a book on Fourier. He did some excellent mathematics in the context of applications, in this case heat diffusion.

[1] *The Fontana History of the Mathematical Sciences: The Rainbow of Mathematics*, 1997.

You also worked on mathematical logic and its history. Presumably that came out of the LSE course.

Yes, quite quickly I also moved into the history of mathematical logic and set theory. In addition it was also, historically speaking, the next stage after Weierstrass's mathematical analysis. Cantor's set theory, which dates from the early 1870s onwards is, he says quite explicitly, 'an extension of mathematical analysis' – so I started to work on him and to do something on set theory. Two people then came into my situation that were very important. The first was Cecily Tanner (1900-1992), the daughter of William Henry and Grace Chisholm-Young. She had done some work on her parents, who were very important figures in the development of set theory in Britain, and they had been personal friends of Cantor (she told me she had even sat on Cantor's knee when she was a little girl). I received a lot of information from her about the Youngs, and indeed later I did a biography and some other work on them. The other person was a young graduate student from Harvard, whom I met, called Joseph Dauben. He was working on Cantor for his PhD, so I decided to leave the Cantor work to him and I would work on something else, which had already turned up, as I shall explain.

Cantor corresponded quite widely; and one of his correspondents was Philip Jourdain, who had put footnotes referring to letters from Cantor in his own papers, but the originals could not be found. I heard that they might be in Mittag-Leffler's Institute in Sweden, so I went there and did find the letters from Cantor to Jourdain, about 15 or 20 of them; they were very interesting, and this made a nice paper. But far more importantly I also came across over sixty letters to Jourdain from Bertrand Russell, about the development of his version of mathematical logic. So I began to work on Russell and from those days, the early 1970s, until today I have been involved with Russell, with two books in and around his mathematical logic and lots of papers. I have also been a collaborating editor on the edition of his papers containing his manuscripts and his essays apart from his books. I also act similarly for the edition of C. S. Peirce.

Mathematical logic is a good subject to study with some reference to history, and in fact that has been done, I think, to a much greater extent than happens in a normal maths course. But on general Popperian grounds any subject could be profitably taught with reference to its historical background.

Are the histories of mathematics and logic distinct?

I have worked in the history of both of these fields, and they are really pretty different from each other. They both have a long history but each history has very little to do with the other one. For example, among the Ancient Greeks you have rigorous mathematics with Euclid's *Elements*, and you have Aristotle's systematic treatment of valid and invalid modes of reasoning, yet the two had nothing to do with each other at the time, and they have not had that much to do with each other, until the nineteenth century with a few people such as De Morgan, Hilbert and Pieri.

There is a phrase of mine which has been quoted quite a lot, about the predicament of the history of mathematics: that it is too historical for mathematicians and too mathematical for historians (including, and especially, historians of science). Well, the situation with the history of logic is very similar; too historical for the logicians, too logical for the historians, too much of both for the philosophers. This situation obtained to the extent that people working in the field had difficulty publishing their work anywhere. I sought to remedy this situation by starting the journal 'History and Philosophy of Logic' in 1979, first volume in 1980, and now 27 years later, we're still going strong. It's not a big journal, about 250 pages a year, but it does fulfill a need and it has encouraged other journals to take material in this field.

Of course, logic is a much broader field than mathematics, and one can do plenty of things in logic that do not have to have anything to do with mathematics at all. Mathematicians still do not recognise much merit in the field of logic, although it has grown with the development of computer science. However, some people in logic do not know much about mathematics. I have just worked on the reception of Gödel's famous theorems in 1931 and their receptions by logicians and mathematicians, and it is two different stories: the logicians got Gödel's point fairly quickly, while the mathematicians took about 25 years or so before they became generally aware of the theorem.

Besides the more specialised research you have done in the history of mathematics, including mechanics and economics, you have also contributed to some more general history of mathematics.

This has mostly been by invitation from other people. I have already mentioned the book for Fontana. There are lots of general history books but they all seem to me to be rather unsatisfactory. The mathematics which is often taught, particularly at undergraduate level, starts from the late eighteenth century onwards; but general histories are often weak on work from that time onwards. In my book I tried to change the balance: half of it is from 1800 onwards and stops at about the First World War. I am pleased I tried out this policy and I would like to see some other people try it out as well.

I was also invited to edit a general encyclopedia on the history and philosophy of mathematics for Routledge; it came out at the end of 1993.[2] The subject had developed just about far enough that one could find authors for such an encyclopedia, and I really wanted it to be wide-ranging and comprehensive, not just pure maths but at least as much on applied maths, including engineering. There was also a substantial section on probability and statistics, about which very many historians of mathematics seem to know nothing, and one on the history of mathematics education and institutional development in various countries. I still feel the book could be useful, even though it is now twelve years old; it was reprinted in 2003. (I was very pleased that, the last time I saw Popper, early in 1994 – he died later that year – I noticed that he had actually bought a copy of it.) It was a major job for five years. I had about 185 articles from 133 authors: almost all of them were splendid to work with.

Are contacts with other people working in the field of the history of mathematics important?

Very. The community of people in the field as a whole are pretty friendly and good to each other; there's far more work to be done than any one of us is ever going to get round to, so you do not seem to get too much of the nastiness and rivalries that obtain, for example, in the natural sciences or in the humanities, where people may often be chasing exactly the same goals.

