International Study Group on the Relations Between
the HISTORY and PEDAGOGY of MATHEMATICS
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Interview: Leo Rogers – Historian and Pedagogue of Mathematics, the first editor of the HPM Newsletter

I found Leo in Oxford during the British Society for the History of Mathematics two-day conference on Mathematics Textbooks: History, Production and Influence. I wanted to find out who came up with the idea of setting up HPM and how he came to be the first Editor of the HPM Newsletter.

Leo: It was in a conversation with David Wheeler who was on the programme committee of ICME in 1972, which was to be held in Exeter that the idea first came about. He was my supervisor at the time, and I was writing a thesis on 'The History of Mathematics and its Implications for Teaching'. When I knew David was on the programme committee, I approached him about starting up a working group about the History and Pedagogy of Mathematics. He encouraged me and showed me how to make an official proposal to ICME and that was the beginning of HPM. Once the proposal had been accepted, it was advertised, and I found that people from all over the world wanted to join by presenting a paper.

I was very concerned that nobody would know me, as I was a young lecturer in my first teacher training job, so I asked Philip Jones if he would be the chair of the meeting of the HPM group. Philip had written the introduction to the NCTM 39thYearbook in 1969, called, *Historical Topics for the Mathematics Classroom*.

How did the organisation go – both of the meeting and of the first Newsletter?

Leo: I was at the time at the Roehampton Institute of Higher Education (now Roehampton University). I produced the first newsletter, which were really notes about who made contributions to the HPM sessions, the programme organisation and the summary of the presentations at the end of the meetings.

This was submitted to the programme committee at the end of ICME, and this became the first newsletter.

The next ICME was due to happen in Karlsruhe in 1976, and Roland Stowasser first became my collaborator and eventually the chairperson of HPM.

There was a big gap between the first few newsletters – but I carried on trying to do the newsletter over the first 4 years. From Karlsruhe it became more organised and a bit more formal and I really faded out in terms of the basic organisation, handing over to other people who were around about that time. It was the period when I was finishing my dissertation so I was preoccupied with that. Then in 1975 I had a Fellowship at the University of Warwick, which took me out of the organisation and writing other things for a time being.

How many people were contributing on a more permanent basis and to how many people was the newsletter sent?

Leo: The newsletter was obviously then distributed by mail. There weren't a lot of people involved at the time – probably no more than a dozen. A lot of the people that we have now in the history of maths came into it later. Of course there was a little group in England at the time already, which was my nucleus, who were also members of the British Society for the History of Mathematics. BSHM already existed, but it hadn't been going for very long. A number of things came together in those few years.

In your opinion has something changed since that time in the way the history of mathematics is perceived?

Leo: Originally, in the early days, there was a clear distinction between mathematicians who were historians and the teachers who were interested in the history. There was the attitude that if you want to be involved in the history of mathematics you have to do it properly, you have to be involved in serious research. Teachers, on the other hand, who were interested in using history in the classroom, clearly did not have the time or the expertise to engage in research, but had to rely on the 'history books' that were available at the time, and not all of them were of good quality.

It was all part of what I personally call the business of 'academic snobbery', that mathematicians don't see mathematics education as a proper subject. There were struggles, and a lot of effort by mathematicians who were involved in teaching, to persuade other colleagues. There wasn't a complete division, and certainly there were a number of mathematicians in the universities who were very passionate about mathematics teaching. But gradually I think the divisions, the borders, became fuzzier and some kind of coherence came through. Come to think of it, in my opinion a change in mood began about the time that John Fauvel became involved in HPM. In the BSHM, I think a significant development was the idea of the 'Research in progress' days, where encouragement is given to young researchers (many of whom are teachers) to talk about their work.

Do you think there is still this tension between the historians of mathematics and the mathematics educators?

Leo: There still is this attitude: 'If you are going to use the History of Mathematics, for goodness sake get it right!' And I do think it is important – it's something which one can worry about quite a lot. I mean, what are the available sources and what is there for teachers? There is still little of sensible, reliable material which is written with teachers in mind. Another aspect of this, certainly the way it is in England now, is that teachers just don't have the time to look at the things.

How would young or trainee teachers find out more about how to use the history of mathematics?

Leo: When I talk to teachers, they are always fascinated by the ideas, the methods that you can talk about, all sorts of things about the history, but it is a big problem for them to see how to use this in the classroom. Leaving aside the problem of time, one of the important things about teaching mathematics is about being able to link different areas of mathematics, so that students see maths as a whole, rather than as a series of topics. The problem as I see it, is the way mathematics is presented in the curriculum – you 'do' a topic one week and then you move on. Also, there are so many demands on the time of mathematical educators, and on their research time. There are so many aspects of education that teacher trainers and teachers have to attend to, so history is really a very small part of this vast array of things.

On the other hand, there ought to be a way of being able to write some material which gets to maths educators to at least make them aware that history has a very important part to play, not just for the purposes of curiosity. My contention is, that what has not been taken seriously, is that history helps people to develop deeper interest into people, the circumstances, into the 'why' something happens.

Going back to your mention of the limitations of time in teacher training, what do you think the answer is?

Leo: When you think of what maths education is, you have to attend to an enormous amount of material – the learning theory, philosophy of education, sociology, anthropology, all these aspects are being brought into maths education, so maths education as such has become such a very wide subject. Quite properly then, as with any academic subject, each of us would like to have a particular specialisation, something that we are very interested in, so there are few of us that are specialising in history.

Another aspect of doing the history of mathematics is the popular conception of history – learning facts about dates and names, kings and queens of England etc. I don't know how this is perceived in other countries – and I do know that English teachers don't still teach like this – but we still have a lot to do to make the history of mathematics dramatic, making sure in the process that the drama and the excitement we are talking about has to do with mathematics and not just something to make it interesting to pupils.

How important do you think is the role of teachers of mathematics in the history of mathematics?

Leo: The main interest for the history of mathematics has been the mathematicians themselves and there has not been much research into mathematics teachers as historical figures in their own right. What was their position in promulgating new ideas? My real interest is what happened outside the universities. Certainly the contributions of the famous mathematicians need to be recognised and are outstanding, but what is often not talked about is the way in which these changes, these ideas, these views, got into the schools and the colleges of the times.

What would you like to see change in current mathematical education?

Leo: In school, very little, if anything, has ever been done on projective geometry, and also perspective, which is very important culturally. I think there is another thing going on here – pretty universal with mathematics curriculum organisers (who are, let's face it, civil servants) – that they see mathematics as a tool, and so not enough attention is paid to develop a curriculum which talks about mathematics as a cultural activity. What is the contribution that mathematics makes to the culture? That is not visible in the current mathematical curriculum in England.

Do you think the trainee teachers are aware of the cultural aspect of mathematics?

Leo: Probably not. The way that various training programmes of teachers are organised ends up with the primary teachers having so much to manage that they really don't have much time for anything like the mathematical contribution to culture. There is an exception to this rule in England, and there certainly existed a positive movement by a lot of teachers' centres recognising that there was mathematics from the other parts of the world, and that the contribution of ethnic minorities was important. This was partly motivated by large groups of people from our former 'empire' coming to England. But much of it was seen as a curiosity, rather than a serious effort to talk about the cultural contexts.

In secondary schools again, the time that we have to talk to trainee teachers is so limited, that you can really only do the essential things. Things like history seem like a nice luxury. It is a matter of trying to find a medium (and I'm not necessarily thinking of purely technical solutions) – but material at the right sort of level, or a good way of motivating – that gives the teachers a reason for becoming involved in some aspect of the history of mathematics. It might be nice to have a series of essays on particular aspects of mathematics, tracing the development of an idea which is easily accessible for secondary teachers.

Are you maybe doing something about it?

