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This and earlier issues of the Newsletter can be downloaded from our websites
http://www.clab.edc.uoc.gr/hpm/ and http://www.mathedu-jp.org/hpm/index.htm

Message from the Chair

With this short message, I would like to let you know about the work that has been done in the last few months, or is still in progress.

As is nowadays clear, an international community like the *HPM* Group may and should benefit as much as possible from the Internet. In fact the *HPM* website can be an important tool for making it easier for members to keep in contact and it also improves our public image. Our website seeks to present the most recent information about the group and its activities, as well as activities organized by its members and interesting links etc. To improve access to our website, we have created a second URL at *www.clab.edc.uoc.gr/hpm* that is available along with the existing one at *www.mathedujp.org/hpm/index.htm*. The new website has practically the same structure (with a somewhat different design) as the original one. In this way, together with the site of the Americas *HPM* section at *www.hpm-americas.org*, there are now at least three main sites to access *HPM*, which we hope will prove more convenient (some people may have easier access to one site than another one). In particular, the *Newsletter* is available for download from all three sites.

Please let us know if there any errors in the information given on the website, as well as offering any suggestions for improving its design or content.

At present all issues of the *Newsletter* from number 45 on are available in electronic form for download from the *HPM* website. Currently, we are trying to collect in electronic form issues earlier than number 45 so as to make them available online as pdf files. Victor Katz has already kindly provided numbers 29–38. We would be very grateful if anyone could provide us with the other Newsletters in electronic form (numbers 1–28 and 39–44).

At ICME 10, there were two *Topic Study Groups* related to the *HPM* perspective in Mathematics Education: TSG 17 on *The role of the history of mathematics in mathematics education*, and TSG 29 on *The History of the Teaching and Learning of Mathematics*. I am pleased to tell you that the contributions to both groups will be published in revised form in forthcoming special issues of different journals. There will be a double issue of the *Mediterranean Journal for Research in Mathematics Education* (published by the Cyprus Mathematical Society) for papers based on the presentations in the TSG 17, with M-K. Siu & C. Tzanakis as guest editors, and a special issue of another journal planned for the next summer for papers based on the presentations in the TSG 29, with G. Schubring as the guest editor.

An important aspect of the *HPM* Satellite Meetings of the ICME meetings has always been the publication of their Proceedings. In the latest of these meetings, in July 2004 at Uppsala, Sweden, the Proceedings were available in advance and were distributed to the participants at the time. Thanks to the initiative of Sten Kaisjer, chair of the Organizing Committee and one of the Proceedings' editors, and with the collaboration of Fulvia Furinghetti, former *HPM* chair, re-editing these Proceedings is under progress, in order to include a few papers that had been left out for technical reasons, as well as revised versions of other papers and a full text for some presentations or workshops for which only an abstract had originally appeared.

Continuing the tradition of organizing the *European Summer University* (ESU) *on the History and Epistemology in Mathematics Education* every three years, it is a great pleasure to announce that the next (5^{th}) ESU will take place in July 2007, at the Faculty of Education, Charles University in Prague, Czech Republic. The international *Scientific Program Committee* and the *Local Organizing Committee* will be announced soon. A preliminary announcement of this event can be found in this issue. Evelyne Barbin from France and Nada Stehlikova from the Czech Republic have kindly agreed to collaborate in organizing what we hope will be a high quality scientific meeting, which will also have a valuable impact on the Mathematics Education community in Europe and beyond.

Since the birth of the *HPM* Group, its *Newsletter* has been an important tool for enabling contact between members and for making the group's activities more widely known. Perhaps, it is time to think of having a cover for this small, but for us, valuable magazine! Your ideas and suggestions will be much appreciated.

Finally I would like once again to invite you to send directly to the *Newsletter* Editors any material that you think it is relevant to the *HPM* perspective and potentially of interest to the readers of this *Newsletter*. Such material could refer to recently published work, book presentations and/or critical reviews, reports on local or international past events, announcement of local or international meetings, interesting sites on the web, work in progress (e.g. on doctoral theses that are at an advanced stage of their development), short texts with historical and/or educational information, questions and problems you think that might be interesting to study, etc.

Constantinos Tzanakis, Greece

Work in progress

Mathematics Resources for 11 to 16 year olds

Dr. Snezana Lawrence, (St. Edmund's Catholic School, Dover Old Charlton Road, Dover CT16 2QB)
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Abstract

This paper introduces a project, which has been developed as part of the Gatsby Teacher Fellowship (The Gatsby Teacher Fellowships programme was established in 1998 by the trustees of the Gatsby Technical Education Projects, one of the Sainsbury Family Charitable Trusts, UK www.gtf.org.uk), which I have been awarded for the 2004-5 academic year. Its principal aim is production of resources from the history of mathematics to supplement teaching of mathematics to 11–16 year olds.