A key break for me, and the reason why I kept going in the field, was being invited in 1969 to the history of mathematics meeting at the Mathematics Research Centre in Germany at Oberwolfach. In those days much of the most important work in the history of mathematics was being done in Germany and in German-speaking countries. (This is no longer true.) With my contacts at Oberwolfach, for the first time I really came across people in some number, particularly Germans, Swiss and Austrians, who were seriously interested in the field and glad to see me working in it, especially that I was concerned with a relatively modern period: at that time there was very, very little on maths after 1800, and sometimes at the meetings at Oberwolfach I would give just about the only lecture on anything after 1800. I have found most of my main contacts, actually, outside of my own country – initially German-speaking, then Italy, then in Spain and Portugal, some in North America and Canada, and in recent

[2] *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, Routledge, 1994, repr. Johns Hopkins UP, 2003.

years in Central and South America and also Japan and the Far East.

On history in maths education, I edited the ICHM Toronto Workshop (1983) proceedings.[3] I have also done some lecture tours, especially a month in 1987 all round South African universities when apartheid still ruled. But outside of our British Society for Mathematics [BSHM] I really do very little in Britain. It is a pity the interest is so modest here, because the library facilities, in London particularly, are extraordinarily good, among the very best centres in the world.

I would like to add a personal note about when I became President of the BSHM from 1986. My father had taken a degree in maths at King's College London in the late 1920's, and although he did not specifically encourage me to do maths, he was quite intrigued when I did so and got to Oxford and then went on to do this historical research. I had to deal with a rather peculiar coincidence when I became president of the BSHM. This had been discussed in advance and I was formally elected at the Annual General Meeting at Christmas 1985. The meeting took place in King's College, my father's old college. He was then in his mid-seventies, and very far gone in Alzheimer's disease, and in a nursing home, so I had this very odd day of becoming president of a society in a subject which intrigued my father, taking place in his old college, when he had become a 'vegetable'. After the AGM was over I went to tell my father all about what had happened. He was hardly aware of my presence and after about an hour I left him. He died the following morning. Not surprisingly, I'm quite interested in the field of research into Alzheimer's because there are some signs it might be hereditary – well, we'll see!

What do you think of history in the popularisation of mathematics?

It has been well done by only a few people (Philip Davis, for example). But one of the really depressing sides I find about the history of maths among people in general is this absurdly exaggerated emphasis on numbers. My Fontana general history book came out with WW Norton in the States, and the paperback version of it had a cretinous quotation on the cover by someone who had obviously not read the book in the slightest. It says: 'The story of how numbers were invented and harnessed is a passionate physical saga. Grattan-Guinness traces the use of numbers in arenas as diverse as the market place and space exploration.' There are too many stupidities in that sentence for me to go through them all. But the emphasis on numbers is explicitly one which I avoid making. This seems very hard to get through to some people. People do not seem able to understand the difference between digit strings (like telephone numbers) and real numbers (like a stock market valuation). There is also an endless confusion between numbers and numerals, with a perhaps excessive emphasis on the history of numerical representation.

If you want a really interesting and important historical topic, then consider one of the greatest of mathematical problems, map-making: how do you get a shape on a curved spheroidal surface reproduced onto a flat piece of paper and preserve its main properties as well as you can? It's a wonderful problem and has led to mathematics at all sorts of levels, not just numerical mathematics but complex analysis and differential equations and all sorts of things. There's a lot of interesting maths one can do here at school and university level, just based on thinking about maps and charts alone; yet how often does it happen? I should think virtually never. And who has written a really decent and thorough history of mathematical cartography? As far as I know it does not exist though certain individual periods and figures have been well treated.

Recently we have seen an enormous increase in the number of history of mathematics courses being offered by colleges and universities. What do you draw from this, and

[3] *History in Mathematics Education*, Paris: Belin, 1987.

what makes a good history of mathematics course for undergraduates?

The prevalence of history of maths courses is, I think, a welcome feature of British maths education, and I wish that I had had one when I was an undergraduate. On the whole, though, I would prefer instead of that (or as well as that) maths courses in which history plays a role – so you would have an algebra course with some components on the history of algebra and related topics, and the same for analysis, statistics, and so on, being careful to distinguish the history account from the modern version.

What is the role of history in maths and maths education in general? Many people point to using it to explain to students something of the background, where things came from, etc. and, broadly speaking, I agree. But I think there is another area which deserves an equal emphasis but does not get it: its usefulness for the *lecturers* as well as the students, not only in the lectures they deliver but also in planning their courses in the first place. I think that history has an enduring place, and if there had been a better awareness of history in maths teaching I wouldn't have had this stodgy, dull degree I had to do at Oxford – or anywhere else, there was nothing particularly different about the Oxford course. And we also wouldn't have had ridiculous blunders like the New Maths nonsense of the 1960s. I did write a long, quite polemical, article at the time, against this whole approach, calling it 'Not from Nowhere',[4] because if you do the history behind the maths of the New Maths (for example, logic and set theory) it does come from somewhere. As usual of course the paper had no influence whatsoever, but I still subscribe to many of the doctrines I put forward in it.