Leo: I am working on something, which at the moment has a working title 'The Quads paper'. It is a general idea that is forming in my mind about quadratics, about squares, but it starts from activities which can be introduced into any classroom at all sorts of levels, and which end up linking certain 'standard' parts of the mathematics curriculum

This idea is coming out of what Ivor [Grattan-Guinness] referred to as heritage, rather than history. It is really talking about the heritage of mathematics, but what I am trying to write is motivated by my knowledge of the history of mathematics. Trying to make a conjunction, a combination, or even almost a sketching out a series of lessons which might do this is the aim!

What do you think about the monumental work that came out under the auspices of the MAA in the form of Modules for the Teaching and Learning of Mathematics this year?

Leo: That style is not, in my mind, appropriate for the use of the secondary teachers in an English context. Here, people would no doubt find much of it interesting and informative, but you'd have to work at it to adapt it for use in English classrooms. Other countries may find it very useful because their system is different. Similarly, there is an enormous amount of material that has been created by the French, but it is not the kind of stuff that you can use in the English classroom easily. It is erudite, interesting, and I use it myself in all sorts of ways, but again, it doesn't quite fit… I think that what we have to do here in England is try and write something which connects mathematics with some of the very simple ideas that everybody needs to know about. At the same time we have to provide something interesting enough to motivate students to investigate independently and also to provide some insights into the importance of mathematics in the cultural context.

Snezana Lawrence

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Classroom practice

In HPM Newsletter 58 Francesca Bavilacqua from Italy reported on the work she had done on Babylonian mathematics, with the co-operation of her colleague who taught history. Here is another example of cross-curricular work in the classroom, this time from Greece.

The Role of the History of Mathematics in Cross-Curricular Teaching – using Euclidean Geometry in the 1st year of the Greek Lyceum

Introduction

The idea of this cross-curricular project was inspired by the history of mathematics. To be more specific, it came from the cross-curricular activities developed in the French IREM several years ago linking the teaching of mathematics with its history through the study of original sources.

In our school, the Experimental School of the University of Macedonia in Thessaloniki, we had the opportunity to exchange ideas among teachers of different subjects. A cross-curricular project arose from these discussions on Euclidean Geometry, Ancient Greek Language and History. The research project was approved by the Greek Pedagogical Institute that supports attempts to introduce cross-curricular activities in statuary education. This is in contrast with the Greek official curriculum of secondary education, which is characterised by a strong departmentalisation and the absence of links between different subjects.

The research

The project was carried out in two classes of the 1st year of Lyceum (15-16 year old students). Students study worksheets, which contain the ancient Greek text of selected geometric propositions of Euclid's *Element*s, as well as, certain *scholia* on these propositions written by Proclus. In his *scholia* Proclus includes a lot of the criticism that some ancient Greek philosophers had raised against Euclid. The worksheets were the result of collaboration between mathematics and Greek language teachers who participated in the project. During the cross-curricular courses, the students had to analyse the ancient texts from a mathematical, literal and historical point of view, guided by their teachers who taught the corresponding school subjects and who were also members of the research group.

The theorems of these texts were compared with the corresponding theorems of the modern textbook of Euclidean Geometry. In particular, we discussed the concept of proof in ancient Greek and in modern mathematics. For example, we gave the students a worksheet with Euclid's theorem concerning the equality of the two angles of an isosceles triangle, accompanied by its Euclidean proof as it appears in the *Elements*. The students tried to understand and analyse the ancient text and its proof with the help of the teacher of ancient Greek. Afterwards the students compared this ancient proof with the modern one taught at school. With the help of their mathematics teacher, they also had the opportunity to discuss two more simple proofs, given by Proclus and Pappus respectively several centuries after Euclid. Students expressed their ideas about the different ways that these mathematicians proved the same theorem. All students believed that Euclid's proof was the most complicated one, with a lot of details (and too long compared with the modern one). There were students who claimed that Euclid's method of proof was more 'scientific', because all the propositions that Euclid used in his proof had already been proved. Their mathematics teacher helped them to discuss the 'scientific', or 'non-scientific', method of proof. Starting from the students' reactions to this topic, the history teacher provided information about Euclid: who he was, his language and what exactly he wrote. In parallel, Euclid was placed in his historical and social context in the Hellenistic era and we discussed the role of the king, the philosophers and the scientists in Hellenistic Alexandria. The discourse that followed touched on issues ranging from Platonic philosophy (which formed the dominant view of Geometry at that time) to the social-political situation that defined scientists' status and their differences from the Epicureans.

Results

The project lasted for two school years. For the evaluation of the cross-curricular courses, we used the material we had written, which included questionnaires and report sheets, as well as videos we had recorded during the courses. Our conclusions were:

1. Students found this project very interesting because they had the opportunity to discuss a subject from different perspectives, to develop their critical thinking and relate different pieces of knowledge. We believe that this cross-curricular teaching in Euclidean Geometry has helped students to gain a wider perception of mathematical concepts (going beyond their purely mathematical definition).

2. The students' ability to analyse and understand an ancient text had increased substantially with the support of their teachers.

3. The participation of teachers of different subjects at the same time during the teaching

process resulted in students' better understanding of particular concepts.

4. The teachers who participated in the project also had the opportunity to collaborate on different topics. Mathematics teachers managed to overcome their difficulty in approaching ancient texts, and respectively, Greek language teachers succeeded in challenging their fear of mathematics. In general, this interdisciplinary cooperation affected our work positively for the benefit of our students.

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Report of the 4th Colloquium On The Didactics Of Mathematics

(with international participation)
22 & 23 April 2005
University of Crete, Rethymnon, Crete, Greece

The Colloquium was devoted to various aspects of teaching and learning Mathematics. More than 300 participants, mainly mathematics schoolteachers, students of mathematics and education, university teachers and researchers in mathematics education and related areas, attended it. Although most of the participants came from Greece, there were also participants from other countries. In particular, 41 presentations were delivered, 20 by Greek speakers and 21 by speakers coming from 14 different countries. Two out of the four invited talks concerned one of the main themes of the colloquium, which was *The didactical value of the history and epistemology of Mathematics*
These talks were delivered by:
Fulvia Furinghetti & Annamaria Somaglia: *The history of mathematics and teacher education in practice: a case study*
Gert Schubring: *Generalizing the concept of multiplication - Epistemological implications of the relation between quantity and number*

There were also another 6 accepted presentations, directly related to the HPM perspective:
R. Bkouche: *La Géométrie entre mathématiques et sciences physiques*
K. Nikolantonakis: *La multiplication dans le cadre de la formation continue des professeurs d'écoles*
Y. Thomaidis & M. Stafylidou: *A research on the perspectives and possibilities of a cross-curricular teaching approach: The case of Euclidean Geometry in the 1st class of the Greek Lyceum*
E. Theodorou & Ch. Lemonidis: *A new interdisciplinary proposal for teaching Geometry to lower elementary school*
P. Strantzalos: *A new approach to the teaching of Euclidean Geometry to students of the 1st class of the Greek Lyceum*
A. Strantzalos: *A proposal for a "change of framework" of the reasoning procedures used in high-school Euclidean Geometry, motivated by Archimedes' work "On Plane Equilibriums"*
The program and the abstracts of all presentations can be found at the Colloquium website **http://www.edc.uoc.gr/4colloquium**
The proceedings will be published in two volumes. The first one will contain all accepted papers of non-Greek speakers, with abstracts in English or French of the presentations delivered in Greek; the second volume will contain the presentations delivered in Greek, with abstracts in Greek of the non-Greek presentations. The proceedings will include both the original (Volume I) and the translation in Greek (volume II) of the texts of the invited talks.

The proceedings are expected to be ready by December 2005 and will be available upon request (10€ per volume plus postage). For more information please write to tzanakis@edc.uoc.gr

C. Tzanakis, Greece

Work in progress

We encourage young researchers in fields related to *HPM* to send us a brief description of their work in progress or a brief description of their dissertation.

