International Study Group on the Relations Between the HISTORY and PEDAGOGY of MATHEMATICS
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http://www.clab.edc.uoc.gr/hpm/

Ubiratan d'Ambrosio was one of the early chairs of HPM. In an extended interview with Maria Laura Magalhães Gomes, he talks about the early days of HPM and about his own growing interest in what has become known as ethnomathematics. We shall publish more detail of what Ubi has to say about the Program Ethnomathematics in a future issue of the Newsletter.

Interview: Ubiratan D'Ambrosio — Historian and Pedagogue of Mathematics, Former Chair of the HPM Group (1984-1988)*

How did you first get acquainted with the HPM? Were you present to the 1972 ICME in Exeter when a working group on History and Pedagogy of Mathematics was organized by Philip Jones and Leo Rogers?

Ubi: I did not go to the 1972 ICME in Exeter. I was involved with mathematics education up to 1958. But after that, I devoted myself almost entirely to research in pure mathematics. In 1963 I completed my doctorate and went to the United States. I only became involved again with mathematics education after my return to Brazil, to become the Director of the Institute of Mathematics, Statistics and Computer Science, of the recently founded State University of Campinas/UNICAMP. My participation in the Third Interamerican Conference on Mathematics Education/IACME-CIAEM, in Bahia Blanca, Argentina, was a sort of debut, internationally, in the field of mathematics education. Then came ICME 3 in Karlsruhe.

What do you remember about ICME3 at Karlsruhe when HPM was founded?

Ubi: Not only was I present at ICME 3, in 1976, I was also a member of its International Program Committee and I prepared a paper on "Objectives and Goals of Mathematics Education", for discussion by a Working Group. My paper was, and still is, very controversial, and relies much on the history and sociology of mathematics. Indeed, my

* Founder and Former Chair of the International Study Group on Ethnomathematics (ISGEM) , Chair of The Brazilian Society for the History of Mathematics, Winner of the Kenneth O'May Medal in the History of Mathematics (2001)

approach to history, and also to sociology, draws attention to what I now like to refer to as "the mathematics of non-mathematicians". Indeed, referring to the mathematics of "mathematicians" and of "non-mathematicians" is the generating idea that leads into my approach to ethnomathematics. This is what I call the Program Ethnomathematics, a research programme of history, sociology and philosophy of mathematics, with pedagogical implications. Indeed this is the way I presented ethnomathematics in a short paper published in the *Notices of the American Mathematical Society* (December 1992, vol. 39, n°10, pp.1183-1185). This led quite naturally to supporting the creation of the International Study Group on the Relations Between History and Pedagogy of Mathematics (HPM), and I became very involved with it.

In 1979, I got very involved with the Pugwash Movement, the result of Russell-Einstein Manifesto of 1955. My participation in the Pugwash conferences and seminars was always focused on the relations between science, hence also mathematics, and global issues—particularly, the relation between mathematics and war. The pursuit of peace became, for me, the top priority. And I am concerned with the fact that the enormous progress of science, technology, and now, techno-science, all strongly depending on mathematics, do not represent progress in the global quality of life. The state of the world is deplorable. Can we, as mathematicians and mathematics educators, do anything to avoid the growing threat facing humanity? What we do, as mathematicians and mathematics educators, have obvious implications for what scientists do.

I believe the challenge is not only to advance more mathematics and to teach it better, but to restore human values and ethics to our practices as mathematicians and mathematics educators. How can this be done? Has this ever been done in teaching the history of mathematics? What have been the bridges between science and mathematics and other cultural practices? This requires a broader look into the history of mathematics, as well into the history of science, since they were, practically, the same thing up to mid-19th century. I use the word "broader" to mean the recognition of the mathematics done by non-mathematicians.

The approach to this kind of history must be cross-curricular and transcultural. I believe that history benefits much from multicultural readings of narratives lost, forgotten or suppressed.

In 1984, in Adelaide, you, from Brazil, and Christian Houzel, from France, were elected co-chairs of the HPM for the period 1984-1988. Can you say something about this time and also about the Newsletter?

Ubi: I don't remember how and why this idea of having two chairs, one from Europe and one from America, was accepted when HPM was founded. It does not make sense. When I was elected, in Australia, there was felt a need for someone from Europe. I have a vague recall that it was Jean-Pierre Kahane, then the President of ICMI, who proposed Christian Houzel. Christian is a distinguished mathematician and historian of mathematics, with important contributions to the history of medieval mathematics and of Bourbaki. Working with him was a pleasure. He is a very open minded colleague, and he was always very receptive to my proposals. But the concept of two chairs was a mistake.

During my tenure as co-chair, there were many opportunities to promote the presence of HPM. I was very close to the North American section, which had been established in Adelaide, in 1984. I used to go to all the major meetings of NCTM, MAA and AMS, always participating in sessions organized by HPM. Since I attended major international meetings, I always used the opportunity to organize special sessions of HPM, with the full agreement of Christian Houzel. In the XVII International Congress of History of Science in Berkeley, California, 1985, it was a session on "The Relation between History and Pedagogy of Mathematics". In 1986, during the International Congress of Mathematicians, in Berkeley, a session on "The time lag between innovation in mathematics and its incorporation into collegiate and university curricula". And in 1988, the Second Latin-American Congress on the History of Science and Technology, in São Paulo, the objective was to make HPM better known in Latin America.

In July 1988, ICME 6 was held in Budapest. It was then decided to hold a satellite meeting in Florence, Italy, chaired by Florence Fasanelli—a memorable and unique meeting organized in Florence by Florence! There, Florence Fasanelli was nominated for the chair of HPM for the next four years and it was decided that the concept of co-chairs should be dropped.

As immediate past-chair, my involvement with HPM continued to be very intense. With full the agreement of Florence Fasanelli, I organized in 1990 an HPM conference in Campinas, Brazil at the Center of Logic, Epistemology and History of Science at the State University of Campinas, Brazil, about "Using History in the Teaching of Mathematics", and, in the same year 1990, I arranged for a meeting of HPM during the ICM 1990 in Kyoto. In 1994, I arranged a meeting of the Americas Section of HPM, as a satellite of the Second Iberoamerican Conference on Mathematics Education/II CIBEM, in Blumenau, Brazil. And I continued to be an assiduous participant of HPM events in the USA and Europe.

The publication of the Newsletter, under the responsibility of Charles V. Jones, was a very important step in consolidating HPM. In 1988, Charles V. Jones resigned and Victor Katz took over the editorship of the Newsletter. Both set the high standard of the Newsletter, which prevails. In the long period of the editorships of Charles Jones and Victor Katz my involvement with HPM was very strong. Being a sort of Latin American correspondent was helpful in enlarging the interest on HPM in Latin America. In the transition of Charles Jones to Victor Katz and to the successive editors, its quality was maintained and the Newsletter is, in my judgment, the main factor for the international visibility of HPM.

I will ask you now a question similar to one answered by Leo Rogers in his interview for the HPM Newsletter. In your opinion, has something changed since you were co-chair of HPM in the way the history of mathematics is perceived around the world? What would you say about this perception in Brazil as Chair of the Sociedade Brasileira de História da Matemática (Brazilian Society for the History of Mathematics)?

Ubi: I have no doubt that HPM made a change in the history of mathematics, as well as in mathematics education in general. Of course, to have ICMI recognizing History and Pedagogy as a permanent study group was a major step. The area became central in mathematics education, with an important presence in major international meetings.