The attitudes of mathematicians to history varies as to what counts as historical work in the first place, and I've been interested in this difference for a long time, and I used my Popperian training to come up with a meta-theoretic distinction which I hope is valuable: to distinguish between the history of some mathematics in the past, as opposed to another completely legitimate way of looking at the mathematics of the past, to do what I call the 'heritage' of mathematics, that is to look at its effect on later figures. I've written on this recently.[5] To use an example, look at Euclid's *Elements*. There's been a long supposedly historical tradition of reading it as covert algebra because you can rewrite it as $(a + b)^2 = a^2 + 2ab + b^2$, etc. But historically speaking, in Euclid's *Elements* most of the books are about geometry and the rest are about arithmetic, and there is no sign of algebra whatsoever in the texts, which is one of the reasons why you don't get any algebra among the later Greeks. (Diophantos is a different matter.) But from a heritage point of view it's a perfectly legitimate, indeed essential, way to read Euclid's *Elements* as algebra because it is the form in which some of the Arabs transformed and rewrote the *Elements* as they created algebra in the first place. This heritage view of algebra in Euclid is fine, but as history it is nonsense, and there's no contradiction between the two situations. However, I think historians often work intuitively with a distinction like this. Mathematicians are usually only interested in heritage and think it was history. I find out they often do not even do the heritage very well because they orient it around themselves. I introduced a phrase, that a lot of historians have picked up, about mathematicians looking at the development of mathematics as 'the royal road to me'.

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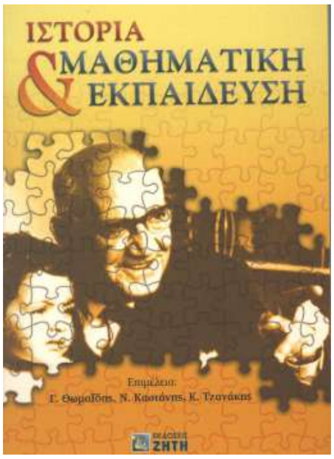
[4] *Int. J. Math. Educ. Sci. Technol.*, 4(1973), 421–453.

[5] The mathematics of the past: distinguishing its history from our heritage', *Historia Mathematica*, 31(2), May, 2004. 'History or Heritage? An Important Distinction in Mathematics and for Mathematics Education', *The American Mathematical Monthly*, 111 (1), 2004.

Conference reports

History of Mathematics, History of Mathematics Education and their didactical implications

A regional meeting of the HPM Group: 14&15 April 2006 Department of Mathematics, University of Thessaloniki, Thessaloniki Greece



The meeting took place last April, at the Aristotle University of Thessaloniki, Greece, and attracted the attention of different groups of people – mathematicians, historians and philosophers of science and mathematics educators working at all levels of education and students of mathematics, both graduates and undergraduates – who spent two days working over a broad and dense agenda, sharing theoretical views and practical concerns, but mostly sharing their excitement about making historical understanding of mathematics and mathematical education relevant for modern curricula. To such mixed audiences the question of relevance is always one that has to be negotiated and it was indeed a good fortune to have Vassilis Kalfas in the opening presentation talking difficult things, such as the Platonic ontology of numbers, and making it intelligible even to the non-experts in the audience, thus proving not merely the historical bond existing between philosophy and mathematics, but also the tremendous intellectual possibilities that the didactical transformation of our historical knowledge offers to the questioning minds. Some of these possibilities were being further explored by the rest of the speakers whose presentations had been gathered, reviewed, corrected and published in a volume that had being distributed to the participants before the official opening (Ziti publications, Thessaloniki 2006, ISBN 960-431-997-3).

C. Tzanakis. Y. Thomaidis. N. Kastanis

Apart from being a collection of research papers of specific interest, this volume – aesthetically attractive and thematically well-structured in three autonomous and yet interconnected units entitled (a) approaches to the history of Ancient Greek Mathematics, (b) approaches to the history of mathematics education, (c) didactical approaches of the history of mathematics – can be read best as an index of both historical and pedagogical questions and metaquestions. The element of surprise, which is a major driving force for every historian, was being highlighted by Vaggelis Spandagos' presentation of ancient Greek mathematical texts which have been published recently and which, nevertheless, form a small part of the total production of antiquity, since most of the ancient texts have been lost – probably for ever. The metaquestion of what is known and what is knowable is immediately inviting the pedagogical question of how we use the notion of the historical record in the classroom. A solid answer to this question is provided by Yannis Thomaidis and Constantinos Tzanakis's analysis of what is being gained by teaching mathematics through original mathematical texts. Apart from being an excellent exposition of interdisciplinary in action, this work is full of insights about how we can trigger the questioning activity in the classroom by taking advantage of the surprise caused by the strangeness of past mathematics, and how we can reshape the students' newly gained historical awareness into a working knowledge of mathematics.

The same element of surprise can be used in the education of teachers – both pre-service and in-service. Eleni Dimitriadou's work on historical notions of proof and on the ways of handling the teaching of proofs in the classroom brings into the open an interesting pedagogical paradox, which may sound shocking to may Geometry teachers, namely that the teaching of proofs in Geometry does not necessarily enhance the learning of what proof means and how it is being handled by mathematicians. Apart from bringing into focus the pedagogically significant polarity between results and processes, Dimitriadou's work leads to interesting questions concerning the multiplicity of meanings that any notion may carry either because of its historical development or because of the students' active reconstruction, especially the question of how this multiplicity affects possible misunderstandings between teacher and students. The same question of multiple meanings is being dealt with in the paper co-authored by Katerina Verykaki and Nikos Kastanis which has the additional advantage of contextualizing not only the plurality of meanings and the conceptual changes that the term "number" underwent in the course of time, but also our own discussions about conceptual changes. The reflexive character of the text is a good example of what historical understanding truly means and invites us to seek new ways for making this mode of thinking available for students of all ages.