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Book reviews

Djebbar, Ahmed, *L'algèbre arabe: Naissance d'un art,* Coéd. Adapt-Vuibert, 2005

After his *Une histoire de la science arabe*, published in 2001 by *Les Editions du Seuil*, this is another important work by Ahmed Djebbar in which he presents some results of his research into the development of algebra in the Islamic civilization, unravelling, in particular, the decisive contributions which came from the Maghreb.

The first part of the book systematically treats the origins of Arab algebra in the Islamic Orient; it presents the major scholars and their impact on the emerging issues of algebraic theory. After this, the author describes how the practical work of solving equations, treating polynomials, studying Diophantine problems, etc. evolved. These developments in the Orient are presented in chronological order for the 9th, 10th, 11th and 12th centuries, and how these devlopments continued in the 13th century and after.

The book's second part deals with the Occident, studying the contributions by scholars in the Maghreb and in Al-Andalus, at first for the 10th and the 13th centuries, and then for the period from the 14th century and later, for which important texts on issues of algebra have been discovered. Particularly noteworthy are the developments concerning the symbolical aspect. These historical innovations are presented with the emphasis they deserve.

The final part of the book provides an overview on how these developments in algebra were transferred to medieval Europe. Several appendices offer useful information, e.g. on the biographies of the mathematicians mentioned, and on terminology.

Gert Schubring

The views expressed in this section are the views of the reviewers and may not necessarily be those of the *HPM* Advisory Board.

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Have you read these?

Ascher, Marcia: *Mathematics Elsewhere: An Exploration of Ideas Across Cultures*. Princeton, NJ and Oxford: Princeton University Press, 2005.

Azzolini, M.: Anatomy of a Dispute: Leonardo, Pacioli and Scientific Courtly Entertainment in Renaissance Milan, *Early Science and Medicine*, 9 (2), 2004, pp. 115-135.

Babb, Jeff: Mathematical Concepts and Proofs from Nicole Oresme Using the History of Calculus to Teach Mathematics, *Science & Education*, 14 (3-5), 2005, pp. 443-456.

Bartolini Bussi, M. G./ Mariotti, M. A./ Ferri, F.: Semiotic Metiation in Primary School: Dürer's Glass, in Hoffmann, M.H.G. et al. (eds.): *Activity and Sign-Grounding Mathematics Education,* Springer, 2005, pp. 77-90.

Bergsten, Chr.: Beyond the representation given –The parabola and historical metamorphoses of meanings, http://www.math.uncc.edu/~sae/dg7a/DG7-Bergsten.pdf

Cohen, P.C.: Democracy and the Numerate Citizen: Quantitative Literacy in Historical Perspective, http://www.maa.org/Ql/pgs7_20.pdf

Cohen, P.C.: The Emergence of Numeracy, http://www.maa.org/Ql/023-30.pdf

Corry, Leo: *David Hilbert and the Axiomatization of Physics (1898–1918).* Archimedes New Studies in the History and Philosophy of Science and Technology, Kluwer Academic Publishers, 2004.

Dass, P. M.: Understanding the Nature of Scientific Enterprise (NOSE) through a Discourse with its History: The Influence of an Understanding "History of Science" Course*, International Journal of Science & Math Education*, 3(1), 2005, pp. 87 - 115 .

Dubinsky, Ed, Weller, Kirk, Mcdonald, Michael A. *et al.*: Some Historical Issues and Paradoxes Regarding the Concept of Infinity: An Apos-Based Analysis: Part 1, *Educational Studies in Mathematics*, 58 (3), 2005, pp. 335-359.

Duke, Dennis: The Equant in India: The Mathematical Basis of Ancient Indian Planetary Models, *Archive for History of Exact Sciences*, 59 (6), 2005, pp. 563 – 576.

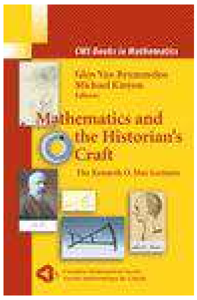
Fried, M. N.: The Problem of Mathematics Education and History of Mathematics from a Saussurean Point of View.
HTTP://WWW.MATH.UNCC.EDU/~SAE/DG7A/MICHAEL%20FRIED.PDF

Giucciardini, N.: Dot-Age : Newton's Mathematical Legacy in the Eighteenth Century, *Early Science and Medicine*, 9 (3), 2004, pp. 218-256.

Ihmig, K.-N.: Newton's Program of Mathematizing Nature, in Hoffmann, M.H.G. et al. (eds.): *Activity and Sign-Grounding Mathematics Education,* Springer, 2005, pp. 241-261.

Jahnke, H. N.: Hilbert, Wayl, and the Philosophy of Mathematics, in Hoffmann, M.H.G. et al. (eds.): *Activity and Sign-Grounding Mathematics Education,* Springer, 2005, pp. 215-228.

Kinyon, Michael; Brummelen, Glen van (Eds.): *Mathematics and the Historian's Craft***.** *The Kenneth O. May Lectures*, Springer Verlag, 2005
Contributions by



Amy Shell-Gellasch, Ivor Grattan-Guinness, Alexander Jones, Jim Bennett, Judith V. Grabiner, Ruediger Thiele, Thomas Archibald and Louis Charbonneau, Karen Hunger Parshall, Volker Peckhaus, Joseph W. Dauben, Stuart Shanker, Ann Hibner Koblitz.

Mormann, T.: Mathematical Metaphors in Natorp's Neo-Kantian Epistemology and Philosophy of Science, in Hoffmann, M.H.G. et al. (eds.): *Activity and Sign-Grounding Mathematics Education,* Springer, 2005, pp. 229-239.

Parker, John : *R.L. Moore: Mathematician and Teacher*, Washington, DC: Mathematical Association of America, 2005

Presmeg, N.: Ancient Areas: A Retrospective Analysis of Early History of Geometry in Light of Pierce's Commens
http://www.math.uncc.edu/~sae/dg7/presmeg.pdf

Radu, M.: Did Hermann and Robert Grassmann Contribute to the Emergence of Formal Axiomatics?, in Hoffmann, M.H.G. et al. (eds.): *Activity and Sign-Grounding Mathematics Education,* Springer, 2005, pp. 263-274.

Richards, J. L.: Connecting Mathematics with Reason,
http://www.maa.org/Ql/031-36.pdf

Saiber, Arielle: *Giordano Bruno and the Geometry of Language. Literary and Scientific Cultures of Early Modernity*. Aldershot: Ashgate, 2005.
Schubring, G.: *Le retour du refoulé - Der Wiederaufstieg der synthetischen Method an der École polytechnique*, Augsburg: ERV Rauner, 2004.

Schubring G.: A Case Study in Generalization. The Notion of Multiplication, in Hoffmann, M.H.G. et al. (eds.): *Activity and Sign-Grounding Mathematics Education,* Springer, 2005, pp. 275-285.

Sidoli, N.: Heron's *Dioptra* 35 and Analemma Methods: An Astronomical Determination of the Distance between Two Cities, *Centaurus,* 47, 2005 pp. 236–258.

Sieg, W. / Schlimm. D.: Dedekind's Analysis of Number: Systems and Axioms, *Synthese*, 147, 2005, pp. 121–170.

Silva da Silva Dynnikov, C. M.: Some German Contributions to Mathematics Research in Brazil, in Hoffmann, M.H.G. et al. (eds.): *Activity and Sign-Grounding Mathematics Education,* Springer, 2005, pp. 287-298.

Thomaidis, Y. (2005). A Framework for Defining the Generality of Diophantos' Methods in Arithmetica, Archive for History of Exact Sciences, *59(6), 2005, pp.591-641.*

Vinel, Nicolas: Un Carré Magique Pythagoricien ? Jamblique précurseur des témoins Arabo-Byzantins, *Archive for History of Exact Sciences*, 59 (6), 2005, pp. 545 – 562.