A very important consequence is, what we might call, the social/professional aspect. We became known to each other, exchanging ideas, methods, materials, thus acquiring a sort of professional standing, instead of an amateurish attitude, in our practice. Frequent meetings, in several places of the World, helped us to raise the consciousness of being a community, with common interests, and similar problems. New friendships developed. It was possible to help each other. HPM was responsible for much interest developed in countries with little or no tradition in the history of mathematics. Personal friendships, as a result of HPM meetings, were a big help for the interchange of scholars, particularly locating students working for a graduate degree.

I have to remind, mainly the younger colleagues, that communication in those years was very different. We depended on the ordinary post. To send the Newsletter to interested individuals was costly. There were no membership dues for HPM, just a mailing list of interested people, normally those that used to attend meetings. A strategy to have more people involved was to have correspondents in different countries, which, after receiving a master copy of the Newsletter, would reproduce and distribute it.

You ask, specifically, about the *Sociedade Brasileira de História da Matemática/SBHMat* (Brazilian Society for the History of Mathematics). I have no doubt that the participation of Brazilians in national and international meetings of HPM was an important factor for the creation of the SBHMat. We have also to consider that the founding of the *Sociedad Latinoamerica de Historia de las Ciencias y la Tecnologia/SLHCT* (Latin American Society of History of Science and Technology), in 1982 and of the *Sociedade Brasileira de História da Ciência/SBHC* (Brazilian Society of the History of Science), in 1983, created a strong group working on the relations between history and pedagogy of mathematics. HPM was a strong supporter of this interest. This helped to develop, as I said above, national communities with common interests. The Brazilian Society of the history of mathematics may be considered a natural consequence of these developments.

In his interview Leo Rogers pointed out that there is still little of sensible, reliable material in history of mathematics written with school teachers in mind. Besides, he emphasized the limitations of time in teacher education since mathematical education has become a very wide subject. In fact, as teachers have to study mathematics, psychology, philosophy, sociology, anthropology etc, there seems to be small room for the history of mathematics and its relations with pedagogical issues. What is your opinion about these limitations of material and time in teachers' preparation?
Ubi: Leo is right. History of mathematics is a fascinating subject, but it is in competition with other established traditional fields in Mathematics Education. Contents continue to be the core in teacher preparation. In contents courses, the subjects are taught in an unhistorical style, which favours obsolete and uninteresting teaching. But administrators, faculty, students, and even parents, value contents.

The interest in history is very high, and an evidence of this is the increasing number of books in history of mathematics being published. Leo is right. We need more. I also mention that good fictional or semi-fictional books are selling well. These can be very interesting material. A critical review of these books is both interesting and can enrich contents. But both, academic and fictional books on history are, in general, non-linear. This is very unappealing for the traditionally trained teacher.

Of course, the crux is the preparation of teachers. As I said above, there is a rapidly increasing number of good books and of excellent sites in the web. But there is a great distance between availability and effective use in the classroom. The reason is the fact that teachers are unprepared. Regrettably, some schools offer no more than a token course on history of mathematics. Unless the amount of time devoted to traditional contents courses is reduced, and an historical approach in the courses is adopted, it will be very difficult to have relevant history of mathematics in schools. The same happens with teachers of elementary and middle schools. Another obstacle is the testing system. Even if teachers are well prepared to teach history of mathematics, they are under pressure to cover traditional material, which is suitable for testing. The dominance of testing interferes with education.

In 1988 there were special talks in honour of your 65th birthday organized by Victor Katz and Karen Michalowicz. One of the speakers was Dirk Struik, who was then 104 years old. I think the readers would like to know something about your relation with this famous historian of mathematics.
Ubi: I was born in 1932. The commemoration of my 65th birthday was in 1998, in the joint meetings AMS/MAA in Baltimore. Indeed, Struik gave an emotional talk. I was so happy! He was 104. When I thanked him for being there and for his talk, he said something like "It was natural for me to come. You flew from Brazil to my 100th birthday party, in Providence. And I hope to come also to your 100th birthday".

My relation with Dirk Struik has a very interesting story. Of course, I knew him from a long time, through his books and talks. Since the international symposium on the "The History of Mathematics Methods", which I had organized in 1979 at UNICAMP, I was trying to strengthen the interest in history of mathematics in the university. I had some money available and I dared to invite

Dirk Struik. It was in 1984. I had no idea of how old he was. He very kindly replied, saying he was close to 90, but he would very much like to visit South America, particularly Brazil. He was curious about knowing something more about the presence of Mauricio de Nassau, in Brazil, during the Dutch occupation, in the XVII century. But there was a problem. He could not travel without his wife. It was clear to us that he could not come alone. Such an old man! Since Mrs. Struik had a degree in Mathematics, I managed to send him two airline tickets, and they came as two invited visiting professors. But when they arrived, to our big surprise, he was very agile and in good health. Then we learnt about the real reason: she could not stay alone. She needed his support. Indeed, he was caring for her all the time. Later, I went to their home in Massachusetts and I could see how she was, increasingly, dependent on him. When I visited him, after her death, he gave me a beautiful lesson on how to understand life and death.

He was very interested in Latin America, and was very receptive to my broader approach to the history of mathematics. He reassured me that this approach is needed to study the history of science in countries which were not in the main stream of scientific development. In one of my visits to his home, he was so kind in offering me a first edition of his books *Yankee Science in the Making* and the *Source Book of Mathematics*.

I want to add something that gave me much emotion. When I was driving them back to the airport, in the visit to Campinas, he said something like "I could not imagine that, in this advanced age, I would still make such good friendship". He exerted great influence on me.

Professor D'Ambrosio, what else would you like to say to the Newsletter readers?
Ubi: I would like to add something about my involvement with the history of mathematics, and how this became, later, the guiding motivation for the Program Ethnomathematics. While preparing my doctorate and later when I was living in the United States, I was much interested in the history of mathematics, but as support for my research in mathematics. The calculus of variations has an exciting history. I was always attending the sessions on history of mathematics in the AMS/MAA joint meetings. I had good relations with Kenneth May and I followed the idea of creating the ICHM. I attended the history sessions of ICM 1970, in Nice, and of ICM 1974, in Vancouver, and of ICM 1978, in Helsinki, and all the quadrennial congresses of mathematicians, up to ICM 2000, in Tokyo. From 1985, up to Mexico in 2001, I also attended all the congresses of history of science. The International Commission of History of Mathematics was in charge of organizing sessions in both series of congresses, and I was a member of commission. Indeed, I later became very active also in the International Commission of History of Science and I have served as member of the Executive Boards of both the ICHM and the ICHS. Of course ICMI, HPM, ICHM and ICHS are all related, and my line of work, the Program Ethnomathematics, relies on the intimacy of these four groups

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Conference reports

Novembertagung in the History and Philosophy of Mathematics

At the beginning of November 2005, the 16th *Novembertagung* on the History of Mathematics took place in Paris, at the Ecole Normale Supérieur, 45 rue d'Ulm.

The event, now an annual international conference, started in 1990 and is organized each year by a new committee of PhD students and those who recently completed their PhDs in the History and Philosophy of Mathematics. It is a great opportunity for young historians and philosophers of mathematics to meet, network and give presentation in a relaxed and friendly atmosphere.