Introduction to the Project

The project grew out of the practice of teaching mathematics in the context of its history in the secondary St. Edmund's Catholic School in Dover, England. Its aim is to produce an ICT based resource base in the history of mathematics with worksheets and practical guidance to place Key Stages 3 and 4 (11 to 16 year olds) mathematics topics into a wider historical context and therefore reinvigorate the interest in the subject.

The prevailing modern view of mathematical ability is one which entails creativity and transcends the more limited concept of technical ability. There is, however, little widely available material which, in a simple and accessible way, introduces the secondary school age children to the world of 'creative' mathematics. The approach adopted will hopefully inspire young mathematicians to recognise this creative nature of mathematical enquiry and to gain an insight into the various techniques of research, analysis and synthesis of mathematical thought through the study of the subject's history. This would be achieved through producing the material on, firstly, re-occurring topics in mathematics through history, and secondly, development of mathematical techniques relevant to KS3 and KS4 mathematics.

Existing resources

In order to produce material and to link it with the existing resources, a study and an evaluation of these was needed in the first instance. There is a wealth of resources in the history of mathematics currently being posted on the World Wide Web. The problems that secondary school teachers and children both face, however, can be defined as:

- the majority of the current resources are aimed at university and post-graduate level students; neither of which is wholly suitable for either secondary school teachers or pupils
- some resources can be used by teachers, but they would require combination of time dedicated to do original research, simplification of the material, and finally time, energy and skill to produce 'digestible' learning resources for their students
- there is no way of telling the accuracy of the material on the internet because of the lack of peer assessment, although this is probably the fastest diminishing problem as a number of professional organisations now offer guidance on the matter through links and reviews.[a]

Apart from these initial difficulties, one can however venture into the ever expanding universe of the World Wide Web. There are easily distinguishable groups among the existing sites in the history of mathematics:

- **Primary resource sites** are sites with the primary historical material, such as that of Gallica and the Cornell University Library. Material presented on these sites is of most value as it does not become obsolete at any point.[b]
- **Accumulative sites** are such as that of the famous University of St. Andrew's History of Mathematics site, which is not only ever more growing in terms of added topics, but also of the internal links and references.[c]
- **Mutating sites** are the least reliable and most ambivalent, although at times interesting, as they may introduce a view of a mathematical topic, or an application to it which may be of interest to the age group at which the project is aimed. These may be sites such as those which deal with 'seasonal' mathematics – giving links and advice on topics such as cryptography for example as a cipher breaking code challenge approaches.[d]
- **List sites** can be very helpful as they inevitably contain some links that one has not seen before, and at the same time offer guidance, and often professional assessment of the site's accuracy.[e]

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[a] For English speaking peoples, the BSHM and CSHPM sites are the most useful in this respect http://www.bshm.org/ and http://www.cshpm.org/.

[b] http://gallica.bnf.fr/ Gallica, part of the Bibliothèque National, Paris; http://historical.library.cornell.edu/math/ Cornell University Library Historical Math Monographs; and the Götiinger Digitalisierungs-Zentrum University of Göttingen http://gdz.sub.uni-goettingen.de/en/index.html. The Jahrbuch Project, is an Electronic Research Archive for Mathematics (ERAM) http://www.emis.de/projects/JFM/.

[c] Others certainly worth mentioning are http://mathworld.wolfram.com/, part of the Wolfram Research sites. David Wilkins, Trinity College Dublin http://www.maths.tcd.ie/pub/HistMath/; and of course, the famous MacTutor History of Mathematics archive http://www-gap.dcs.st-and.ac.uk/~history/.

[d] Good example is Simon Singh's site and the www.simonsingh.net/, in particular historical background to cryptography linked with past and currently running competitions.

[e] The Math Forum @ Drexel University is an excellent example of this http://mathforum.org/; as is the Mathematics Archives site at http://archives.math.utk.edu/topics/history.html.

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We encourage young researchers in fields related to *HPM* to send us a brief description of their work in progress or a brief description of their dissertation.

Book presentation

Apollonius of Perga's Conica: text, context, subtext,

(Leyden, Brill 2001, pp. xii+499), by Michael N. Fried & Sabetai Unguru:

This book constitutes a new interpretative approach of *Conica*, the famous work of the Greek geometer Apollonius' from Perga, written around 200 BC. This work endeavoured to provide systematic study of the conic sections, using the methods Greek geometry available at that time. From the 8 books of the *Conica*, the first four have been preserved in ancient Greek, the following three survived in Arabic translation of the 9th century A.D., whereas the last one has been lost.

The traditional historiographic approach to the *Conica* goes back mainly to H.G. Zeuthen, who in his work "Die Lehre von den Kegelschnitten im Altertum" (1886) made systematic use of the term "geometric algebra" to characterise some of the methods employed by Euclid to prove geometrical propositions in the 2nd Book of the *Elements*, and subsequently, to show that Apollonius' work is an extension of these methods to the study of the conic sections. The term "geometric algebra" means mainly that the corresponding propositions of the *Elements*, are nothing more than "geometrical proofs of algebraic identities", or "geometrical solutions to algebraic equations", whereas, the propositions in the *Conica* are nothing else than geometrical formulations of the equations of modern Analytic Geometry. Zeuthen's approach, that has as its starting point the practices of modern mathematics, instead of ancient Greek Mathematics, established a particular approach to the *Conica* in contemporary works on the history of Mathematics.