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A new journal

Aestimatio: Critical Reviews in the History of Science provides critical, timely assessments of books published in the history of what was called science from antiquity up to the early modern period in cultures ranging from Spain to India, and from Africa to northern Europe. This review, which is published by the Institute for Research in Classical Philosophy and Science (Princeton, NJ), is now in its second volume.
Its primary aim is to promote the study of pre-modern science by allowing reviewers the opportunity to engage critically both the results of recent research in the history of science and how these results are obtained.

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Have you been here?

Societies and organisations

African Mathematical Union: Commission on the History of Mathematics in Africa (including newsletter)
http://www.math.buffalo.edu/mad/AMU/amuchma_online.html

Association des Professeurs de Mathematiques de l'Enseignement Public [APMEP] History site:
http://www.apmep.asso.fr/BMhist.html

British Society for the History of Mathematics [BSHM]
http://www.bshm.org

HOMSIGMAA - History of Mathematics Special Interest Group of the MAA
http://home.adelphi.edu/~bradley/HOMSIGMAA/

HPM Americas
http://www.hpm-americas.org/

Italian Society of History of Mathematics
http://www.dm.unito.it/sism/indexeng.html

Topics and Resources

Ethnomathematics on the Web

http://www.rpi.edu/%7Eeglash/isgem.dir/links.htm

About Medieval Arabic Numbers
http://www.geocities.com/rmlyra/Numbers.html
http://www.geocities.com/rmlyra/arabic.html

Annotated Bibliography on Proof in Mathematics Education
http://fcis.oise.utoronto.ca/~ghanna/educationabstracts.html

BibM@th
http://www.bibmath.net/dico/index.php3?action=rub&quoi=0

Centro Virtual de Divulgación de las Matemáticas, esta siendo desarrollada por la Comisión de Divulgación de la *Real Sociedad Matemática Española (R.S.M.E.)*
http://www.divulgamat.net/index.asp

History of Statistics
http://www.stat.ucla.edu/history/

Images of Lobachevsky's context
http://www.ksu.ru/eng/museum/page0.htm

Images of Mathematicians on Postage Stamps
http://members.tripod.com/jeff560/index.html

Numdam-Digitization of ancient mathematics documents
http://www.numdam.org/en/ressnum.php

Convergence: an online magazine of the MAA providing resources to teach mathematics through its history
http://convergence.mathdl.org/

The editors welcome information about other sites.

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Notices

Problématique-Two Questions on Historical Conceptions on Teaching and Learning Mathematics

It has rightly been observed that multiformity is a basic characteristic of the relation between history and pedagogy of mathematics. Let us examine for example an important issue in this area – the study of historical sources aiming to get insights into didactical problems. The study of an historical text, which is closely related to the genesis or evolution of a mathematical concept, usually reveals aspects that cannot be found in its modern, refined definition.

From the learner's point of view, this revelation may help him/her to attain a broader understanding of the concept. Moreover, from a teacher's point view, this broader understanding may offer ideas for creating new didactic situations, or even help the teacher to foresee some of his/her students' difficulties and errors when they struggle to grasp the meaning of the concept's modern definition.

In this note we aim to point out yet another aspect of studying historical sources for the benefit of teaching.

In the history of mathematics, several mathematical works are recorded as landmarks, either for having been vehicles of a radical epistemological change, or for having exerted a determinative impact on the development of a field, or simply because they have been widely disseminated and adopted. More often than not, these works are mathematical textbooks written by great mathematicians, who thought their duty to expose their general ideas on the teaching and learning of the subject matter.

Let us cite firstly an example coming from the late antiquity.

Studying Diophantus' *Arithmetica* (written about AD 250) we grasp the meaning and difficulties of what could be called, very briefly, 'algebraic problem-solving in the pre-symbolic era of algebra.'

In the introductory part of this work Diophantus addresses some psychological, didactical and methodological remarks on problem solving to an eager student. Diophantus stresses the fact that beginners' minds are doubtful of success, but the desire for learning followed by teaching, assures a rapid reception of the new knowledge. On the other hand, however, the mass of the subject matter slows down assimilation and obstructs memorising.

To cope with this difficulty, Diophantus emphasises the importance of two presuppositions: Firstly, he exhorts his student to undertake a lot of practice on the subject matter of *Arithmetica* for mastering both the basic theory (i.e. operations with the powers of the unknown) and the basic method (i.e. formulating equations from the problem's data). Secondly, he announces that the subject matter should be divided so that easier things come first, followed by the more difficult ones. Diophantus claims that these presuppositions assure easier understanding and impression on the memory.

Evidently, Diophantus highlights here some traditional values of the teaching and learning process: smooth organisation of the subject matter followed by teaching, along with eagerness for learning followed by practice, will provide firm knowledge. It is well known, however, that modern didactics, under the influence of constructivist theories, seriously calls into question this model of teaching and learning.

Our second example comes from the 18th century.

An important lesson that we learn from the study of Leonhard Euler's *Introductio in Analysin Infinitorum* (1748), is that it is possible to obtain correct results for the expansion of trigonometric and logarithmic functions into infinite polynomials (i.e. series), without having to resort to a theory of convergence. Euler obtained these results, which changed drastically the way of calculating these functions, after extending algebraic operations with finite magnitudes to infinite and infinitesimal ones. In the context of this 'algebraic analysis', Euler opens his monumental work with the following didactical remarks:

> Often I have considered the fact that most of the difficulties, which block the progress of students trying to learn analysis, stem from this: that although they understand little of ordinary algebra, still they attempt this more subtle art. From this it follows not only that they remain on the fringes, but in addition they entertain strange ideas about the concept of the infinite, which they must try to use.

Euler's remarks could serve well the critics of the 'New Math' movement of the sixties of the 20th century, when the teaching of analysis entered upper secondary curricula at the expense of teaching hours devoted to the traditional course of elementary algebra. How could one cope with theorems about convergence of functions, or the techniques of differentiation and integration, without a firm knowledge of and experience with the rules of algebraic transformations and the calculus of inequalities?

The previous examples of didactical interventions found in great mathematical textbooks of the past, raise the following questions:

– Is it possible to form, in a similar way, a broad outline of the conceptions held by mathematicians of a specific era on the issue of teaching and learning mathematics?

– How does modern didactical theory value these conceptions, especially from the point of view of the relations between the history and the teaching of mathematics?

It is our opinion that further inquiry and discussion on these issues will broaden the current *problématique* in the context of the HPM.

Bibliographical note and guide

Arcavi, A. & Bruckheimer, M. (1989). The Didactical De Morgan: a Selection of Augustus De Morgan's Thoughts on Teaching and Learning Mathematics. *For the Learning of Mathematics* 9 (1), 34-39.

Carlton, L. V. (1959). *An Analysis of the Educational Concepts of Fourteen Outstanding Mathematicians, 1790-1940*. Dissertation. Evanston, Illinois: Northwestern University.

Euler, L. (1748/1988). *Introduction to Analysis of the Infinite*. Book I (translated by J. D. Blanton). New York: Springer-Verlag.

Fauvel, J. (1989), Platonic Rhetoric in Distance Learning: How Robert Record Taught the Home Learner. *For the Learning of Mathematics* 9 (1), 2-6.

Tannery, P. [ed.], (1893-95/1974). *Diophanti Alexandrini Opera Omnia Cum Graecis Commentariis*. Vol. I & II. Stuttgart: B. G. Teubner.

Thomaidis, Y. *(2005). A Framework for Defining the Generality of Diophantos' Methods in Arithmetica*, Archive for History of Exact Sciences, *59(6), 2005, pp.591-641.*

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Mathematics Education in the Age of Enlightenment – a few words on Condorcet

Marie Jean Antoine-Nicolas Caritat (1743-1794), the Marquis de Condorcet, is a remarkable man for his works in mathematics

and philosophy, as well as for his political action before and during the French Revolution. Condorcet is usually called the last of the *philosophes* and is also a very important character in the history of education because of his work in support of public instruction.

Condorcet is considered a pioneer in mathematics thanks to his studies in a field of knowledge, that he himself called social mathematics, which deals with the application of the theory of probabilities to social issues like legal decisions and elections.