The 16th Novembertagung's committee consisted mainly of French PhD students and researchers – Frédéric Brechenmacher, Jean Delcourt, Matthieu Husson, Guillaume Jouve, Brendan Larvor, Juliette Leloup, Sébastien Maronne, Amirouche Moktefi, Norbert Verdier and Elodie Vieille Blanchard hosted the event in Paris, and were supported by the Ecole Normale Supérieure, Institut de Mathématiques de Jussieu, Université Paris 6, Laboratoire R.E.H.S.E.I.S. CNRS, GHDSO, Université Paris 11, Ecole doctorale MathIF-Université Claude Bernard Lyon 1 and the Université de Cergy Pointoise.

The conference ran from the morning of Friday 4th to Sunday 6th November 2005. It saw 45 papers from participants from around Europe, from France, UK, Germany and Netherlands, to Romania and Turkey.

The theme of the conference was originally defined through the question of 'How does one identify and analyze a movement in the history of mathematics?', although the conference saw presentations which didn't always respond to this initial question. The opening lecture, under the same title, was given by Norbert Schappacher from the Université Louis Pasteur, Strasbourg, who on some previous occasions was one of the student presenters, and is now a lecturer at Strasbourg.

The first day of the conference was entirely devoted to the original question, while Saturday and Sunday saw papers on themes divided more broadly into history and philosophy of mathematics. Furthermore, the historians narrowed their interests chronologically, and presented papers from ancient mathematics to developments in the 20th century. Two sessions on the final day concerned the history of geometry.

To see the paper abstracts and find further details about the presenters, see http://irist.u-strasbg.fr/nov2005/ and http://www.novembertagung.info/ .
Next year's Novembertagung is being hosted by the British researchers. For an initial expression of interest to present a paper contact Snezana Lawrence, member of the 17th Novembergatung Committee.

Snezana Lawrence
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22nd International Congress of History of Science, Beijing (China)

The 22nd International Congress of History of Science (ICHS) was held in Beijing from 24-30 July 2005 with the general theme: "Globalisation and Diversity. Diffusion of Science and Technology throughout History". Organised by the International Union of History and Philosophy of Science, Division of History of Science (IUHPS/DHS) there were almost 800 participants – specialists in different fields of the history of science – from all over the world. There were numerous interesting papers on the history of mathematics given in many symposia organised by the DHS Commission and other groups. The author of this report draws attention to the following symposia:

Ten Classics of Ancient Chinese Mathematics

This symposium was devoted to the major classical texts of ancient Chinese mathematics, namely the Shi Bu Suan Jing (Ten Books of Mathematical Classics), and the oldest known Chinese precursor to these classical texts, the Suan Shu Shu (A Book on Numbers and Computation), only recently discovered in 1983 in the tomb of a Han dynasty nobleman in Hubei Province, China. We had the participation of established experts on the various ten classical texts, as well as younger scholars from both China and abroad who have also published analyses of these works. The symposium covered the major contributions Chinese mathematicians have made to the subject in ancient times, and discussed aspects of theory and applications.

Mathematical Practitioners and the Transformation of Natural Knowledge in early modern Europe

The sixteenth and seventeenth century have long been seen as fundamentally important to an understanding of the changing study of nature. Where once historians argued for a philosophical change, "from a closed world to an infinite universe", more recently historians of science see the intrinsic interconnections between scientific understanding and practice. Most important to the development of an understanding of nature is that group known as mathematical practitioners. This symposium emphasized the fact that mathematics and mathematical practitioners played an essential role in the transformation of science in this early modern period, by examining the role of mathematics and mathematical practice, utility, commerce and trade on the changing ideology and methodology of science. It shed light on the cross-cultural comparisons between the situation of mathematics in early modern England and other European countries and in the process placed the more traditional stories of the scientific revolution within this new and more complex narrative.

Multicultural Transmission of Mathematical Knowledge

This symposium shed light on problems concerning the transmission of mathematical knowledge, e.g. from Greek to Islamic cultures, from Islamic to medieval European cultures, from early modern European to East Asian cultures, and among East Asian countries.

Along the Silk Road: mathematical and astronomical exchanges between East and West in ancient and medieval times

Modern mathematics has its historical roots in diverse civilizations. Knowledge exchanges and the blending of cultures have been and will always be an important stimulus for the progress of mathematics and of science and technology in general. Encouraged by the spirit of the Silk Road, which was an historical cultural link between East and West, this session discussed related aspects and problems about mathematical and astronomical exchanges between European and Asian countries, as well as between Asian regions, in ancient and medieval times.

Speciation in Science: Historical-Philosophical Studies on the Emergence and Consolidation of Scientific Disciplines

The topic of scientific disciplines, their historical emergence, their differentiation, and their regroupings, has been intensively studied in recent decades, but usually from a sociological or purely historical perspective. This symposium considered cognitive and epistemic factors, their roles in the processes of "speciation" or discipline formation, and their interaction with other non-cognitive factors. This symposium offered an ideal opportunity to combine historical-sociological studies and philosophical analysis. The participants in the symposium have all devoted themselves to studying the historical development of the physical and mathematical sciences.

The associations of scientists in Europe: from the Republic of Letters to the national scientific societies (XVIIth-XIXth centuries)

During the 17th, 18th and 19th centuries, science takes on a more social aspect. Out of the *salons* of wealthy patrons grew a loose association of scientists throughout Europe who were in touch through correspondence, indifferent to national boundaries. These 'invisible' colleges of the Republic of Letters gave rise to scientific academies which themselves later took on a more national flavour.

In the 19th century, we see national academies withdraw from taking the leading role in the process of organisation of scientific research in favour of the newly created universities and higher technical schools. National research communities were also grouped around learned societies of all kinds, which, from the second half of the 19th century, regularly organise their own congresses. In doing so they became more international in outlook and this tendency was strengthened by the increased migration of scientists, caused both by political events and by local academic conditions. Thus, after three centuries of evolution and experimentation of various forms of institutionalisation, the scientific community returned again to its starting point, except that the Republic of Letters, cosmopolitan and uni-European, has now given way to an international scientific community founded on national academies, universities, schools, companies and congresses.

The symposium "Associations of scientists in Europe: Republic of Letters at the national learned societies (17th-18th-19th centuries)" explored the forms of emergence of the national scientific communities, using examples from various countries, by revealing their specific features, as well as the role of these communities in the emergence and the organisation of the international community which has become effective only in the 20th century. The organisers also discussed the nature of the influence of these communities on other social institutes and on society in general.

Science in Islam and its interactions with other Scientific Traditions

This symposium received papers concerning the interactions of Islamic scientific traditions with other scientific traditions (Latin, Hebrew).

Early Modern Mathematics

In this symposium there were contributions on various topics in the history of mathematics including: methods of practical geometry in France, differential equations of J. Riccati, Nicolas of Cusa, Arab commentaries on the Xth book of Euclid's Elements, medieval mathematical doctrines, numerical systems and mathematical development, the concept of zero, Leibniz, Sangaku of Mie Prefecture, western music and its relationship with mathematics and cosmology.

Modern Mathematics

The symposium had contributions on various topics: Fermat numbers, statistics in Spain, mathematical journals of the Lisbon Academy, Peter Sergescu (historian of Sciences from Romania), the Jordan canonical form, the 19th century Saint Petersburg Mathematical School, the teaching of analysis in Colombia.

Ancient Mathematics

The symposium considered various subjects in the history of mathematics: al-Tusi, Archimedes, Pythagoras' Theorem, perspective theory, linear congruence, indefinite equations in calendars in China.

For more details see the http://2005bj.ihns.ac.cn/pdf/ICHSAbstracts.pdf

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Work in progress

We encourage young researchers in fields related to *HPM* to send us a brief description of their work in progress or a brief description of their dissertation.