The characterisation of certain works of the ancient mathematical Greek tradition as "geometric algebra", which caused heated debates between mathematicians and historians[1], constitutes both the starting point and the conclusion of the authors of the present book, since the first and last chapters of the book are devoted to this issue. Their approach to the *Conica*, an alternative to Zeuthen's, aims at being guided only by the ancient text and interpreting it with the aid of the methods of ancient Greek mathematical tradition; that is, not to be influenced by the radical transformation of the Greek mathematical tradition that took place in the 17th century, through the arithmetisation-

[1] For a survey, see D.E. Rowe, "New trends and old images in the history of Mathematics", in R. Calinger (ed.) *Vita Mathematica: Historical research and integration with teaching*, MAA, Washington 1996, pp.3-16.

algebraisation of synthetic geometry that finally led to modern analytic geometry. This new interpretative approach leads to interesting historical results, like, for instance, the revision of traditional points of view on the major, or minor significance of the content of certain of the Books of the *Conica*, which have been evaluated in the past according to the modern standards of analytic geometry.

Fried & Unguru's book is valuable, both for looking at the *Conica* from a different perspective and for the important methodological issues in mathematics historiography raised by its authors. Although the volume is intended primarily for historians of ancient mathematics, its approach is fresh and engaging enough to be of interest also to historians, philosophers, linguists, and open-minded mathematicians.

A final remark I would like to make in this short note, is that in their survey of the various editions of Appolonius' works in modern times (pp.10-11), the authors do not mention the 4-volume edition of the ancient text of the *Conica* in modern Greek that was done by E. Stamatis and was published in 1975-76 by the Technical Chamber of Greece.

Yannis Thomaidis, Greece

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The views expressed in this section are the views of the reviewers and may not necessarily be those of the *HPM* Advisory Board.

Ph.D. theses

Dr. Snezana Lawrence:

Geometry of Architecture and Freemasonry

Open University, 2002

This thesis describes the process of the establishment of geometrical concepts as perceived through architectural education, theory and practice, and examines their ontological significance in terms of social relations, which are placed in the wider context of the development of Freemasonry.

The first emphasis is on the establishment of the architectural profession and the search for a technique of graphical communication through which the profession could become a given and strictly defined social institution. The history of the establishment of the first architectural schools in London as well as the influence Freemasonry played on these developments are traced. This theme is contained within the period beginning roughly with the publishing of the Descriptive Geometry in France in 1795, its translation into English in 1809, and finishes with the explanation of how the technique of Descriptive Geometry was replaced in England with other techniques invented for the same purpose in the 1840s.

The history of the Descriptive Geometry in France and its utilisation in the educational system of France since the 18th century has already been well documented in the work of Taton (1951), and more recently by Sakarovitch (1989, 1995). The history of the technique in England, however, makes a captivating story, particularly as it relates not only to the technique itself, or how the treatises relating to it were translated into English,[i] but also because it was closely related to the establishment of the architectural profession in Britain[ii].

It was however, through another door that the Descriptive Geometry would enter the stage of the development of educational system in England and play an important role in the development of the graphical communication related to architecture. Various methods of graphical operations which were incorporated into the system of descriptive geometry were already written about in works published since the first half of the 17th century both in England and in France.[iii] Records exist which point that

[i] See Booker (1963).
[ii] See Crison and Lubbock (1994).
[iii] Jousse - Secrets de l'Arcitecture 1642
Bosse - La practique des trait pour la Coupe des Pierres 1643
Derand - L'Architecture des Voutes 1643

From the front page of Lawrence's thesis.

Monge was aware of this connection and he taught descriptive geometry at Mézières in conjunction with the practice on stone-cutting. It was here, in seeing the origin of and the application to the stone-cutting, that Descriptive Geometry attracted interest from the architectural educationalists in England.

Freemasonry grew in London and was officially instituted some fifty years after the Great Fire of 1666. That the various Masons Companies and their associated lodges existed to provide professional services and institutional control for their members practising the trade of building ('operative masons') is well established.[iv] The myth of continuity of the ancient wisdom related to Geometry and Architecture and the relationship between the divine architecture (of the cosmos) and earthly architects, was invented, defined and publicised by the Masonic establishment with the first edition of the *Constitutions of Freemasons* published in 1723 in London.[v] It shows how Freemasonry claimed to be the natural and contemporary heir to the hidden wisdom as all the elements were successfully assembled together in Masonic ideology: building, geometry (Great Geometer) and the associated secrets. Stone-cutting was, through this association with the Divine Geometry and the Great Geometer elevated from a technique used within a profane craft to a technique whose applicability could be contemplated upon; the principles of stone-cutting and all the mathematical concepts employed became imbued, within an ethical, or even theological context, with a spiritual significance.