As for philosophy, Condorcet's most famous influence rests on his *Esquisse d'un tableau historique des progrès de l'esprit humain*, a book describing the progress of the human race during nine stages, from the union of families into tribes to the time of Descartes and up to the American and French Revolutions in 1776 and 1789. According to Condorcet, the ninth stage would be followed by an age where there would be equality among nations and social classes. Condorcet thought there could be no end to human progress because men would always seek for improvement.

During one of the phases of the revolutionary government, the Legislative Assembly (from 1 October, 1791 to 20 September, 1792), Condorcet was the chairman of the Committee on Public Instruction and the author of a report proposing a national system of education under the responsibility of the State. In this system – universal, free, equal for both sexes, and removed from ecclesiastical influence – mathematics and science would play a prominent role. Condorcet's plan is known as the *Rapport sur l'instruction publique* and although it was presented to the Legislative Assembly on 21 April, 1792, the revolutionay wars that followed prevented its implementation. Yet the *Rapport* was an inspiration to the system of central schools created in France after Condorcet's death.

When considering the contents of the instruction to be offered to the citizens of the new republic of France, Condorcet was emphatic in pointing to mathematics as an essential subject. To him, mathematics provides an indispensable contribution to children's cognitive evolution and at the same time plays an essentially political role in public instruction.

Condorcet's ideas on the role of mathematics in children's education can be found in his early works on education (published by Manuela Albertone in 1983). They also belong to his *Cinq Mémoires sur l'instruction publique*, a work written before the *Rapport*.

To show the importance Condorcet attached to mathematics in public education, this what he has to say in *Premier Mémoire – Nature et l'oject de l'instruction publique*:

> Anyone who cannot write, and is ignorant of arithmetic, is actually dependent on the more educated man, to whom he must continually have recourse. […] But the man who knows the rules of arithmetic necessary for every-day life, is not dependent on the wise man posessing a much greater skill in the mathematical sciences, and [his] talent will be a very real utility to him, without it ever being a hindrance to the enjoyment of his rights. [1]

But what is even more interesting for the history of mathematics education is that Condorcet did more than just assert the importance of mathematics in public instruction, for he is also the author of a textbook of arithmetic for French children. As a matter of fact, as Schubring (1988) writes, after the banishment of the Jesuits from France in 1762, French educational politics promoted the use of textbooks as the main means of transmission of knowledge. During the revolutionary period every project concerning public instruction, including Condorcet's, stressed the importance of writing elementary textbooks (*livres élémentaires*) to be used in the national school system to be established by the new republic.

Condorcet's arithmetic textbook was written at the same time as the *Esquisse* while the author, his arrest having been ordered by the Convention on 8 July, 1793, was being hidden by the widow Madame Vernet in Paris. Condorcet managed to send his wife, Sophie de Grouchy, the parts of the book he was able to finish before having to flee Madame Vernet's house, shortly before his death in prison in March, 1794. Jean-Baptiste Sarret, a mathematics teacher to whom Condorcet entrusted some of his papers, borrowed some of Condorcet's ideas to write an arithmetic textbook of his own, in order to enter a competition for elementary textbooks held by the National Convention. Sarret was in fact one of the few winners when the results of the competition were announced more than one year after Condorcet's death. Although Sarret was accused of using Condorcet's work, when Sarret's book was later compared to the text that Condorcet had sent his wife, a commission formed by Lagrange, Laplace, Legendre et Bossut decided the two works had been written by different authors.

In 1799 Condorcet's widow had the manuscript of the arithmetic published under the title *Moyens d'apprendre à compter sûrement et avec facilité*. Some months later a Council for Public Instruction authorised the use of the book and another edition appeared. Nothing is known, however, about how and by whom the *Moyens* was used in France (Schubring, 1988).

Condorcet's arithmetic book is in two parts: the first consists of twelve lessons to be studied by the children, while the second, a set of instructions guiding the work with each lesson, is addressed to the teachers who would act in public instruction.

The first three lessons cover the basic idea of number, names and symbols for numbers, and the properties and rules of the Hindu-Arabic numeral system. The following three lessons deal with addition, subtraction and proofs of the results of both operations by using their inverse operations. The last six lessons are aimed at teaching the ideas and algorithms of multiplication and division. The algorithms for the four operations explained in Condorcet's book are the same techniques still taught in primary schools all over the world today.

The sequence followed by Condorcet in each one of the four operations is:
1. he presents the ideas associated with it;
2. he introduces its algorithms as way to facilitate a task that would otherwise be too long and tiresome;

[1] [*tr.*] Condorcet, 1994, p. 62.

3. he illustrates the algorithm through worked examples.
Thus, instead of starting with a definition or a rule followed by some worked examples, Condorcet always prefers the opposite order. Given this choice, he usually places rules and generalisations at the end of the lessons.

The second of Condorcet's lessons deserves a special mention since it presents a great innovation proposed by the author of the *Moyens à compter sûrement et avec facilité*. He changes the French names of some of the numbers in order to make a closer link between spoken numbers and written symbols. Examples are: *dix-un* and *dix-deux* for *onze* (eleven) and *douze* (twelve), *duante* for *vingt* (twenty), and *septante* and *octante* for *soixante-dix* (seventy) and *quatre-vingts* (eighty). This reform, based on the structure of the decimal number system, was a failure in France. As Nicole Picard (1988) comments, even today children face difficulties which stem from the lack of analogy between the French names of some numbers and their decimal numeration.

As we said before, the second part of Condorcet's arithmetic textbook is addressed to teachers. Some of Condorcet's pedagogical concerns are[2]:

1. He worries about the teaching of mathematics in schools with many different students. His words show he is aware of the difficulties involved in a 'common instruction':

> I present here the method one could follow to reveal the system of numeration, without however insisting on it. In a common instruction one cannot follow a procedure that is as rigorous in this respect as it could be individual teaching.(p. 75)

2. Teachers should be aware of the needs and interests of the sudents in order to choose suitable examples.

3. Teachers should attend to the different natures and abilities of children.

4. He stresses the utility of arithmetic knowledge and wants children's learning to be effective, but he insists that the students' ability to be able to use arithmetic independently is not secured by memorisation and rote learning. Having the difficulties of beginners in mind he writes:

> But it is essential therefore for them to solve things by themselves, so that they do not get into the habit of repeating the words 'I write', 'I learn', without thinking, and by the use of a sort of mechanical memorising. (p.96)

5. He attacks dogmatic teaching styles and tells teachers to try to show students the links between different parts of arithmetic, and to avoid using arbitrary terminology and methods.

Condorcet's ideas, expressed in words written more than two hundred years ago, in the context of the French Enlightenment, are still valid for mathematics education today.

Bibliography

Albertone, Manuela. *Introduzione a Condorcet. Réflexions et notes sur l'éducation*. Napoli: Bibliopolis, 1983.

Badinter, Elizabeth & Badinter, Robert. *Condorcet (1743-1794). Un intellectuel en politique*. Nouvelle édition révue et augmentée. Paris: Fayard, 1991.

Condorcet, Marie-Jean-Antoine-Nicolas Caritat. *Esquisse d'un tableau historique des progrès de l'esprit humain*. Paris: Éditions Sociales, 1971.

Condorcet, Marie-Jean-Antoine-Nicolas Caritat. *Réfléxions et notes sur l'éducation*. A cura di Manuela Albertone. Napoli: Bibliopolis, 1983.

Condorcet, Marie-Jean-Antoine-Nicolas Caritat, *Moyens d'apprendre à compter sûrement et avec facilité*. Appareil critique – études, notes, commentaires, bibliographie. Paris: ACL éditions, 1988.

Condorcet, Marie-Jean-Antoine-Nicolas Caritat. *Cinq Mémoires sur l'Instruction Publique*. Présentation, notes, bibliographie et chronologie par Charles Coutel et Catherine Kintzler. Paris: Flammarion, 1994.