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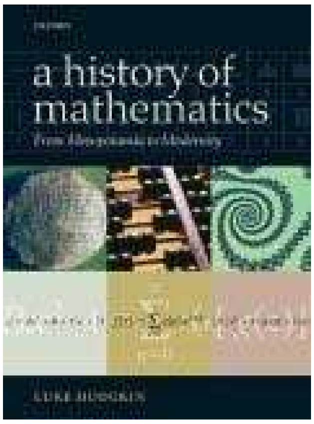
Book reviews



Hodgkin, Luke, A History of Mathematics: from Mesopotamia to Modernity, Oxford University Press, 2005 (281 pages)

For the English speaking world, the history of mathematics, as a subject of study for undergraduate students, has been served well by a number of standard texts, mostly from the USA. Among the first to be noted is the two-volume work by D. E. Smith of 1923, reprinted by Dover in 1958 and still in print today. Although there were other works on the history of mathematics, this was the first that was written specifically for students and teachers of mathematics. Smith's textbook has been followed by many others, again mainly written for the large American university and college market, the most recent and hugely successful being the comprehensive, detailed and carefully referenced textbook of V. Katz.

In the introduction to his first volume, Smith remarks that the subject "has come to be recognized as an important one in the preparation of teachers of mathematics and in the liberal education of students in colleges and high schools." For a variety of reasons (one could hope that it was with the intention of providing a liberal education) the number of mathematics courses that now include a unit on the history of mathematics has grown considerably over recent years. (The website of the British Society for the History of Mathematics lists twenty-seven such courses currently offered by British university mathematics departments, not counting courses for teachers and other institutions of higher education.) The market for textbooks is certainly there, but it could be said to be already well served, so a new text needs some justification.

Textbooks on the history of mathematics are usually written by mathematicians or teachers of mathematics, and not by historians. Hodgkin is certainly a mathematician but he is also well read in philosophy and his book is not like other textbooks on the history of mathematics. He devotes an introductory chapter to the purpose of the book and to a discussion of the wider issues of what might be involved in a study of the history of mathematics. For it is a dismal fact that whereas students are traditionally offered a rich diet of facts about mathematical ideas and techniques used at different times and in different places, they are rarely provided with information about the culture or power structure of the societies in which the mathematical activity takes place, and even less are they challenged to think about what may bring about changes to accepted perceptions or what might stimulate new enquiry.

History of mathematics courses, it has to be said, are not like 'normal' undergraduate history courses. Not only would history students be expected to relate a study of, say, the medieval field system, to its social and political context, but they would also be introduced to theories of history that serve to explain or interpret the past. Facts alone are not enough. Hodgkin attempts to remedy this lacuna in standard history of mathematics texts. He warns against the Whig historical perspective that sees the past only in terms of contributing to our (superior) understanding and equally warns against 'presentism', the tendency, perhaps at times unavoidable, to present the mathematics of the past in modern form. He raises the question of the extent to which mathematics is driven, on the one hand by 'internal' considerations, and on the other, by 'external' factors, as Marxists would claim. He describes the Kuhnian model of paradigm shift that might be used to explain 'revolutions' in mathematical thinking, and he warns against Eurocentrism. When he discusses particular topics—Greek geometry or the mathematics of Islam—he reminds the reader of theories of history as an aid to interpretation. (In the chapter on Islam, for example, we are not only introduced to developments in algebra but also asked to consider what were the factors that favoured mathematics under Islam. Did Islam differ from Christianity, for example, in the value it placed on knowledge? Or can we go even further to claim, as does Rashed, that the values of Islam are particularly favourable to science?)

It is not feasible for a history of mathematics of under 300 pages to be comprehensive and, like all teachers of history courses, Hodgkin makes a selection of topics. This selection will not suit everyone (there is nothing on Egypt and almost nothing on European eighteenth and nineteenth century mathematics), but unusually for such a book, consideration is given to the mathematics of the twentieth century and raises important questions about the status of mathematics in relation to physics. Each chapter can be taken as a more or less self-contained study unit. The book is meant to be a guide, a text to read alongside other material and Hodgkin devotes a section of each chapter to identifying where sources may be found and, importantly, recommends texts that are likely to be easily obtainable, for example the reader by Fauvel and Gray. Some extracts of original writing that are not easily available are appended to each chapter. The detail of the mathematics is not denied its place; each chapter contains exercises on the mathematics and solutions are given, although the level of expertise expected of the reader is uneven.

This is a serious book that demands intelligent reading. But it is immensely readable, in the way that facts-driven texts are not, and also witty (Nietzsche, Marx and Shakespeare are called on for support, but so are Chuck Berry and Tom Lehrer). And if the author offers a balanced interpretation of history in the main text, we are in no doubt about his sympathies, nor should we be, in his concluding remarks. Hodgkin's book is to be recommended to any student of the history of mathematics who wants to think seriously about the subject.

As a final remark, it is sad to say that the author has not been served well by the publisher. These are technical matters, but reading is not aided by a longer than usual line length, as well as narrow gutters so that the opened book does not easily reveal the text. The choice of font (Photina?) is also unfortunate and does not display mathematics well. This will not, I hope, deter readers and if the book sells well enough a second printing would be able to correct a number of minor typographical errors.

Chris Weeks

The views expressed in this section are the views of the reviewers and may not necessarily be those of the *HPM* Advisory Board.

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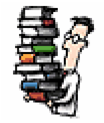

Have you read these?

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A brief summary in French and English:

Ouvrage collectif, sous la direction d'Évelyne Barbin et d'Anne Boyé, Centre François Viète de l'Université de Nantes.

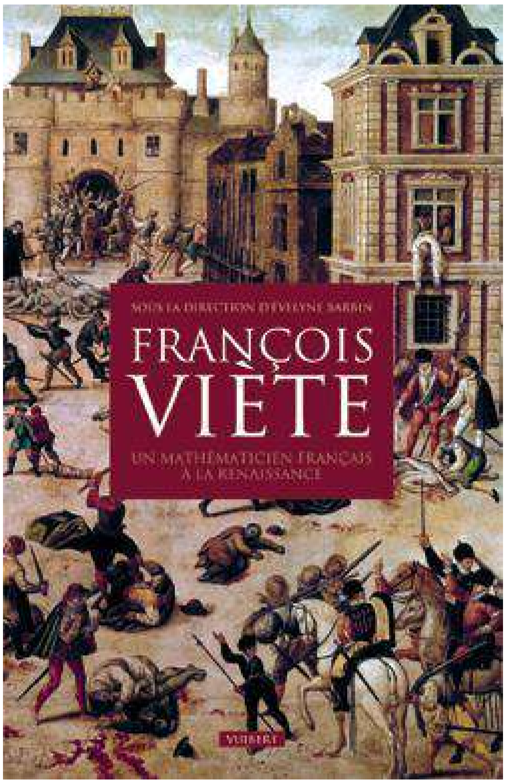
François Viète (1540-1603) aurait sans doute marqué plus profondément l'histoire des mathématiques s'il avait eu le temps d'éditer ses travaux et d'y accorder plus de loisir. On découvrira ici non seulement une vue complète de son œuvre mais aussi – et surtout – la carrière d'un homme politique dans une époque extrêmement troublée.

Savant très ambitieux, il produisit néanmoins son œuvre dans des conditions très particulières; aussi ne fut-elle que partiellement publiée de son vivant avant d'être poursuivie par quelques disciples puis oubliée en faveur des mathématiciens du XVIIème siècle qui succèdent à Viète, comme Descartes ou Fermat.