The thesis shown that while any direct influence from the concept of Sacred Geometry as a Masonic ideal appeared to have entirely evaporated by the end of the 19th century, an indirect influence has been maintained through a certain stratum of contemporary architectural theory in England, through the role of geometry in Masonic lore and the link established between Freemasonry and architecture, which brought about a concept of 'meaningful space'[vi].

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Of these, Frezier's work was the first to employ the horizontal and the vertical projections for the definition of voussoirs (Cunningham, 1868, p. 12); descriptive geometry later adopted the system of two projections without necessarily employing terms or concepts of what was horizontal and/or vertical.

[iv] For original sources ee Conder (1894), also London, Court of Common Council (1694).

[v] The first Constitutions of Freemasonry were written by Rev. James Anderson (1680-1739), who was a Scottish Presbyterian educated at the University of Aberdeen. He was commissioned by the Grand Lodge to provide a history of the Fraternity.

[vi] See Perez-Gomez (1983).

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* * *

George Zoumpos:
Mathematics in the Times of the Ionian Academy (1824-1864),
Ionian Univ. (Corfu, Greece), 2004

During the years of the British "Protection" (1814-1864) the Ionian Islands (Greece) started enjoying for the first time some degree of educational autonomy, with all three levels of public education functioning while their social and intellectual potential was being fully extended. In this way, the foundations for further development were being gradually laid and with the establishment of the Ionian Academy (1824-1864) a whole new scientific world was added which, in the long run, did also influence the history of the islands.

During the *first period* (1824-1832) of the Ionian Academy, which coincides with the term of Lord High Commissioner Adam, the figure of Ioannis Karandinos is dominant. Karandinos is the last great Ionian mathematician of the era of the Neo-Hellenic Enlightenment. In his relatively short academic career he translated some of the most important French books on elementary mathematics of the time and through the influence of his students he is the man who introduced French mathematics into Greek education curricula.

It is during this period that the academic productivity of the Academy in the field of mathematics reaches its peak, a fact reflected in the timetable schedules and the number of taught subjects. The graduate students went on to work in the secondary education schools of the Ionian Islands and also in the newly constituted Greek state.

The *second period* (1833-1844) starts with the retirement of Karandinos and is characterised by a decline of the activity previously achieved in the Academy. The main figure in this period is Ioannis Koundouris, who succeeds in mainatining the teaching of elementary mathematics at a relatively good standard up until the arrival of Ottavio Mossotti.

In 1835 many Italian professors joined the staff of the Academy. The leading figure among them was Mossotti, who taught Advanced Calculus, Mechanics and Astronomy.

The establishment of the new university in Athens in 1837 meant the start of the end for the importance and influence of the Ionian Academy as a major educational institution in Greece, although the process of decline was long and some good moments were still to be enjoyed.

The *third period* (1845-1855) was characterised by a general decline in all levels of education in the Ionian Islands that went on until the end of the British protection, with a simultaneous shortfall of its liberal characteristics. This period started with Andreas Mavromatis who taught advanced mathematics but his premature death deprived the Academy of a first-rate scientist.

In the *fourth period* (1856-1865) the main figure is Spyridon Katsaitis. Regarding this politically difficult period that led to the unification of the Ionian Islands with Greece, the retrieved data about the teaching of mathematics (but also those regarding the overall function of the Academy) are limited and sometimes only hypothesis can be advanced as a tool of reasoning.

In conclusion, the main characteristic of mathematics teaching in the Ionian Academy during the forty years of its existence is a steadfast orientation towards France and its main mathematical personalities. Apart from the local mathematicians, even the Italian Mossotti was deeply influenced in his methodology and conception by the leading French analysts and figures like Laplace, Poisson and Ampere.

In 1865 the Ionian Academy was closed and its historical role ended. Some forty years of service that can only be evaluated as positive not only for the field of mathematics but for the benefit of education in general all over the Greek state.

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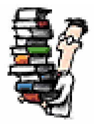

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[Chapter 6 in the book may be of interest to members of HPM. It is by SIU Man Keung, with the chapter title "*Official Curriculum in Mathematics in Ancient China: How Did candidates Study for the Examination*?" (pp.157-185).

The chapter abstract says: This chapter starts with a brief general account of mathematics education in ancient China, then discusses in detail the official curriculum and the state examination system in mathematics in the Tang Dynasty. In the second part of the chapter some examples of examination questions are re-constructed with "circumstantial evidence" to offer an alternative viewpoint from a traditional one, to argue that study in mathematics in ancient China did not proceed in an examination-oriented, rote-based learning environment. This "animated" historical account may help to shed some light on the comparative study of mathematics education in the East and West.