The same firm orientation towards contextualization is also obvious in other papers and provides a true measure of the authors' historical understanding. Nikos Kastanis' study on the influence that Pestalozzi's methods exerted in the educational systems of different countries highlights the ways in which current ideologies affect educational choices; Tina Zorbala's analysis of 19th century mathematical textbooks elucidates the fact that educational systems change and mature within particular political and social contexts which allow or hinder specific transformations of knowledge; the political and social implications of mathematical education have been also highlighted in Andreas Kastanis' study of mathematical education in the Greek military academy in the early 19th century; while in Dimitris Patsopoulos' analysis of Byzantine textbooks one can find an interesting comparison of different styles of teaching and presenting geometrical knowledge. A significant pedagogical question seems to underlay all these texts; a question which has been always considered urgent: why do we teach what we teach? At the metacognitive level these texts seem to encourage a simple answer to this same question: each society's choices are unintelligible unless there are put into their specific historical context.

What is the role of history then, if both theory and praxis is context dependent? What can we learn from history if the past is unique, totally irreversible and mostly irretrievable? Is the insistence on details that characterises the historians' work a useless exercise in esoteric language or is there anything important to be learned if we manage to connect the small brushstrokes referring to individual lives with the broader picture which emerges through the collective effort of written history? Biographical studies like those presented by Christina Phili (of Nikolaos Nikolaidis) and Giorgos Zoumbos (of Ottavio F. Mossotti) provide a firm reassurance that the transition from the level of individual details to the level of larger interpretations of social change is not only a matter of course for historians but also a reflexive exercise which may have significant pedagogical implications, especially in the context of interdisciplinary teaching which attempts to encourage students to accept and understand reality in its irreducible complexity. In the case of Yannis Thomaidis' text on Diophantos, we are led to realize, through a careful discussion of minute details, how closely interwoven the pedagogical aims and the philosophical prerequisites may be. In the case of Yannis Petrakis' paper, the detailed analysis of the narrative structures used by Euclid and Proclos highlights the interconnection between mathematical and linguistic understanding.

But the most interesting advantage of history emerged during the discussions which followed the oral presentations: the puzzling realization that the detailed historical understanding of the past can offer us a different language in which we can talk about current pedagogical issues and make them intelligible at levels which go far beyond the profane caused much excitement among the non-expert participants of the sessions. Constantinos Tzanakis, in his capacity as the chair of the International Study Group on the Relations between History and Pedagogy of Mathematics, attempted to put the whole discussion within its broader context and to connect the efforts of the Greek academic community with those of the scholars abroad. His talk created a feeling of optimism that the interconnection between history and pedagogy, which has been actively pursued for some decades now, has been already productive but has not yet exhausted its potentialities. The determination of the participants to keep this line of thought alive and to explore the new possibilities offered by this synthetic approach was enthusiastic and very much in the spirit of Nikolaos Sotirakis, the pioneer Greek thinker and academic who attempted to bring together history of science, psychology and pedagogy, actively transplanting new lines of thought to Greek society, as Andreas Poulos explained in his detailed analysis of Sotirakis' major works, and to whose memory this meeting was dedicated.

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Pre-PME Meeting on The promises and problems of a semiotic approach to mathematics, the history of mathematics and mathematics education

Prior to 2006 PME in Prague, Falk Seeger, Gert Schubring and Michael Otte organized, from July 13 to 15, at Ohrbeck near Bielefeld (Germany) a meeting reflecting the present state of the semiotic approach to mathematics, mathematics history and mathematics education.
Besides papers on semiotics and on semiotic analyses of mathematics teaching, there were papers on the relation between philosophy and semiotics, by Paul Ernest and Michael Otte, and on semiotic aspects in analysing the history of mathematics.
Regarding the relation to history, there were two papers: by Michael N. Fried, on *Similarity and Equality in Greek Mathematics: Semiotics, History of Mathematics and Mathematics Education*; and by Gert Schubring, on *The Process of Algebraization in the History of Mathematics: The Impact of Signs*.
A publication of the papers is in preparation.

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Work in progress

We encourage young researchers in fields related to *HPM* to send us a brief description of their work in progress or a brief description of their dissertation.

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Have you read these?

Actes du 7ème Colloque Maghrébin sur l'Histoire des Mathématiques Arabes

The proceedings of the seventh Maghrebian colloquium on the history of Arab mathematics (COMHISMA7) held at the École Normale Supérieure in Marrakech May 30-June 1, 2002.
Edited by Abdellah El Idrissi and Ezzaim Laabid. 2 vols. Marrakech: École Normale Supérieure, 2005.
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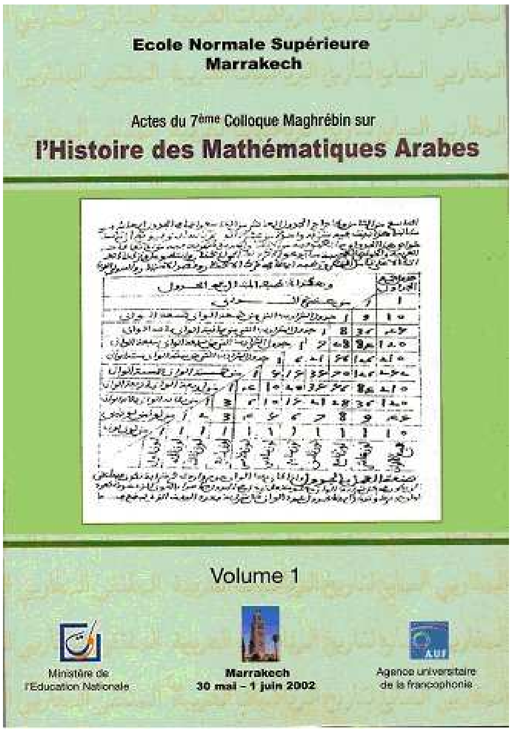

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Proceedings of TSG 29 at ICME 10

The Proceedings of the Thematic Study Group 29 at ICME 10 in Copenhagen, the first international meeting on the history of mathematics education, is published now, as a special issue of the journal:

Paedagogica Historica. International Journal of the History of Education. Special Issue: History of Teaching and learning Mathematics (ed. by Gert Schubring)

Volume XLII, Numbers IV&V, August 2006

Within the three main dimensions of ongoing research in this area, eight papers give access to recent findings:

1. Transmission and Modernizations of Mathematical Curricula

Iason and Nikos Kastanis on Greece (1800-1840), Shinya Yamamoto on the process of adapting German pedagogy in Japan, and Kristín Bjarnadottir on modernization in Iceland.