La première partie retrace sa vie. La deuxième concerne son algèbre: le but, les applications et l'héritage. La troisième est consacrée à des aspects de ses mathématiques qui n'ont pas toujours été retenus par l'histoire.

La dernière partie situe l'homme dans son époque. On y lira le récit d'un parcours complexe, non celui du seul cryptographe, mais celui de l'homme politique, du négociateur ou encore du conseiller au parlement de Rennes. Chargé par le roi de « missions spéciales » au plus haut niveau, François Viète se mêlera aux luttes qui opposeront catholiques et protestants et finira par s'y engager.

Cet ouvrage rétrospectif pose ainsi la question de l'ambition de l'homme politique. Un « politique » qui placerait les intérêts de la France avant ses propres convictions religieuses, au point que celles-ci restent aujourd'hui mystérieuses.



A collective work under the direction of Evelyne Barbin and Anne Boyé, Centre François Viète, University of Nantes.

François Viète (1540-1603) would doubtless have made a more profound mark on the history of mathematics if he had found the time to publish his works and given more attention to them. Our book provides the reader with, not only a complete account of Viète's mathematical work but also, importantly, an account of the career of a man of politics in a troubled time.

While Viète was a very ambitious *savant*, his work was produced in unusual circumstances. Furthermore his work was only partially published during his lifetime, before being taken up by some of his disciples, and then becoming eclipsed by the work of later 17th century mathematicians such as Descartes and Fermat.

The first part of our book is biographical and the second part deals with algebra, which concerns applications including inheritance. The third part deals with those topics of Viète's mathematics which history has not always remembered.

The final part of the book locates the man in his time. This is a convoluted tale, not only of a cryptographer, but also of the man of politics, the negotiator and a member of the Rennes *parlement.* Charged by the king with "special missions" at the highest level, Viète became embroiled in the struggles between the catholics and protestants and ended up by taking part in them himself.

This retrospective work thus poses questions about the ambition of a man of politics. A "politician" who placed the interests of France above his own religious convictions to the point that these remain unclear today.

Contents:

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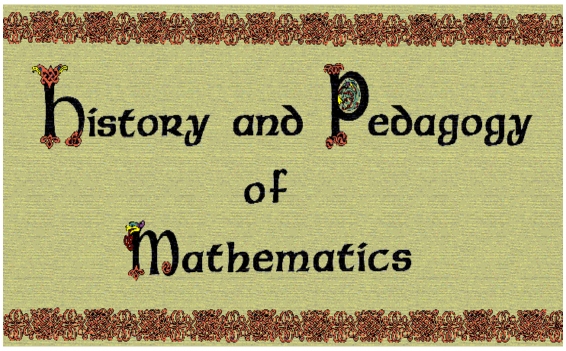
Societies and organisations

African Mathematical Union: Commission on the History of Mathematics in Africa (including newsletter)
http://www.math.buffalo.edu/mad/AMU/amuchma_online.html

Association des Professeurs de Mathematiques de l'Enseignement Public [APMEP] History site:
http://www.apmep.asso.fr/BMhist.html

British Society for the History of Mathematics [BSHM]
http://www.bshm.org

HOMSIGMAA - History of Mathematics Special Interest Group of the MAA
http://home.adelphi.edu/~bradley/HOMSIGMAA/



HPM Americas
http://www.hpm-americas.org/

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http://www.dm.unito.it/sism/indexeng.html

Association pour la Recherche en Didactique des Mathématiques:
http://www.ardm.asso.fr/

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http://www.cfem.asso.fr/

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http://www.univ-irem.fr/

Topics and Resources

Ethnomathematics on the Web

http://www.rpi.edu/%7Eeglash/isgem.dir/links.htm

About Medieval Arabic Numbers
http://www.geocities.com/rmlyra/Numbers.html
http://www.geocities.com/rmlyra/arabic.html

Annotated Bibliography on Proof in Mathematics Education
http://fcis.oise.utoronto.ca/~ghanna/educationabstracts.html

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http://www.divulgamat.net/index.asp

History of Statistics
http://www.stat.ucla.edu/history/

Images of Lobachevsky's context
http://www.ksu.ru/eng/museum/page0.htm

Images of Mathematicians on Postage Stamps
http://members.tripod.com/jeff560/index.html

Numdam-Digitization of ancient mathematics documents
http://www.numdam.org/en/ressnum.php

Convergence: an online magazine of the MAA providing resources to teach mathematics through its history
http://convergence.mathdl.org/

Documents for the History of the teaching of mathematics in Italy
http://www.dm.unito.it/mathesis/documents.html

Ethnomathematics Digital Library
http://www.ethnomath.org/

Some Japanese Mathematical Landscapes*:* The results of wandering in a beautiful country, with a mathematical eye, aided by a digital camera, by A. Arcavi http://math.criced.tsukuba.ac.jp/museum/arcavi/arcavi_english/index.html

The editors welcome information about other sites.

* * *

Notices

Problématique: On the Concept of Beauty inherent to Mathematics and Physics

What does it mean to say that mathematics or physics are beautiful? More specifically, what is the meaning of expressions like "beautiful theory", "nice theorem", "elegant proof" and so on? Such expressions have often been used to characterize theories and results both in mathematics and physics. If such expressions have a meaning at all, then any aesthetic element could constitute a motivation for studying these disciplines! Talking about beauty I am referring to *the intrinsic aesthetic content of mathematics and physics*; in particular, those *elements of research in these disciplines that give the flavour of beauty to their results*.

That beauty has always been considered an essential ingredient of research in mathematics and physics is reflected in the writings of many great philosophers, mathematicians and physicists. For instance:

- Discussing solid geometry, Plato remarks, "...because of its *exceptional beauty* [its study] presents great interest" (*Republic* Z 528).
- H. Poincaré notices, "the scientist does not study nature because it is useful to do so. He studies it because he takes *pleasure* in it and he takes pleasure in it because it is *beautiful*" (Poincaré 1914/1996, p.22; (my emphasis).
- G. Hardy writes in his "Apology" that the "...mathematician's patterns, like the painter's or the poet's *must be beautiful*. The ideas, like the colours or the words must fit together in a harmonious way. *Beauty is the first test*" (Hardy 1969, p.84-85: (my emphasis).
- Along the same lines, E. Wigner remarks that "if a mathematician is asked to justify his interest in complex numbers, he will point … to the many *beautiful* theorems in the theory of equations, of power series and of analytic functions in general, which owe their origin to the introduction of complex numbers" (Wigner 1967, p.225)
- Referring to his work, H. Weyl says that "my work always tried to unite the true with the beautiful; but when I had to choose one or the other I usually chose the *beautiful*" (quoted in Chandrasekhar 1987, p.65).
- Similarly, P. Dirac suggests "The research worker, in his efforts to express the fundamental laws of Nature in mathematical form, should strive mainly for *mathematical beauty*. He should take simplicity into consideration … It often happens that the requirements of simplicity and beauty are the same, but *when they clash the latter must take precedence*" (quoted in Kragh 1990, p.277, my emphasis).
- S. Chandrasekhar goes further, claiming that "*what is intelligible is also beautiful*' (Chandrasekhar 1987, p.66, my emphasis).
- Finally, R. Penrose has remarked that "… It is hard to doubt the remarkable role that aesthetic judgements play, both mathematical and physical, in making decisions as to the most fruitful lines to follow in research into theoretical physics" (Penrose 2004, p.1040).