Man-Keung Siu, China

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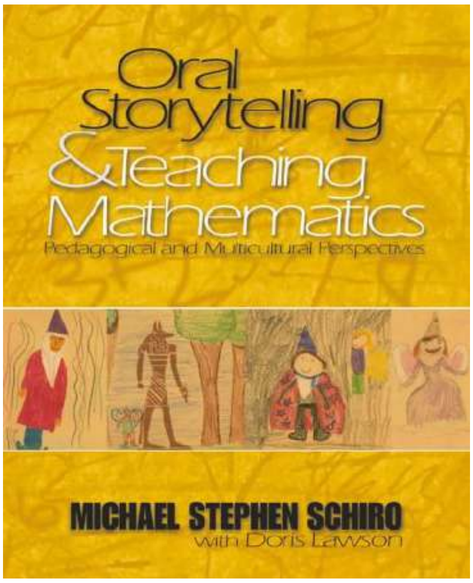

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* * *

Have you been here?

The Euler Society

http://home.adelphi.edu/~bradley/EulerSociety/

Homepage of Prof. Michael S. Mahoney

http://www.princeton.edu/~mike/

La galerie de portraits des mathématiciens

http://trucsmaths.free.fr/images/matheux/matheux_complet.htm

Teaching With Original Historical Sources In Mathematics

by REINHARD LAUBENBACHER and DAVID PENGELLEY
http://emmy.nmsu.edu/~history/

Ethnomathematics on the Web

http://www.rpi.edu/%7Eeglash/isgem.dir/links.htm

Mathematical Laboratory

In the Mathematical Laboratory of the University Museum of Natural History and Scientific Instruments there are about 160 mathematical machines built by teachers of Liceo A.Tassoni in the scope of an innovation project for the teaching of geometry developed by the Nucleo di Ricerca in Storia e Didattica della Matematica (University of Modena) . The project , coordinated by prof. Mariolina Bartolini Bussi (bartolini@unimo.it) has produced mathematical machines, didactical itineraries, animation films and *simulations* by computer. With these teaching aids we introduce a historical dimension and a manipulative and visual one in our classrooms. Our didactical research purpose is the historical contextualization of problems, of theories and of methods. Many "machines" realize projects or ideas of mathematicians, from the Ancient Greek up to now. For the use of these "machines" in the classroom, students have to elaborate abstract themes and proofs.
http://www.museo.unimo.it/labmat/usa1.htm

Links for the History of Mathematics

http://www.math.muni.cz/~sisma/history/internet.html

The British Society for the History of Mathematics web site at
www.dcs.warwick.ac.uk/bshm/
has many links to related sites.

The Italian Society of History of Mathematics web site is
www.dm.unito.it/sism/index.html

The HPM-Americas web site is
www.hpm-americas.org

AMUCHMA newsletter on the history of mathematics in Africa can be found at
www.math.buffalo.edu/mad/AMU/amuchma_online.html

History and Epistemology for the Teaching of Mathematics has been activated at the address:
www.syllogismos.it

Iris Gulikers' website (a unit for schools on surveying):
http://members.home.nl/gulikgulikers/WiskundePagina.htm

Convergence - where mathematics, history and teaching interact, [A Magazine of the Mathematical Association of America]
http://convergence.mathdl.org/convergence/1/

La Matematica in Italia (1800-1950), [Il Giardino di Archimede,
Un museo per la matematica]
http://www.math.unifi.it/matematicaitaliana/

Euclid in several languages are available at
www.euclides.org

The HPM History by Florence Fasanelli is at
http://www.clab.edc.uoc.gr/HPM/HPMhistory.PDF

ICMI is at http://www.mathunion.org/ICMI/

The editors welcome information about other sites.

* * *

Reports on past events

COMHISMA8 in Tunisia

by Randy K. Schwartz
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The Eighth Maghrebian Colloquium on the History of Arab Mathematics (COMHISMA8) was held successfully in Radès, Tunisia on December 18-20, 2004. This latest meeting was the eighth in a series of biennial conferences on the theme, which have rotated between Algeria, Morocco and Tunisia since 1986 (no meeting was held in 1996 or 1998).

For conferences treating the overall history of Arab mathematics, these biennial gatherings are "where the action is": they were the first such international meetings, and they remain the leading ones. Their purpose is to uncover and disseminate information about the historical contributions made by mathematicians in the Arab world. The colloquia are meant not only for researchers but also for educators and other people who want to help spread and popularise this knowledge.

COMHISMA8 was organized by the Tunisian Association of Mathematical Sciences (ATSM) and the Higher Institute for Continuing Education and Training (ISEFC), a unit of the University of Tunis. The International Organizing Committee was led by Ahmed Djebbar, and the National Organising Committee by Mohamed Souissi.

The meeting took place on the hilly campus of the National Centre for the Training of Trainers and for the Science of Training (CENAFFIF), about five miles south of the capital city of Tunis. The official languages were French, English, and Arabic. Roughly 40 people participated, arriving mainly from the countries around the Mediterranean. The narrative below weaves together details from many of the presentations.