2. Teaching Practice, Textbooks, Teacher Education

Eileen F. Donoghue on David Eugen Smith (USA) and Harm J. Smid on the emergence of a realist school system in the Netherlands.

3. Cultural, Social and Political Functions of Mathematics Instruction

Livia Giacardi on the crucial period 1867 to 1923 in Italy, Alexander Karp on reform debates in Russia, and Mahdi Abdeljaouad on Arab countries.

In addition, there is a synthesis giving the state of the art, regarding research into the history of teaching and learning mathematics, by Gert Schubring.

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* * *

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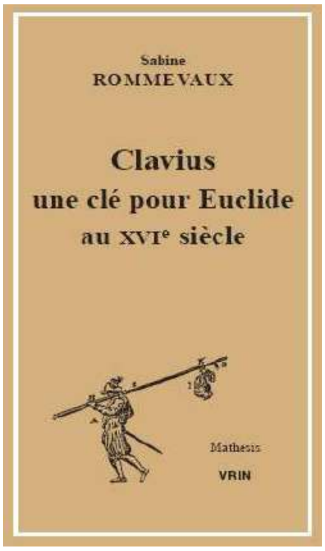

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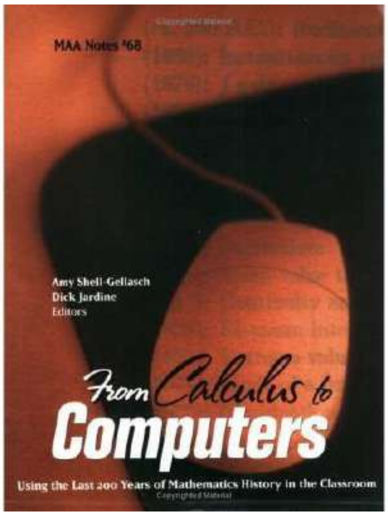

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Luis María Lleras
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Fred Rickey's History of Mathematics Page
http://www.dean.usma.edu/math/people/rickey/hm/default.htm

Canadian Society for History and Philosophy of Mathematics
http://faculty.umf.maine.edu/~molinsky/cshpm/

Culture*MATH*. Ressources pour les enseignants de Mathématiques
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Brazilian Society for History of Mathematics
http://www.sbhmat.com.br

The French INRP (National Institute for Pedagogical Research) is
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Societies and organisations

African Mathematical Union: Commission on the History of Mathematics in Africa (including newsletter)
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Association des Professeurs de Mathematiques de l'Enseignement Public [APMEP] History site:
http://www.apmep.asso.fr/BMhist.html

British Society for the History of Mathematics [BSHM]
http://www.bshm.org

HOMSIGMAA - History of Mathematics Special Interest Group of the MAA
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About Medieval Arabic Numbers
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Convergence: an online magazine of the MAA providing resources to teach mathematics through its history
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Homepage of International Journal for the History of Mathematics Education
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* * *

Notices

Problématique-Three Questions on the History of Mathematical Theories and its Didactic Value

The modern mathematical perception of "theory", as Alan Schoenfeld also points out from the domain of didactics of mathematics,[6] refers to a technical and specific use of the

[6] Schoenfeld, A. H.: "Purposes and Methods of Research in Mathematics Education", in *Holton, D. A. (ed.): The Teaching and Learning of Mathematics at University Level*, Kluwer Academic Publishers, 2001, pp. 221-236.

term that of a formal language with rules of deduction and axioms.

This logico-syntactical view of mathematical theories confines their educational understanding to a cumulative learning of theorems and of handling of their proofs, without promoting the awareness of their close relation to the idea of theory in its historical and cognitive dimensions.

At this point, it is worth mentioning the relative views of two eminent mathematicians, who evoke a different attitude to the issue of mathematical theories. The first view is of George Polya, who, in an interview, claimed that:

> "In order to understand a theory really, you must know how it was discovered."[7]

The second reference is due to Jean Dieudonné, who, in his book, *Mathematics–The Music of Reason*, noted that:

> The evolution of mathematical theories "does not consist solely in the accumulation of new theorems; for these are not simply superimposed on the old ones, but they absorb them, transforming them into "corollaries" which in the end are sometimes not even explicitly mentioned any more, unless by historians. Even the formulation of theorems can change completely… other transformations may appear even stranger: a system of axioms is replaced by an "equivalent" system, in which a statement which was a theorem in the old system becomes an axiom in the new one, while the old axioms become theorems".[8]

In recent years, a special interest in the nature of scientific theories and their cognitive process has been developed. From this new perspective, a scientific theory constitutes an explanatory system of knowledge, which is being renewed and reconstructed in the process of time.