This is just a small sample of assertions, suggesting the existence of an aesthetic aspect inherent to both mathematics and physics that seems to be valuable and important for the progress of these disciplines. The list of these assertions could be considerably extended and there have been several authors, who have tried to make this aesthetic aspect more explicit (e.g. Poincaré 1914/1996, Chandrasekhar 1987, Huntley 1970, Wechsler 1978, Curtin 1980, Penrose 2004 §34.9; see also Kragh 1990 ch.14, Schweiger 1989, 1994, Tzanakis 1997). Thus, the question naturally arises of what exactly constitutes the aesthetic content of mathematics and physics. This aesthetic appeal occasionally appears as a sufficient reason for studying a scientific domain, or as a source of enjoyment and pleasure, or as a principal criterion for judging the value of a scientific result, or even as a concept essentially equivalent to truth!

There appear to be the following six relevant points that constitute the essential ingredients of this aesthetic content:

(a) C*onceptual clarity* in the foundations and the development of a theory. This concerns pure ideals, mainly in mathematics, a typical example being Euclidean geometry. However, it is often also a requirement in physics; Einstein's Special Theory of Relativity is such an example, especially after Minkowski revealed its deep geometrical content.

(b) *Elegant and concise reasoning*. This is an element often explicitly demanded in mathematics, in particular for proofs of mathematical statements (e.g. various proofs of the infinity of primes, or of the irrationality of $\sqrt{2}$). However, it is often also considered to be an important criterion in physics, so as to encourage a positive response towards a particular result or theory. A typical example is Einstein's 1917 mathematically almost trivial derivation of Planck's blackbody radiation law from first principles; this was a most important result, since all derivations up to then were based on logically contradictory ideas, or on heuristic claims which were not always conceptually clear.

(c) *Unification* of a priori unrelated concepts, methods, theories or phenomena, into a more general setting. For instance, abstract algebraic concepts and theories emerged in this way; and example is the emergence of the concept and theory of linear spaces (Dorier 2000, Part I). This is also a most basic feature of "beautiful" physical theories; just think of the unification of physical phenomena that resulted from Newtonian mechanics, as compared to Aristotelian physics with its clumsy geocentric model of the universe.

(d) The "*naturalness*" of a theory, in the sense of the unforced and unexpected explanation or description of formerly incomprehensible facts, or the inference of new ones. This is a basic characteristic of physical theories and models that are considered to be beautiful and elegant, but it also appears in mathematics. It means that a theory is capable of explaining particular but formerly unintelligible facts, or can be used to make specific predictions of new ones that were not originally in the mind of the founders of the theory, and whose starting point may be quite independent of these facts (the term *naturalness* has been used in a somewhat different sense by Einstein; Einstein 1949, p.23). Einstein's General Relativity Theory is a most characteristic example of this kind; it was conceived as extending the geometrical ideas of the Special Theory to include accelerated systems and implied, without any conceptual bias, the most unexpected consequences that either were unexplained in the context of Newtonian Physics, or were totally unimaginable in that context! Similarly, the Theory of Functions starts as a straightforward generalization of differentiable real-valued functions, but leads to many elegant and most unexpected consequences (e.g. Cauchy's theorem, existence of derivatives of all orders, uniqueness of the analytic continuation etc).

(e) *Symmetry*, not as an object of study, but as a guiding pattern of reasoning. This is equally important both in mathematics and physics. For example F. Klein's *Erlangen Program* is a typical example in mathematics. Maxwell's introduction of an additional term in *Ampère's Law* so that in the laws of electrodynamics the electric and magnetic fields play a completely symmetric role, led to the establishment of classical electrodynamics and the prediction of the existence of electromagnetic waves.

(f) *Analogy*, as a basic mode of thinking (distinct from deductive, or inductive reasoning), either in the sense of strict similarity (isomorphism) of structures or of loose similarity of this kind, with which objects of an *a priori different* nature are equipped. It is a basic mechanism for heuristically formulating conjectures, motivating the introduction of new concepts and establishing new methodological frameworks (Polya 1954, Tzanakis & Kourkoulos 2000, §2.1.3). Hamilton's search for his quaternions was motivated by the relation between complex numbers and plane rotations; he looked for generalized complex numbers that bear a similar relation to space rotations (Tzanakis 1995)! In physics there are many such important examples; e.g. Schrödinger's formulation of wave mechanics and his equation were motivated by the analogy between classical mechanics and geometrical optics (Tzanakis 1998).

Further analysis of these six elements, together with detail of the supporting examples given above, are given in Tzanakis (1997). It appears that any body of knowledge in mathematics or physics, which incorporates any of these elements, possesses a nontrivial aesthetic content.

Question: Nevertheless, there remains a deeper study of the nature of the above six elements, through elaborating more examples, and a further examination as to whether other distinct, but complementary, elements can be identified as ingredients of the aesthetic aspect inherent to mathematics and physics.

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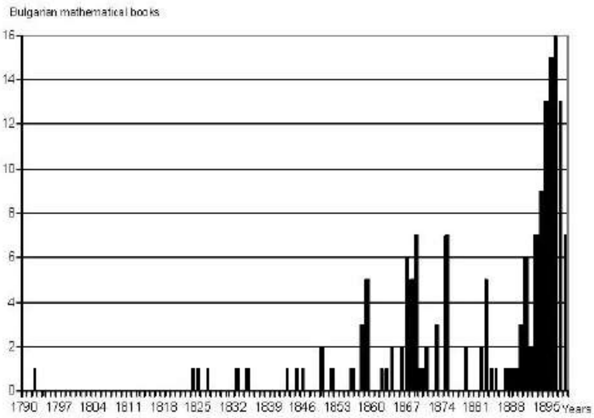
Bulgarian Mathematical Culture in the 19th century

Bulgaria adopted Western mathematical culture later than the other Balkan countries, being further away and also because it was part of the Ottoman Empire until 1878. Mathematical knowledge began to increase, with the support of the church. The Bulgarian Church had secured the inheritance of religion, language, script, the enlightenment and mathematical culture in the Middle Ages and also led developments in the nineteenth century, in contrast to the disruption between church and secular science in the West.

Up to the eighteenth century the only schools in Bulgaria were church schools existed. They provided some mathematical education and used a unique method for writing numbers, based on the Cyrillic alphabet. Modern mathematics entered Bulgaria through Greek schools and literature (from end of the 18th to the beginning of the 19th century) and through the influence of Russia (after the 1830s) and Western Europe (after the Crimean war).

More than 165 Bulgarian mathematics books were printed in the nineteenth century. These were mainly primary, secondary and university textbooks. Few of them were reference books or scientific publications. Some teachers (68), five university lecturers (A. Shourek, S. Ganev, A. Tinterov, M. Bachevarov, and E. Ivanov), four members of Bulgarian Academy of Sciences (I. Gjuzelev,



E. Ivanov, V. Kirilov and G. Kirkov) and two publishers (H. G. Danov and D. V. Manchov) were the authors. Textbooks in mathematics provided practical, economical and geodetic instructions and descriptions of apparatus and instruments for calculation, measurement and drawing. There were no vocational schools at that time. Before the Liberation of 1878, it was the textbooks that provided the syllabus and some guidance on teaching method. After the Liberation, however, there were only small changes to the mathematical syllabus.