Among the overall themes addressed were these:

- the relation between mathematics and the social, economic and cultural needs of society
- the application of mathematics to other domains of knowledge, such as astronomy
- the history of the dissemination and teaching of Arab mathematics
- the contributions of mathematicians from North Africa and Andalusia
- the discovery, publication, and translation of important manuscripts.

The recovery of manuscripts remains an important ongoing task in this subfield of the history of mathematics. Medieval Islamic treatises were composed or copied by hand, and thousands of these precious manuscripts have been lost over the centuries. Some key ones have never come to light in modern times. Many were destroyed in the Spanish Inquisition or other spasms of violence; others, their significance not grasped, were discarded, or might still be languishing somewhere. Still, of those that have been found, many have yet to be studied closely or published in redacted editions. All of this helps explain why the Arab contributions to mathematics that we know of so far, already very considerable, represent just the initial recovery of an even larger treasure.

Key Arab Breakthroughs

Several presentations helped deepen our understanding of contributions to such basic areas of mathematics as arithmetic, algebra, geometry, and trigonometry.

Today, we think of algebra and geometry as two distinct but complementary ways of exploring mathematical relationships. The fact that the word "square" can designate both a geometric shape and a formula for finding its area seems very natural to us. And when we see the algebraic equation $(x + y)^2 = x^2 + 2xy + y^2$, we can visualise it as a large square subdivided into two smaller squares and two rectangles. But Gert Schubring, from the Univ. of Bielefield, Germany, pointed out that what seems simple and natural to us today was an ingenious innovation over a thousand years ago. In his presentation on “The Process of Algebraization as a Determinant in the History of Mathematics”, he made the interesting observation that the Arabs invented algebra around 800 AD by first “returning” to ancient Greek geometry. They figured out how to represent a whole series of geometric propositions as algebraic equations like the one above. Although they justified their key steps with geometric arguments, the resulting equations were much easier to manipulate than were geometric figures. This enabled them to solve new types of problems such as cubic equations. They went on to complete the Greek theories of ratio and proportion. Al-Kashi (c. 1380–1429) figured out how to represent rationals and irrationals with decimal fractions, paving the way for modern concepts such as limits and real numbers.

Of course, algebra as initially devised by the Arabs didn’t use symbols like our $(x + y)^2$. It was rhetorical, with equations written out in words, including *shai’* (“thing”) to designate an unknown quantity. Rashid Bebboushi (from Algiers), in his talk on “Algebra and Algorithm, Same Origin, Different Evolution”, described how later, in the period 1100–1500, mathematicians from the Maghreb (such as al-Hassar, Ibn al–Yasamin, and al-Qalasadi) played a leading role in inventing algebraic symbols. They also helped free algebra once and for all from its geometric trappings. Especially important in the latter regard was Ibn al-Banna’ of Marrakesh (1256–1321), whose work spanned algebra and geometry as well as arithmetic, fractions, proportionality, combinatorics and number theory. Three other presenters – Anissa Harbili (Kouba, Algeria), Farès Ben Taleb (Rennes, France), and Ahmed Djebbar (Lille, France) – detailed the influence of Ibn al-Banna’ across all of North Africa based on manuscripts they’ve recently analysed by al-Ghurbi, Ibn al-Ha’im, and Ibn al-Majdi, respectively.

Mathematicians in the Middle Ages made important strides in solid geometry, driven in part by a keen interest in astronomical investigation based on the notion of a celestial sphere. One of the first scientific treatises translated into Arabic after the birth of Islam was the Greek work *Spherics* by Menelaus of Alexandria, which had been the first important work on spherical trigonometry. Pierre Pinel described to us how he and his colleagues in Toulouse, France are studying the ways that Arabs extended the work of Menelaus. For example, al-Mahani and others grappled with a problem involving anharmonic ratios on a sphere and their relation to a sine law. Emilia Calvo and Roser Puig (Barcelona, Spain), in their talk on “Andalusian Improvements in the Field of Astronomical Instruments”, summarised 25 years of research carried out by their team at the University of Barcelona. They have investigated the geometric theories behind the Arab astrolabe and many related instruments, which form a very important chapter in the history of applied mathematics.

Al-Mu’taman ibn Hud, a mathematician and philosopher who was king of Saragossa, Spain from 1081 until his death in 1085, wrote a text in which he explained the workings of plane and solid geometry as well as arithmetic and combinatorics. A presentation by Abdelmalek Bouzari (Kouba, Algeria) focused on al-Mu’taman’s treatment of conic sections. His colleague Youssef Gergour, in a separate talk, took up al-Mu’taman’s proof of the Pythagorean Theorem and compared it with other Arab proofs of the same result, such as that of al-Kashi, who was able to find and prove the generalisation that we know today as the cosine law.

How Religion Shaped Science

Western cultures tend to segregate matters of faith from matters of reason. By contrast, Islam holds that everything in the world is knowable and rational, and that Muslims should study every nook and cranny of the universe in order to understand God. In the Middle Ages, mathematics was viewed as a key to unlock such knowledge. This is a fundamental reason why the Muslim world became pre-eminent in mathematics and science at that time.