In philosophy of science, the nature of theories is regarded on the basis of general epistemological characteristics that qualify it to be a theoretical, coherent, systematic, explanatory knowledge, which is historically evolving, according to various patterns of scientific change.[9]

Moreover, in Science Education, there is a growing interest in the role of the systematic and explanatory character of theories in the construction and conceptual change of students' cognitive base during learning procedure.[10]

Concerning all this, the following questions are posed:

How the historical understanding of mathematical theories can contribute to the didactic analysis and approach of theoretical knowledge of mathematics?

The understanding of the nature and the change of mathematical theories through their historical course and the awareness of their influence in the development of mathematical knowledge ascribes to theories their important role in cognitive development. Consequently, the value of their explanatory character comes to the fore, as a useful instrument for the overcoming of the epistemological obstacles in the transition from the empirical perception to the theoretical level of mathematical thought.

Boero, A.: Approaching Mathematical Theories in Junior High School, http://www.wcape.school.za/malati/Boero.htm

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[7] Alexaderson, G. L. / Polya, G.: "George Polya Interviewed on his Nineteenth Birthday", *The Two-Year College Mathematical Journal*, 10(1), 1979, pp. 13-19, especially p.17.

[8] Dieudonné, J.: *Mathematics-The Music of Reason*, New York, Springer, 1992, p.162.

[9] For instance, see Boero, P., Pedemonte, B. & Robotti, E.: "Approaching Theoretical Knowledge Through Voices and Echoes: A Vygotskian Perspective", in *Proc. XXI PME*, 2, 1997, pp. 81–88, and Gopnik, A. & Wellman, H. M.: "The Theory Theory", in *Hirschfeld, L. A. & Gelman S. A. (eds.): Mapping the Mind: Domain Specificity in Cognition and Culture*, Cambridge University Press, 1994, pp. 257-293.

[10] For instance, see Schwitzgebel, E.: "Children's Theories and the Drive to Explain", *Science & Education*, 8, 1999, pp. 457-488, and Gilbert, J. K.: "On the Explanation of Change in Science and Cognition", *Science & Education*, 8, 1999, pp. 543-557., as well as the rest articles of the same issue.

http://www.mat.ucm.es/deptos/am/guzman/quebec/originevol.html
(Also see HPM-Newsletter, 28, 1993, pp. 2-3)

What does historical development of mathematical theories reveal about their explanatory function?

The standard treatment of mathematical theories within their technocratic contexts abstracts from them their historicity concerning the procedures of their formation and development, thus, assigning a fixed and invariable existence to them, which serves as a deposit to the addition of new knowledge. Contrary to this absolutist pattern of cumulative knowledge, the theories of conceptual change assert that concepts and theories do change in the historical course of mathematics, affecting their understanding.[11] So, this way, from the reconstruction of the inner structure of knowledge, in the context of a theory change, a new way of thinking arises.

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Radner, M.: "Possible Theories", *Synthese*, 41, 1979, pp. 397-415.

What is the didactic significance of the historical understanding of mathematical theories as explanatory structures and of proofs as their explanatory components?

Theorems and proofs are means of expression of theoretical knowledge, which is historically being modified, differentiating its explanatory contexts. Moreover, the very notion of proof is tied to that of theory (in the sense of "systematic structure").[12] This viewpoint sets off the dynamics of the explanatory aspect of proof in classroom. In this way, the pedagogical approach to proof becomes more fruitful when its aim is towards a mainly relational understanding rather than a merely operational one. In different words, when its focus is not only to convince that the results are true, but also to explain *why*.

Hanna, G.: "Proof as Explanation in Geometry", *Focus on Learning Problems in Mathematics*, 20 (2-3), 1998, pp. 4-13.

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[11] Knowledge is independent neither of theory (theory-free), nor of values (value-free). See Confrey, J.: Conceptual Change Analysis: Implications for Mathematics and Curriculum Inquiry, *Curriculum Inquiry*, 11, 1981, pp. 243-257.

[12] Hanna & Jahnke, regarding the educational evolution of proof, also point out: "One cannot really discuss proof without discussing theories". See Hanna, G. & Jahnke, N.: "Another Approach to Proof: Arguments from Physics", *ZDM*, 34(1), 2002, pp. 1-8, p.2.

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* * *

Babylonian Mathematics: Some history and heuristics

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For mensurational geometry, a practical method of Old Babylonian Mathematics (1600 B.C.) consisted in using specific fixed 'coefficients'. Two examples are as follows (see K. Muroi's paper in *Historia Scientiarum* Vol. 2, No. 1, 1992):

(i) Diagonal d of a square of side s

$$D = cs \quad (1)$$

where coefficient c = 1;25 (2)

(Here 1;25 is written in the usual sexagesimal form)

(ii) Height h of an equilateral triangle of side a

$$H = ka \quad (3)$$

where coefficient k = 0;52;30 (4)

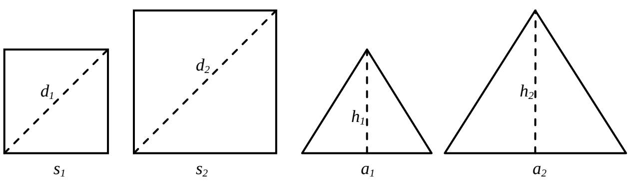

Figure 1

The recognition of constancy of the coefficients (c and k) imply that the Babylonians knew the linear proportionality of the variables involved. That is, they were aware that for two (or more) similar figures $d_1/s_1 = d_2/s_2$ = constant, and $h_1/a_1 = h_2/a_2$ = constant.

Geometrically, this proportionality of linear dimensions of plane figures imply the similarity-property namely $d_2/d_1 = s_2/s_1$ and $h_2/h_1 = a_2/a_1$.