The number of Bulgarian mathematics textbooks increased in the nineteenth century. The level of mathematical content was elementary up until 1824. Primary school mathematics texts were published before the Crimean War (1824-1846) and secondary mathematics texts began to appear in the period 1846-1878. Academic mathematical culture began to make progress in Bulgaria following the Liberation.

Elementary education in mathematics (1792-1824)

At the beginning of the nineteenth century in Bulgaria secular schools only taught reading, writing and arithmetic. Arithmetic was found in the *ABC book*, published by Marko Teodorovich in 1792, and the *ABC book with various teachings*, written by Pierre Beron in 1824. In both books this was only nine pages. Pierre Beron introduced Arabic numerals and the operations of addition, subtraction, multiplication and division. A manuscript arithmetic textbook, written around 1822-1828, is preserved in the museum in the town of Vratsa.

Primary education in mathematics (1824-1846)

Situated on the river Danube, the town Svishtov became centre of primary mathematics education. Emanuil Vaskidovich (1795-1875) opened the first Bulgarian school with two levels (elementary and primary) in 1815. The school's chronicle dating back to 1824 is still preserved. The curriculum included arithmetic, algebra, physics, history, political economy, geography, anatomy and the Greek language. Emanuil Vaskidovich had studied in the Greek school of his native town Melnik, then in a school on the island of Hios, and at Bucharest Bay academy. A second primary school was later established, also in Svishtov, by Hrisant Georgiev Pavlovich (1804-1848) in 1831. The curriculum included arithmetic, Bulgarian history, geography, logic, rhetoric, Slavonic and Bulgarian languages and grammar, catechism and God's sermons. Born in

Dupnitsa, Pavlovich studied in Rila monastery and in the Greek school of Melnik and in Sjar. Svishtov schoolmasters were responsible for the creation of a mathematical literature in Bulgaria.

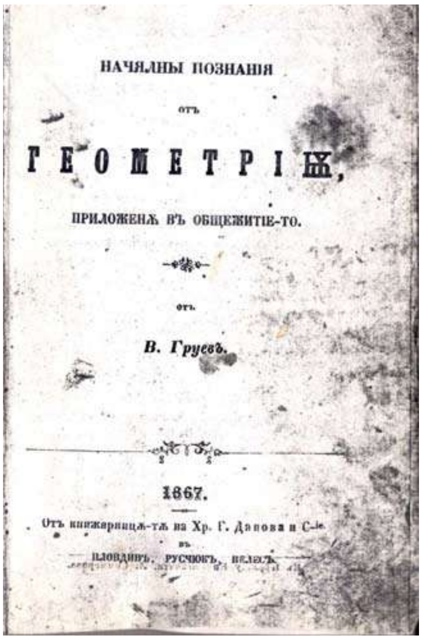
Hrisant Pavlovich wrote the first Bulgarian language mathematics book (*Arithmetic or science calculating*), published in 1833 (see illustration). It contained 118 pages and presented new mathematical procedures for fraction and ratio. There are also a few solved mathematical problems in it. The content of the book corresponded to the mathematics curriculum for the first and second classes in 1890 (fifth and sixth classes according to today's classification).


АРІѲМЕТІКА
НАУКА
ЧИСЛИТЕЛНА
ХРИСАНѲА ПАВЛОВИЧА,
Сега перво издана и, помощь земан- от любоученны тр единородны.
ХРИСТОДУЛ КОСТОВИЧ
В БЕЛГРАД,
1833.

Neofit Hilendarski and Emanuil Vaskidovich wrote the next textbook "*Guidance in Arithmetic*" (1835) which also included the theorem of Pythagoras, calculating interest, and the terms multiplicand, multiplier, product, dividend, divisor, as well as the use of letters as mathematical symbols. Neofit Hilendarski Bozvely (1785-1848) was born in Kotel. He studied in Hilendar monastery and taught in Svishtov from 1814.

A new school organization with three or four classes above the elementary level was created in 1835 (fifth, sixth, seventh or eighth classes according to today's classification). Abstract mathematical concepts were introduced in the textbooks as well as traditional practical knowledge.

The first textbook, corresponding to the mathematics programme for the third class in 1890 (seventh class according to today's classification), was *Arithmetic* (343 pages), written by Hristodul Sichan-Nikolov in 1845. Powers, square root and cube root were introduced for the first time, together with their symbols. Hristodul Sichan-Nikolov (1808-1889) was born in Samokov. He had also studied in Rila monastery, and later in schools in Austria and Bucharest.


НАЧЯЛНЫ ПОЗНАНІЯ
отъ
ГЕОМЕТРІѪ,
ПРИЛОЖЕНѪ ВЪ ОБЩЕЖИТІЕ-ТО.
отъ
В. Груевъ.
1867.
Отъ книжарницѫ-тѫ на Хр. Г. Данова и С-іе
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ПЛОВДИВЪ, РУСЧЮКЪ, ВЕЛЕСЪ.

Geometry was introduced as a new primary subject at this time. There were elements of geometry in a textbook by D. V. Manchov (1862) which contained chapters on grammar, geography, natural history, arithmetic, geometry and physics. The first textbook devoted entirely to geometry was by V. Gruev (1867) published in Vienna, containing 72 pages (see illustration). It dealt with the line, angle, triangle, polygon, circle, radius, diameter, chord, segment, surface and capacity, as well as describing geometrical instruments. The content of the book corresponded to the syllabus for geometry of the first, second and third classes in 1890 (fifth, sixth and seventh classes according to today's classification).

Secondary education in mathematics

(1846-1878)

Bulgarian secondary mathematics education began with the school founded by Nayden Gerov (1823-1900). Gerov was born in Koprivshtica and graduated at the Richelieu Lyceum. He set up the first Bulgarian normal school in Koprivshtica (1846), and then decided to site it in Plovdiv (1850) as the central regional school "St. Kiril and Methodi". Initially, the school had only higher fourth, fifth and sixth classes (eighth, ninth and tenth classes according today classification). Nayden Gerov himself taught high school physics for three years and he wrote a textbook in physics. The first part of the physics textbook was printed in 1849 and the manuscript of second part can be found in the National Library in Sofia. It for this reason that we can assume that higher education in mathematics had been taught from 1846. The archive of Nayden Gerov school contains documents of examinations in algebra, geometry, physics and French (1846-1849). Emanuil Vaskidovich, mentioned above, taught algebra in Pleven at the same time (1848).

The first high school textbook in mathematics, containing proofs, was the popular Greek study guide "*Elemental arithmetic*", written by Atanasios Gerakis and translated into Bulgarian by Sava Radulov in 1843. New mathematical items now appeared in Bulgarian included some algebraic concepts. For example, mathematical identities with letter symbols were introduced in the book, including the quadratic identity $[(a + b)^2 = a^2 + 2ab + b^2]$, the cubic identity $[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3]$ and their proofs. Powers, square roots, cube roots, surds and arithmetic and geometric progressions also appeared. Sava Iliev Radulov (1817-1887) was born in Panagurishte. He had studied in Sveta Gora monastery, a Greek school in Smirna (Izmir), and the Richelieu Lyceum in Odessa.

The first Bulgarian algebra textbook (*First algebraic lessons)* (149 pages) was a translation from the French by Hristo Vaklidov in 1859. It contained descriptions of algebraic expressions, algebraic inequalities, raised to the second power and square roots, linear equations with three unknowns, quadratic equations in two unknowns and 103 solved mathematical problems. Hristo Vaklidov (1841-1891) was born in Kazanlak. He studied at the French college in Tsarigrad (Istanbul) and typography in Odrin (Edirne).