Condorcet, Marie-Jean-Antoine-Nicolas Caritat. Informe sobre la organización general de la instrucción pública. In: *Bosquejo de un cuadro histórico de los progresos del espíritu humano y otros textos*. Tradução de Francisco González Aramburo. Cidade do México: Fondo de Cultura Económica, 1997.

[2] All the quotations from Condorcet's *Arithmetic*, given here in translation, are taken from Condorcet, 1988.

Coutel, Charles. Condorcet ou l'exigence didactique a l'oeuvre. In: Condorcet, Marie-Jean-Antoine-Nicolas Caritat. *Moyens d'apprendre à compter sûrement et avec facilité*. Appareil critique – études, notes, commentaires, bibliographie. Paris: ACL éditions, 1988.

Gomes, Maria Laura M. *Quatro visões iluministas sobre a educação matemática : Diderot, d'Alembert, Condillac e Condorcet.* Ph. D. Thesis, UNICAMP, Campinas, Brazil, 2003.

Kintzler, Catherine. *Condorcet. L'instruction publique et la naissance du citoyen*. Paris: Folio-Essais, 1987.

Picard, Nicole. Notes et commentaires sur les "Moyens..." In: Condorcet, Marie-Jean-Antoine-Nicolas Caritat. *Moyens d'apprendre à compter sûrement et avec facilité.* Appareil critique – études, notes, commentaires, bibliographie. Paris: ACL éditions, 1988.

Schubring, Gert. Essais sur l'Histoire de l'Enseignement des Mathématiques, particulièrement en France et en Prusse. *Recherches en Didactique des Mathématiques*, v. 5, n. 3, p. 343-385, 1985.

Schubring, Gert. Introduction: Un savant des lumières. Un livre élémentaire pour la république. In: Condorcet, Marie-Jean-Antoine-Nicolas Caritat. *Moyens d'apprendre à compter sûrement et avec facilité*. Appareil critique – études, notes, commentaires, bibliographie. Paris: ACL éditions, 1988.

Schubring, Gert. Análise histórica de livros de matemática: notas de aula. Campinas: Autores Associados, 2003.

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Ottoman Mathematical Culture in the Nineteenth Century

Spurred on by increasing military, political and economic needs, the Ottomans had, by the nineteenth century, already begun to adopt Western science. To be specific, when the Treaty of Karlowitz was signed in 1699, following the first major defeat of the Ottomans, they realised the gulf between themselves and the Europeans, who had made great advances in science and technology. Coming to terms with their losses, and attempting to deal with the apparent superiority of the West, marks the beginning of Ottoman westernisation, the first such transformation of its kind anywhere. In this transition period of westernisation, the adoption of Western mathematics and related military techniques, like firearms and cartography, played an important role.

As a first step, military engineering schools were established so as to better prepare the Ottoman Empire against future military attacks. These were the first schools of higher education in the Ottoman Empire and were based on western models One of these schools, the Naval Engineering School, was established in 1773, under the guidance of Baron de Tott, a Hungarian in the service of the Ottoman Empire, and was reformed under Abdulhamid I (1774-1789). The aim of the school was to educate naval officers in geometry and geography. The subjects taught were arithmetic, geometry, geography, trigonometry, algebra, topography, mechanics and astronomy, as well as the Turkish language, Arabic and French. Some of the teachers were from France.

The second engineering school to be established was the Army Engineering School, founded in 1795. The aim was to train artillerymen and military engineers. Among the subjects taught were, again, French, Arabic, geometry, arithmetic, geography, algebra, trigonometry and astronomy, together with fortification.

These engineering schools had an important influence on mathematical activities at the time. Text books for the schools were prepared with translations and adaptations from European sources. The chief instructors and other teachers of these schools wrote and translated mathematical books. Hence mathematical teachers from Europe, and students who had been sent to the West, introduced modern mathematics to the Ottomans. Meanwhile, the printing press, which was established in the eighteenth century in Istanbul, served the Ottoman westernisation in the field of mathematics by printing the mathematical books.

The following were some of the key contributors to Ottoman westernisation in the field of mathematics.

Gelenbevi Ismail Efendi (1730-1790)

Among the chief instructors of the Naval Engineering School, Gelenbevi Ismail Efendi, from Gelenbe, near Aydın in western Anatolia, was of outstanding mathematical ability. Active at the end of the eighteenth century, when Ottoman science was already in decline, he was the last great Ottoman mathematician of the classical tradition, still working in the Islamic practice of algebra. He also used the traditional sexagesimal fractions in trigonometry. He wrote four books on mathematics, three in Turkish and one in Arabic: *Hisab al Kusur* (Calculation of the fractions), *Sharh-i Jadavil al Ensab* (The Explanation of the Sine Tables), *Usul Jadavil-i Ensab-ı Sittini* (Roots of the Sexagesimal Sine Tables), *Adla-i Musallasat* (The Sides of Triangles).

The *Hisab al Kusur*, on arithmetic and algebra, is a detailed work with five chapters. The fifth chapter, the most important one, explains positional notation for numbers and operations with them, as well as the rules of finding the unknown using algebra. The chapter deals with the solutions of different types of equations with integer coefficients, known in Islamic algebra as *masail-i sitte* (six problems, equations). Gelenbevi did not give geometrical justifications of the solutions as earlier mathematicians such as Khwarizmi had done.

Gelenbevi's *Risala fi Sharh-i Jadavil al Ensab* concerns tables of logarithms and their use, a topic becoming very well known in Istanbul at the time. This treatise is the first independent work on logarithms in the Ottoman era.
In his *Adla-i Musallasat*, written in Turkish and one of the rare original Ottoman trigonometry books, Gelenbevi studied the relations between sides and angles of triangles. Here, three kinds of theorems related to the triangles were presented, the Pythagorean Theorem, and the tangent and sine theorems.

Huseyin Rıfkı Tamani (? – 1817)

Huseyin Rıfkı Tamani, from the city of Taman in the Crimea, was among the first teachers appointed by Sultan Selim III to teach at the Army Engineering School. He was employed as the chief instructor at the school for a long time. He tried to organise the mathematical courses and wrote text books for them. His mathematical books were reprinted several times and were used as textbooks at the Engineering School for many years.

Huseyin Rıfkı knew English, French, Italian and Latin, in addition to oriental languages. He was a pioneer for bringing modern mathematics to the Ottoman Empire through translating many books into Turkish. The most important being his translation of a modernised and revised version of Euclid's *Elements,* published by Bonnycastle. He made the translation with the help and collaboration of an English engineer called Selim, who had entered the Ottoman service, and had converted to Islam. Both the British writer and traveller MacFarlane and the engineer Sang, who had been invited to Istanbul to work in manufacturing, highly praised Huseyin Rıfkı's *Elements* and stated that his Geometry was the best book to read and understand, existing in any language. It is thought that the final two sections on plane trigonometry were Huseyin Rıfkı's own work.

Huseyin Rıfkı was the author of *Article on Logarithms*, the third book on logarithms to be published in the Ottoman era. He gives the rules of logarithms and demonstrated the conversion of sexagesimal and decimal fractions to each other in the introductory part. This book was partly original and partly a translation, and was written to support the widespread use of logarithms by the Ottomans.

Work on the translation of western scientific books at the School, which was very active during the time of Huseyin Rıfkı (1806-1817), decreased in the time of Sayyid Ali Pasha who was the chief instructor following Huseyin Rıfkı. However, translations from the West were taken up again under Ishak Efendi who succeeded Sayyid Ali Pasha.