Now if we use the relation (1) with awareness of similarity-property, we get (Fig. 2).

$$D = cs$$

for and from the square *PQRS*.

By the same relation we also get

Diagonal $PS = c$.(side PO),

Or, $s = c(s/2)$, for the square *POST*.

By combining these two results, we have

$$d.s = cs.c(s/2) \quad (5)$$

which gives $c^2 = 2$ (6)

This shows that the relation (1) is equivalent to

$$d^2 = s^2 + s^2, \text{ exactly.} \quad (7)$$

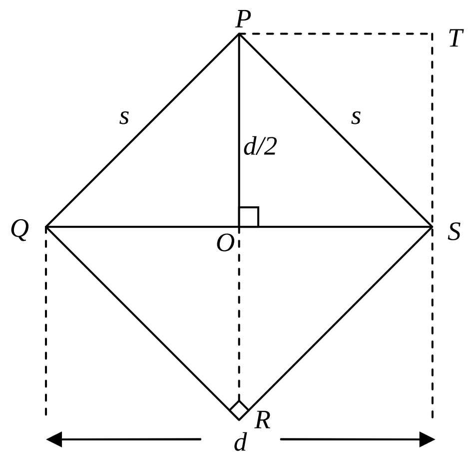

Figure 2

The important point to note is that we have *not* used there the so-called Pythagoras Theorem which, otherwise, directly gives (7) from the triangle *PQS*.

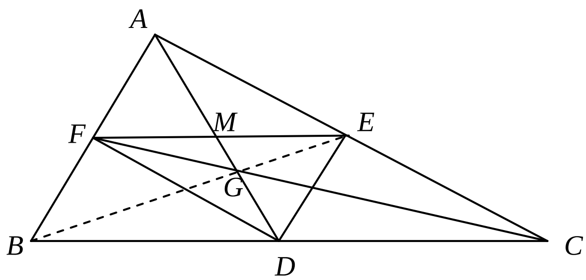

Figure 3

Simple heuristics can also yield some other familiar results. In Fig. 3, D, E, F are mid-points of the sides of a triangle *ABC*. We wish to find the ratio in which the centroid G divides the medians. It can be easily seen that G will also be the centroid of the triangle *DEF*. Let $MG = x$, and $GD = nx$ (where n is to be found out). If we apply this very proportionality to the triangle *ABC*, we get

$AG = n.GD = n^2x$ from above.

Now, $AD = AG + GD = n^2x + nx$

But also,

$AD = 2MD = 2(MG+GD) = 2(x+nx)$

Thus, we must have $n^2x + nx = 2(x + nx)$ which easily yields the admissible value $n = 2$.

That is, we have $AG/GD = 2/1$ (8)

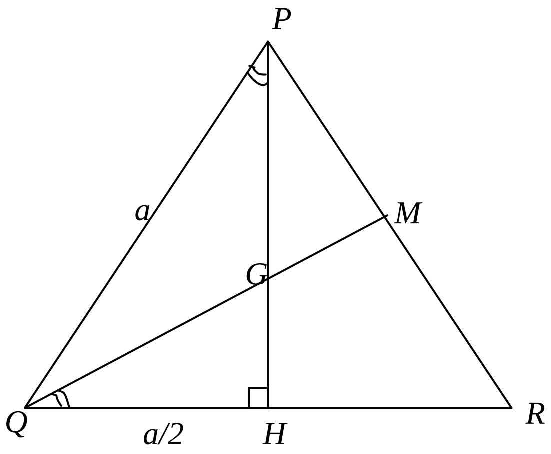

Figure 4

We apply this result and the Babylonian relation (3) to the equilateral triangle *PQR* in Fig. 4. Here we have

$QG = (2/3)\ QM = (2/3)\ PH = 2\ h/3$ (9)

In triangle *PQH* we have relation (3), and in the similar triangle *GQH*, by the same relation, we have

$QH = k.QG$

or, $a/2 = k.\ (2h/3)$ (10)

by putting value from (9). Relations (3) and (10) give

$h.(a/2) = ka.k.(2h/3)$

or, $k^2 = ¾$ (11)

Here also, it should be noted, we have *not* used the Pythagoras Theorem which, otherwise, directly leads to, from triangle *PQH*,

$h^2 = PH^2 = PQ^2 – QH^2 = 3a^2/4$ (12)

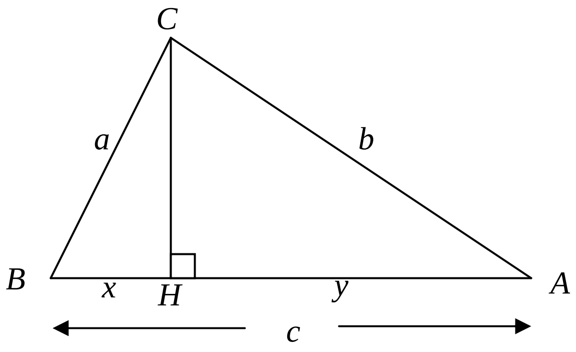

Figure 5

On the other hand we found *k* in (2) as shown in (11), and hence (12).

Our above discussion shows that it is not necessary to presume that advanced or more general theorems (of later times) are implied in some particular results. On the other hand, the general Pythagoras theorem can be obtained by using the similarity-property. In the triangle *ABC*, the angle *C* is a right angle. The perpendicular *CH* from *C* divides the hypothenuse *BA* into two parts *BH* = *x*, and *HA* = *y*. From similar triangles *HBC* and *CBA*, we get $x/a = a/c$.