In 1871, Nestor Markov wrote a textbook (*Geometry, rectilinear trigonometry and statics*) based on his translation of a French high school geometry text by J. Ph. Olivie. Markov added the sections on statics and trigonometry, and this is the first occurrence of trigonometry in a Bulgarian text (the names of the trigonometrical ratios were given into Cyrillic only). The textbook contained proofs.

Ivan Gjuzelev (1844-1916) (see image) was the most eminent Bulgarian mathematician in the nineteenth century. He was born in Gabrovo and obtained university mathematical education in Odessa. He later became a member of the Bulgarian Academy of Sciences. He wrote a textbook in geometry (*Short elementary geometry by A. Davidov*) in 1873. Gjuzelev printed the 165 page textbook in Prague with 177 diagrams. The geometrical theorems were chosen according to high school syllabus in physics. There were two parts (plane geometry and solid geometry). The plane geometry part comprised lines, angles, geometrical figures, parallel lines, proportional lines, similar triangles and polygons, figure surfaces measurement. The solid geometry part comprised lines, planes and angles in space, tetrahedron, hexahedron, octahedron, dodecahedron, icosahedron, capacities measurement, cylinders, cones, spheres. The textbook contained many proved theorems and 100 solved problems.

Academic mathematical culture (1878-1899)

Higher education in mathematics began to develop under West European influence and the role of Russia in this sphere was very small. The first Bulgarian High School was founded in Sofia in 1888 and gradually evolved into a university, similar to the West-European ones. The faculty of Physics and Mathematics was founded in 1889. Almost all the professors who taught mathematics at Sofia University up until the end of nineteenth century were German and French graduates. Emanuil Ivanov (1857-1925) was born in Svishtov. He graduated from Munich Polytechnics. Anton Shourek (1857-1926) was born abroad and studied in Vienna. Atanas Tinterov (1856-1927) was born in Kalofer. He obtained his mathematical university education in Odessa. Spiridon Ganev (1869-1961) was born in Sliven. He studied at the University in Prague and defended his PhD in mathematics (1893). At Sofia University, from 1889, they taught projective geometry, descriptive geometry (*géométrie descriptive*), analytical geometry, advanced geometry (*géométrie supérieure*), differential calculus, integral calculus, algebraic analysis, advanced analysis (*analyse supérieure*), foundations of advanced mathematics (*principles de la mathématique supérieure*), high algebra (*algèbre supérieure*), calculus of probability.

Scientific research in mathematics

Mathematical methods were used in PhD theses in physics (1842). A paper (*De Psychrometria*), written in Latin by Demetrius Mutieff (1818-1864), was defended in Frederick William University in Berlin (Humboldt University). Ivan Salabashev (1853-1924) announced a solution to a mathematical problem (*About curves, described by the vertices of a moving triangle*) at the session of the Czech Academy of Sciences in 5 March 1875. Salabashev was born in Stara Zagora and studied in Prague Polytechnic. The second Bulgarian PhD mathematician was Ivan Dedov who obtained his doctorate from Zurich University (1896).

Mathematicians became members of the Bulgarian Academy of Sciences, including Ivan Gjuzelev (1875), George J. Kirkov (1881), Vasil K. Vasilev (1881), and Emanuil Ivanov (1898). George Jakovliev Kirkov (1848-1929) was born in Pleven. He obtained his mathematical education at Odessa University and studied typography in Austria and Germany. He wrote a textbook in mathematic (1869). Vasil Kirilov Vasilev (1849-1923) was born in Bolgrad, a town in Bassarabia. He also obtained his mathematical education at Odessa. After the Liberation he became professor at the Military school in Sofia (1878-1907). He wrote the textbook in *Algebra* (1879). The Bulgarian Society of Physics and Mathematics was created in 1898.

The author wishes to thank Professor Ivan Ganchev for providing copies of mathematical textbooks from the nineteenth century and Dr. Veselin Krastev for corrections of English text.

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Announcements of events

History of Mathematics, History of Mathematics Education and their didactical implications

April 14-15, 2006
Thessaloniki Greece
A regional meeting of the HPM Group: 14 & 15 April 2006, Department of mathematics, University of Thessaloniki.
This is a meeting of the *HPM* community in Greece. There will be 13 oral presentations divided into three main sessions:
- History of ancient Greek Mathematics
- History of Mathematics Education
- Didactical approaches to the History of Mathematics

All presentations will be in Greek. They will be included in the proceedings, which will appear shortly after the meeting.

The texts are already on the web at http://users.auth.gr/~nioka/Files/sinthesi_ergasion.doc
A short report will be included in the next issue of the *HPM Newsletter*.

History in Mathematics Education

April 16-18,2006
Loughborough
Reflections on Classroom Practice.
Organised by the British Society for the History of Mathematics.
Contact: Martin Perkins
martin@perkinstowers.freeserve.co.uk

Espace Mathématique Francophone : Colloque EMF 2006

May 26-31, 2006
Québec, Canada
http://emf2006.educ.usherbrooke.ca/

3rd International Conference on the Teaching of Mathematics (ICTM-3)

June 30-July 6, 2006
Istanbul, Turkey
Following on the success of earlier conferences held in Samos, Greece (1998) and Crete, Greece (2002), the 2006 conference intends to focus on "new ways of teaching undergraduate mathematics". The conference will be co-sponsored by the MAA. For more information, including information on how to submit a paper, see http://www.tmd.org.tr/ictm3.

International Leibniz Congress - Unity in Plurality

July 24-29, 2006
Hannover, Germany
http://www.gwlb.de/Leibniz/Gesellschaft/Veranstaltungen/Kongress/Circular/

International Symposium on "Policy and Practice in Mathematics and Science Teaching and Learning in the Elementary Grades"

November 8-10, 2006
Beirut, Lebanon
Mathematics Education into the 21st Century Project. For further information contact arogerson@vsg.edu.au

5th European Summer University on the History and Epistemology in mathematics education (ESU-5)

July 19-24, 2007
Prague, Czech Republic
The original deadline for submitting an abstract for all types of activities, which was March 31, 2006, has been extended to **May 15, 2006**.
For more information and the details for the submission procedure, see the HPM Newsletter issues No58, 60 or the ESU-5 website http://www.pedf.cuni.cz/kmdm/esu5.

Mathematics Education In A Global Community (9th International Conference of The Mathematics Education into the 21st Century Project)

September 7 - 13, 2007
Charlotte, North Carolina, USA
The project is dedicated to the planning, writing and disseminating of innovative ideas and materials in Mathematics and Statistics Education. Papers are invited on all innovative aspects of mathematics education. For further information contact arogerson@vsg.edu.au

ICME-11
July 6-13, 2008
Monterrey, Mexico

Models in Developing Mathematics Education *(10th International Conference of The Mathematics Education into the 21st Century Project)*
September 12-18, 2008
Dresden, Germany
For further information contact
arogerson@vsg.edu.au

Please note that the webpage http://www.mathedu-jp.org/hpm/index.htm is currently not updated due to technical reasons. Please use http://www.clab.edc.uoc.gr/hpm/ for the time being.

A note from the Editors

The Newsletter of HPM is primarily a tool for passing on information about forthcoming events, recent activities and publications, and current work and research in the broad field of history and pedagogy of mathematics. The Newsletter also publishes brief articles which they think may be of interest. Contributions from readers are welcome on the understanding that they may be shortened and edited to suit the compass of this publication.

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Table of contents

Items for the Newsletter should be sent to the editors, preferably by email (see addresses below).

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