Ishak Efendi (1748? – 1834)

Ishak Efendi, the famous chief instructor of the Army Engineering School, was a student of Huseyin Rıfkı. He was born in the town of Narta in Yanya (Janina) and was a Jewish convert to Islam. He played an important role in the transmission of modern science to the Ottomans through his numerous books and lectures. Among his treatises, of special importance is the four volume *Mecmua-i Ulum-u Riyaziya* (Compendium of Mathematical Sciences) (1831), written for students of the School. The first and the second volumes are devoted to mathematics. The first volume treats arithmetic, algebra and geometry, while the second volume considers plane trigonometry, geometrical operations, applications of algebra to geometry, conic sections and differential and integral calculus. Higher mathematics was introduced to the Ottomans for the first time with these subjects. Again, for the first time, the terminology of modern sciences was translated into Turkish by Ishak Efendi. Being well versed in the subject matter as well as western languages, he coined many Ottoman Turkish equivalents for scientific terms (including mathematical ones) used in Europe.

Mecmua-i Ulum-u

The second volume of the treatise has two chapters – plane trigonometry and conics, and the calculus. Higher mathematics appears in the third section of the first chapter. Here we find geometric curves, including the loci of linear and higher degree equations, as well as irrational curves, and the helix.

In the second chapter, the first section treats the differential calculus, including methods for finding the differentials of the second and third degree, the differentials of the sine and cosine and the differentials of logarithmic functions.

The second section, on the integral calculus, looks at the possibilities or impossibilities of geometrical integrals of differentials with two terms, the quadrature of curves, calculations of volumes, the integral calculus of sine and cosine functions, approximate integrals, some transformations used in the integral calculus, integrals of surd expressions, integrals of differential equations, and differential equations of second, third and higher degrees..

It is clear that the understanding of the modern treatment of numbers, as well as the differential and integral calculus were introduced to Ottoman mathematics by *Mecmua-i Ulum-u Riyaziya*, the work drawing on several western sources. Ishak Efendi dedicated the first volume of his work to Sultan Mahmud II. The Sultan approved the book and its publication and distribution were provided by the government. The book was also published in Cairo in 1845 for use by the students of a new military school set up by Mehmed Ali Pasha, the reformist governor of Egypt. Ishak Efendi's influence on the transmission of modern mathematical sciences therefore spread to the wider Islamic world beyond Istanbul. The *Mecmua-i Ulum-u Riyaziya* was at the same level as the mathematical books of the technical schools in Europe at that time.

Emin Pasha (d.1851)

Emin Pasha was the son of Huseyin Rıfkı. He was sent to Cambridge University in 1835 after finishing the Army Engineering School.

He returned to Istanbul in 1841 and became the director of the Military School. He tried to develop it as a modern institution of education by bringing in experienced teachers and new methods.

Although he was a brilliant mathematician, Enin Pasha could not write many mathematical treatises, because so much of his time was taken up with administration. At Cambridge, his doctoral thesis was a paper of 27 pages called *Calcul de Variation*. Here, he explains how one could solve problems that could not be treated with calculus. He referred to the works by French Poisson (1781-1840) and Yakoli. His book on mathematics, physics and the art of war, *Memoire sur un nouveau Système de Confection des Fusées de Guerre*, was published in 1840.

Ibrahim Edhem Pasha (1785? -1865)

Ibrahim Edhem Pasha was another of the mathematicians who was in contact with European mathematics. Not much is known about his life since he lived in Egypt and does not appear in Turkish sources. Furthermore, since he wrote in Turkish, he is not found in Arabic catalogues. Ibrahim Edhem Pasha translated some mathematical books into Turkish, the most important being a translation of Legendre's *Éléments de Géométrie*. He copied the theorems of Lacroix, as did some mathematicians in France, rejecting Legendre's theorems, because of their complexity. Also in this translation, he uses the measurements of the meridian circle determined by Delambre (1749-1822) and Mechain (1744-1804), as a unit length, which shows that he also knew something of practical subjects such as geodesy, and the measurement units in France.

Huseyin Tevfik Pasha (1833-1901)

Huseyin Tevfik Pasha, from the city of Vidin in Rumelia, was one of the most important representatives of modern mathematics among the Ottomans. He graduated from the military school of Istanbul in 1859 and was appointed as a teacher of algebra to the same school. He taught higher algebra, analytic

geometry, calculus, mechanics and astronomy.

Huseyin Tevfik could follow the mathematical developments in Europe and America closely, because of his knowledge of English and French languages. He was often sent abroad for official duties. Meanwhile, he found time to devote to mathematics, collected a rich library, and encouraged scientific discussions with talented students close to him (for example Salih Zeki). He also tried to create an intellectual environment by publishing some periodicals.

Huseyin Tevfik was sent to Germany in 1886 to inspect the Mauser rifles invented by Peter Paul Mauser (1838-1914). He gave lectures on the Ottoman State and Islam in French, in the course of appointment to the membership of the Historical Society of New Island.

Although many of Huseyin Tevfik's books were not published, his most important mathematical work, *Linear Algebra* (1882), written in English, was successful enough to be republished in 1892. In this book, which had been written in 1872 while he was in America on official duties, Huseyin Tevfik he extended Argand's multiplication of complex numbers to three dimensions.

The preface to *Linear Algebra* which is the one of the first books on the subject, declares:

> This linear algebra is the product of applying of Argand's complex and imaginary quantities to three dimensional space. The "Quaternions", invented by Sir William Hamilton, was similar to that as well. However, almost there are no features in common between these two systems. The Argand's system, can be applied only to plane geometry, is not a specific condition of Hamilton's system. Hence, after the discovery of Hamilton's "calculus of directed lines", Argand's algebra is far from the being complete.
>
> Cauchy, one of the eminent mathematicians, used the Argand's system in his some

important works. "Methode des Equipollences" by M. Bellavitis was a generalised system of the plane analytic geometry and in reality was a developed form of the Argand's algebra.

The Argand's system provided the best and the most perfect proof for the most important theorem of the theory of algebraic equations. Argand's method, on the imaginary quantities, gave the geometrical commentary of some expressions in ordinary algebra like $a + b\sqrt{-1}$ and $a - b\sqrt{-1}$. So, it can't be thought that the ordinary algebra is not fully completed without Argand's system.

For this reason, I established a new algebra instead of Hamilton's calculus.

The applying of linear algebra to three dimensional geometry and to the plane geometry is possible. When it was applied to the plane geometry, it becomes the Argand's algebra except its notation.

What we can say in behalf of the Quaternions, is also valid for the linear algebra.

The most important point in the view of education is that almost every problem is independent each other.

This preface clearly shows the importance and originality of this book, which was not a translation of an existing European work.

Mehmed Nadir Beg (1856-1927)

Mehmed Nadir Beg was a teacher of Salih Zeki (see below) in Darüşşafaka (high school for orphan children). When a department of Number Theory was established in Istanbul University under the direction of Salih Zeki in 1919, Mehmed Nadir was appointed the head of this department. He published the solutions of many problems in the theory of numbers in the journal *L'Intermédiaire des Mathématiciens*, which first appeared in 1894.

Most of Nadir Beg's works were on Diophantine equations. He also prepared a text book on the theory of numbers and published its first volume.

Salih Zeki (1864-1921)

Salih Zeki, one of the mathematicians who lived in the final period of the Ottoman empire, contributed to mathematical studies and to higher education in Turkey. His mathematical talent was evident when he was a student in high school. He began work as an officer in the Office of Mails and Telegrammes after graduation from Darüşşafaka. He was then sent to Paris by the office to further his study. There he completed his education in electrical engineering and returned to his country. Although he was working in administrative positions, such as director of the observatory, Salih Zeki could find time for his scientific activities, as Huseyin Tevfik Pasha had done, and finally, on appointment as director of Darülfünun (the house of sciences – today's Istanbul University), he had the opportunity to study mathematics. He established the Departments of Mathematics, Astronomy and Physics. He prepared text books for the departments and contributed as a teacher. He wrote numerous textbooks on mathematics, physics and astronomy, and on algebra, theory of numbers, plane geometry, theory of probability, arithmetic, plane trigonometry, and on Lobatchevsky and Riemann geometries.