And from similar triangles *HAC* and *CAB*, we have $y/b = b/c$.

If we put *x* and *y* from these equations in $x + y = c$, we at once get the Pythagorean relation $a^2 + b^2 = c^2$.

This short proof was known to the famous Indian mathematician Bhāskara II (A.D. 12th century) who outlined it in his *Bījaganita* (‘Algebra’). According to F. Cajori (*History of Elementary Mathematics*, N.Y. 1961, p. 123), “this proof was unknown in Europe until it was rediscovered by Wallis” (17th century).

* * *

Announcements of events

Colloquium José Anastácio da Cunha – Time, Ideas and the newly discovered Manuscripts.

December 14-15, 2006

University of Minho

Organized by: Maria Elfrida Ralha, Maria Fernanda Estrada, Maria Paula Mendes Martins and Pedro Patrício.
We are reporting the mathematical manuscripts by José Anastácio da Cunha (1744 -1787) that were recently found in an archive which belonged to the Earl of Barca (1754-1817), a Portuguese foreign affairs minister. These manuscripts have undergone preliminary studies by a group of Portuguese researchers on the History of Mathematics and, in particular, on the life and works of José Anastácio da Cunha. We are now prepared to share this discovery with a wider audience in a Colloquium that will be held at University of Minho on the forthcoming 14th and 15th of December.
This Colloquium is also aimed at bringing together people who are interested in Mathematics and its History, as well as people interested in related fields such as History itself, Philosophy and Literature of the XVIII century.
For more information, please contact us in jac2006@math.uminho.pt
or visit the following website:
www.math.uminho.pt/~jac2006

5th European Summer University on the History and Epistemology in mathematics education (ESU-5)

July 19-24, 2007
Prague, Czech Republic
For more information,see the HPM Newsletter issues No58, 60 or the ESU-5 website http://www.pedf.cuni.cz/kmdm/esu5.

Mathematics Education In A Global Community *(9th International Conference of The Mathematics Education into the 21st Century Project)*

September 7-12, 2007
Charlotte, North Carolina, USA
The conference will open with an evening welcome reception on Friday, Sep 7 and finishes with lunch on Wednesday, Sep 12. There will be an additional social programme for accompanying persons. The chairman of the Local Organising Committee is Associate Professor Dr. David K. Pugalee. For all further conference details and updates please email arogerson@inetia.pl (not arogerson@vsg.edu.au - please delete this from your address book!).
Our conferences are renowned for their friendly and productive working atmosphere and are attended by innovative teachers and mathematics educators from all over the world – for example 25 countries were represented at our last conference! This conference follows on from our eight previous Project Conferences.
The Conference is organised by the Mathematics Education into the 21st Century Project - an international educational initiative whose coordinators are Dr. Alan Rogerson (UK/Australia/Poland) and Professor Fayez Mina (Egypt). In 1992 UNESCO published our Project Handbook "Moving Into the 21st Century" as Volume 8 in the UNESCO series Studies In Mathematics Education.
Chairman:
Associate Professor Dr. David K. Pugalee, (dkpugale@email.uncc.edu)

ICME-11

July 6-13, 2008
Monterrey, Mexico

Models in Developing Mathematics Education *(10th International Conference of The Mathematics Education into the 21st Century Project)*

September 12-18, 2008
Dresden, Germany
For further information contact
arogerson@vsg.edu.au

* * *

In Memoriam

Karen Dee Michalowicz died on July 17, 2006 at the age of 63. She was a nationally recognized mathematics teacher at the Langley School in McLean, Virginia, having received the Presidential Award for Mathematics Teaching in 1994. Karen was especially interested in the use of history in the teaching of mathematics. In that role, she co-directed the NSF-sponsored, MAA administered grant program, *Historical Modules for the Teaching and Learning of Mathematics*, beginning in 1998. That program involved about twenty-five college and high school teachers of mathematics, who produced such a large amount of material for teaching mathematics using history that the MAA was forced to publish it as a CD. In 1998, she was also invited to participate in the study group on History in Mathematics Education, organized by the International Commission on Mathematical Instruction. After an intensive week of discussions, the group ultimately co-authored the ICMI Study, *History in Mathematics Education*, which appeared in 2000. Karen was in charge of the group writing the chapter on "History in support of diverse educational requirements — opportunities for change". Over the years, she had amassed a huge collection of mathematics texts from the nineteenth century and earlier, a collection whose contents she enthusiastically shared with students and colleagues in many presentations at national and international meetings. She will be greatly missed by her numerous friends and colleagues in the MAA, the WME, the NCTM, and the Benjamin Banneker Association, among others.

The Mathematical Association of America,
http://www.maa.org/news/inmemoriam.html

(Karen Dee Michalowicz in discussion with Costas Tzanakis during HPM 2004 in Uppsala.)

More memories of Karen from around the world are available at the HPM webpage.

Please note that the webpage http://www.mathedu-jp.org/hpm/index.htm is currently not updated due to technical reasons. Please use http://www.clab.edc.uoc.gr/hpm/ for the time being.

A note from the Editors

The Newsletter of HPM is primarily a tool for passing on information about forthcoming events, recent activities and publications, and current work and research in the broad field of history and pedagogy of mathematics. The Newsletter also publishes brief articles which they think may be of interest. Contributions from readers are welcome on the understanding that they may be shortened and edited to suit the compass of this publication.

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http://www.clab.edc.uoc.gr/hpm/

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64	12 February 2007	1 March 2007
65	7 June 2007	1 July 2007
66	12 October 2007	1 November 2007

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