Islamic beliefs thus stimulated and shaped scientific discoveries, and mathematics developed differently under Islam than elsewhere. In ancient India, for instance, an interest in astrology had stimulated investigations of astronomy and mathematics. By contrast, Islam did not approve of astrological practices. On the other hand, the Qur'an prescribes that Muslims should pray at five designated times every day, based on the position of the sun, and that they should pray facing toward Mecca. The practical need to determine the correct prayer times and prayer direction, or *qibla*, at any point on the earth's surface, called forth new advances in geometry, trigonometry, and their application to astronomy and geography. Mercè Comes and Mònica Rius (Barcelona, Spain), in their talk "Finding *Qibla* in the Islamic Mediterranean Milieu", discussed the work of al-Tajuri of Tripoli, Libya, whose 15 treatises on astronomical instruments and *qibla* determination are still in the process of being studied. An instrument of the sort described by al-Tajuri might incorporate a compass, sundial, and other devices, allowing the user to determine the current time of day as well as the correct prayer times and direction. Besides instruments, there were calculations, maps, tables and diagrams of various sorts that were brought to bear on these problems.

Since it was considered immoral for Muslims to profit from one another through mere idleness or risk-taking, Islamic law banned interest-bearing loans and financial speculation. Eva Caianiello (Paris, France), in her talk on "Interest and Usury in Fibonacci's *Liber Abaci*: A First Comparison of the Mathematical Tradition of the Western Muslim World", explained to us that this prohibition hindered the development of an Arabic mathematical notation to deal with concepts such as simple and compound interest. This, together with many other factors, she said, might have contributed to European supremacy in business and banking. Eventually, however, Muslim financiers found ways around these prohibitions. Interestingly, the resulting policies on usury and other financial matters varied even between different sects (Hanafite, Malakite, etc.) of the main Sunni branch of Islam. The corresponding mathematics developed in different directions. Elena Ausejo (Saragossa, Spain) showed us elaborate grids used by Malakite judges in Spain for calculating each survivor's share of the estate left behind when a Muslim person died.

The partition of debts, wills and bequests according to Qur'anic law was an early stimulus for the development of algebra, and for centuries this remained one of the most important ways that algebra was used in the Muslim world. Much of al-Khuwarizmi's seminal treatise on algebra (Baghdad, c. 825) was devoted to inheritance problems of the *al-dawr* ("circular") type. As presenter Ezzaiem Laabid (Marrakesh, Morocco) explained, *al-dawr* problems involved several survivors whose portions of an estate had to be found, but determining any one of these unknowns depended on determining the others. This "circularity" could be resolved by such techniques as algebra or double false position. Laabid gave examples of Hanafite and Malakite equations for solving such problems, reflecting differing interpretations of Qur'anic law.

For many centuries, double false position (*hisab al-khata'ayn*), a numerical algorithm for evaluating linearly related quantities, played an important practical role under Islam. But how this technique first entered the corpus of Arab mathematics has been controversial, since similar methods were known in China and India in pre-Islamic times. By examining cross-cultural similarities and differences in approach, applications, and terminology, Randy Schwartz (Livonia, US) concluded that while the possibility of a borrowing can't be ruled out, it's more likely that double false position appeared independently in more than one region, most likely used by merchants and other practical people before it was taken up by trained mathematicians such as Qusta ibn Luqa and Ibn al-Banna'.

Religion and mathematics jointly influenced Islamic art and design. Muslims, believing that representational art tends to usurp the role of God as creator, turned instead to abstract geometry as a source of patterns. Ángel Ramírez Martínez (Calatayud, Spain) showed us how *mudejar* artists in Aragon (*mudejar* = a Muslim living under Christian rule in Spain prior to the Expulsion) used symmetry in their elaborate geometric ornamentations of the brick, wood and plaster on village towers and other public facades.

The Importance of Cultural Contact

Transmission and cross-fertilization played a huge role in the development of mathematics as a global enterprise. Knowledge migrated across cultural and linguistic boundaries, sometimes because of commercial trade contacts, other times because scientists or clerics dedicated their lives to finding and translating important treatises, even staying years in other countries for that purpose. Advances made by one group of people could thereby be adopted, extended and enriched by others.

Gabriela Fernandez Barberis (Madrid, Spain) reviewed the important role played by Spain in this regard, a result of several centuries of Muslim rule there. Christian monks and other scholars in cities like Toledo translated Arabic mathematical works into Latin. This led, in 976 AD, to the first use of "Indian figures" (i.e., Hindu-Arabic numerals with place-value ciphering) in a European language, followed by adaptations of Islamic methods of arithmetic, algebra, geometry, and trigonometry.