Salih Zeki is recognised as the founder of studies related to Turkish history of science and wrote two important works on the history of mathematics and astronomy. The first of these, entitled *Asar-ı Bakiya* (Immortal Works), consists of four volumes, of which only the first two were published. The first volume discusses trigonometrical studies in the Islamic World, while the second volume treats the Muslim contributions to arithmetic and algebra. He wrote the treatise using western sources such as Montucla, Tannery, Delambre and Cantor, and also drew on oriental source material from manuscripts in the libraries of Istanbul.

Salih Zeki's other book on the history of mathematics is the *Kamus-ı Riyaziyyat* (Dictionary of Mathematical Sciences). Here, he explains mathematical and astronomical terms and describes the lives and studies of all western and oriental mathematicians and astronomers. Only the first volume of the ten volumes was published, in 1892.

Salih Zeki was also interested in the philosophy of science. He translated into Turkish Poincaré's *La Valeur de la* Science (1905) in 1912, *La Science et l'Hypothèse* (1903) in 1927, and *Science et Méthode* (1908) in 1928. He also translated *Philosophy of Science* by Alexandre Bertrand in 1914/15. He also published a book on logic, *Mizan-i Tefekkur* (The Balance of Thought), in 1916. Thus he was instrumental in laying the foundations of the philosophy of mathematics as well as philosophy of science in Turkey.

Salih Zeki gave lectures for mathematics teachers and amateurs in Istanbul University in the academic years of 1912-13 and 1913-14. In these lectures, he mentioned modern topics in mathematics, such as non-Euclidean geometries and imaginary quantities.

In conclusion, although Ottoman mathematicians kept faith with the classical Islamic tradition in the period of the empire's growth, when the scientific revolutions occurred in Europe a gulf emerged between Ottoman and Western mathematics.

Ottoman mathematicians followed mathematical developments in Europe and used European knowledge to catch up with the modern world. Modern mathematical concepts and methods slowly began to be absorbed. However, the gulf between them could not be easily bridged. The Ottomans seemed to be informed about Western sciences mainly by means of translations and adaptations. Their most significant source was France, the centre of the Enlightenment, where talented students were sent for their higher education. It is significant to see that Ottoman mathematicians thought that development in mathematics could only be accomplished through adopting Western mathematics. There is still a great deal of mathematical papers to be studied to be able to make clear the success of the last Ottoman mathematicians.

Bibliography

Adnan Adıvar: *Osmanlı Türklerinde İlim*, Remzi Kitabevi, İstanbul 1982.

Bursalı Mehmet Tahir: *Osmanlı Müellifleri*, Vol. III, İstanbul 1342.

Bursalı Mehmet Tahir: *İdare-î Osmaniyye Zamanında Yetişen Kırım Müellifleri*, Kitaphane-î Sudî, İstanbul 1335.

Bursalı Mehmet Tahir: *Aydın Vilayetine Mensup Meşâyih, Ulemâ, Şuarâ, Muverrihîn ve Etibba'nın Terâcüm-ü Ahvâli*, İzmir 1324.

Abdülkuddûs Bingöl: *Gelenbevî'nin Mantık Anlayışı*, MEB, İstanbul 1993.

Melek Dosay: İbrahim Edhem Paşa, *OTAM*, 7, Ankara 1997.

Sadık Erdem: *Mir'ât-ı Mühendîs-Hâne-î Berrî-i Hümâyûn*, İstanbul Teknik Üniversitesi, Bilim ve Teknoloji Tarihi Araştırma Merkezi, No:3, İstanbul 1986.

Osman Ergin: *Türkiye Maarif Tarihi*, 1, İstanbul 1939.

Kerim Erim: Riyaziye, *Tanzimat*, İstanbul 1940.

Gelenbevî: *Risâle fî Şerh-î Cedâvili'l-Ensâb ve Nisbeti'l-Ceybiyye ve'l-Zılliye ve Ceybi'l-A'şârî ve Zılli'l-A'şârî,* Bâyezîd Umûmî, Nr.4516.

Ekmeleddin İhsanoğlu, Ramazan Şeşen, Cevat İzgi: *Osmanlı Matematik Literatürü Tarihi*, Vol. 1, Ircıca, İstanbul 1999.

İshak Hoca: *Mecmûa-i Ulûm-ı Riyâziyye*, Bulak Matbaası, Mısır 1257.

Cevat İzgi: *Osmanlı Medreselerinde İlim*, 1, İz Yayıncılık, İstanbul 1997.

Ercümend Kuran: Müsbet Bilimlerin Türkiye'ye Girişi (1797-1839), *VII. Türk Tarih Kongresi*, II, Ankara 1973.

Aydın Sayılı: Turkish contributions to and reform in higher education, and Hüseyin Rıfkı and his work in geometry, *Ankara Üniversitesi Yıllığı*, XII, Ankara 1972.

Semuhi Sonar: İbrahim Edhem Paşa'nın Kitâbû

Usuli'l Hendese'si Hakkında, *Araştırma*, II, Ankara 1964.

Hüseyin Rıfkı Tamâni: *Logaritma Risalesi*, Süleymaniye Kütüphanesi, Esat Efendi, 2001.

Kemal Zülfü Taneri: *Türk Matematikçileri*, Matbaacılık Okulu, 1958.

Hüseyin Tevfik Paşa: *Hüseyin Tevfik Paşa ve "Linear Algebra"*, prepared by Kâzım Çeçen, İstanbul 1988.

Salih Zeki: *Asâr-ı Bâkiye*, Vol. 2, İstanbul 1329.

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Announcements of events

8th International Conference of The Mathematics Education into the 21st Century Project "Reform, Revolution and Paradigm Shifts in Mathematics Education"

November 25-December 1, 2005
See Newsletter 59 and further information from
arogerson@vsg.edu.au.

Third International Conference on Ethnomathematics – Cultural Connections and Mathematical Manipulations

February 12-16, 2006
Auckland, New Zealand.
http://www.math.auckland.ac.nz/Events/2006/ICEM-3/

Espace Mathématique Francophone : Colloque EMF 2006

May 26-31, 2006
Québec, Canada
http://emf2006.educ.usherbrooke.ca/

3rd International Conference on the Teaching of Mathematics (ICTM-3)

June 30-July 6, 2006
Istanbul, Turkey
Following on the success of earlier conferences held in Samos, Greece (1998) and Crete, Greece (2002), the 2006 conference intends to focus on "new ways of teaching undergraduate mathematics". The conference will be co-sponsored by the MAA. For more information, including information on how to submit a paper, see http://www.tmd.org.tr/ictm3.

International Leibniz Congress - Unity in Plurality

July 24-29, 2006
Hannover, Germany
http://www.gwlb.de/Leibniz/Gesellschaft/Veranstaltungen/Kongress/Circular/

5th European Summer University on the History and Epistemology in mathematics education (ESU-5)

July 19-24, 2007
Prague, Czech Republic
The fifth conference in this series will focus on

- history and epistemology as tools for an interdisciplinary approach in the teaching and learning of mathematics and the sciences
- introducing an historical dimension in the teaching and learning of mathematics
- history and epistemology in mathematics teachers' education
- cultures and mathematics
- the history of mathematics education in Europe
- mathematics in Central Europe

The official languages of ESU-5 are three: English, Czech and French.
For more information, see Newsletter 58 or http://www.pedf.cuni.cz/kmdm/esu5.

ICME-11

July 6-13, 2008
Monterrey, Mexico

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Please note that the webpage http://www.mathedu-jp.org/hpm/index.htm is currently not updated due to technical reasons. Please use http://www.clab.edc.uoc.gr/hpm/ for the time being.

A note from the Editors

The Newsletter of HPM is primarily a tool for passing on information about forthcoming events, recent activities and publications, and current work and research in the broad field of history and pedagogy of mathematics. The Newsletter also publishes brief articles which they think may be of interest. Contributions from readers are welcome on the understanding that they may be shortened and edited to suit the compass of this publication.

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