Pierre Pinel (Toulouse, France) discussed the influence of Leonardo Fibonacci (1170-1250), the Pisan merchant and mathematician. Leonardo learned Arab mathematical techniques at his father's counting house in Bejaia (a port on the Algerian coast) and in his subsequent commercial travels around the Mediterranean, then built on and popularized these in *Liber Abaci* and other books. Pinel tracks this borrowing of ideas by comparing manuscripts meticulously, paying attention not only to their mathematical content but to quirks of notation and to how words are translated or mistranslated. It was revealing, for instance, to find that Fibonacci used the fraction bar that had been invented a few decades earlier by the Maghrebian mathematician al-Hassar. He also adopted al-Hassar's succinct notation for composed ("continued") fractions such as:

$$\frac{4\ \ 6\ \ 8}{5\ \ 7\ \ 9} \quad \text{to represent} \quad \frac{\dfrac{\dfrac{4}{5}+6}{7}+8}{9}$$

$$\frac{7\ \ 1\ \ 2}{10\ \ 10\ \ 10} \quad \text{to represent} \quad \frac{\dfrac{\dfrac{7}{10}+1}{10}+2}{10}, \text{ i.e.,}$$

0.217.

The second example shows how al-Hassar and Fibonacci anticipated modern decimal notation, a step finally taken by al-Kashi.

Jens Høyrup (Roskilde, Denmark) updated us on his ongoing research into the sources of early-Renaissance Italian mathematics. By closely examining the techniques and terminology found in commercial *abbaco* texts of the time, he has found evidence that another strand of Arab mathematics made its way to Italy besides the one taken up by Fibonacci. According to another participant, Mahdi Abdeljaouad (Tunis, Tunisia), a possible source of that strand was the work of Ibn al-Yasamin of Marrakesh, a mathematician of mixed African/Berber origin. Abdeljaouad, in a presentation marking the 800th anniversary of the death of Ibn al-Yasamin, explained that his notation for whole numbers and fractions differed from that of al-Hassar and Fibonacci, yet it appeared subsequently in Italian texts.

Gregg De Young, of the American University in Cairo, has spent decades helping piece together how Euclid's *Elements* "returned to Europe" in the 1100's. This was an important chapter in the history of mathematics, in which Arabs preserved and extended ancient Greek learning that had been all but forgotten in Europe. In the major Latin translations of Euclid, which were made by Adelard of Bath and Gherard of Cremona, occasional tell-tale Arabic words are seen. These two mathematicians are known to have spent time in Muslim-influenced regions of Spain and Sicily. By closely examining text and diagrams, De Young concludes that these translations of Euclid cannot be traced entirely to any single family of manuscripts that survives today, but despite this complexity, he finds strong evidence that Adelard and Gherard had access to versions of the *Elements* not very different from those compiled three centuries earlier in Baghdad by mathematicians al-Hajjaj and Thabit ibn Qurra.

Indian Mathematics Teacher's Meet, Nasik, 2004

R. C. Gupta

The Association of Mathematics Teachers of India (AMTI: website: amtionline.com) held its 39th Annual Conference along with Kaprekar Centenary Celebration at Nasik from December 28 to 30, 2004. It was jointly hosted by the Nashik District Mathematical Association and another organisation devoted to the Development of Mathematics Studies of Teaching.

A beautiful 120-page souvenir entitled *Ankamitra Kaprekar Felicitation Number* was released at that time. D. R. Kaprekar (1905-1986) was a remarkable teacher devoted to mathematics who made significant contributions to the recreational theory of numbers. A number of booklets dedicated to his life and work were released:

(i) *The Wonder World of Kaprekar Numbers*, ed. By R. Athmaraman, AMTI, Chennai, 2004.

(ii) *Kaprekar Talented Mathematician* (in Marathi) by M. V. Dingankar, Pune, 2004 (with bibliography).

(iii) *Ganitanandi Kaprekar* (in Marathi) by S. P. Deshpande. Mumbai, dated 2005.

(iv) *Pearls of Mathematics* (in Marathi) by V. G. Tikekar, Nasik, 2004.

The *Souvenir* and the booklets contain rich historical and pedagogical material useful for learning and teaching mathematics.

The Inaugural Address was delivered by Dr. M. S. Gosavi, Secretary of Gokhale Education Society. He said that good teaching in mathematics can be achieved only by good teachers like G. K. Gokhale (1866-1915), late professor of Fergusson College, Poona.

In his Presidential Address on "Some Noted Properties and Patterns of Integers for Beginners", Prof. R. C. Gupta mentioned several interesting types of numbers whose structural niceties generate curiosity among students. These included Armstrong Numbers, Abundant Numbers, Polygonal Numbers (which were popular among Greeks), Ramanujan Number (1729) (smallest number which can be put as sum of two cubes in two ways), Kaprekar Constant (see below). The diabolic or pandiagonal magic squares were learnt by the English missionary Rev. A. H. Frost (19th century) at Nasik, and so he started calling them Nasik Squares (or even Nasiks!) Through his writings they were popularised in the West.

The P. L. Bhatnagar Memorial Lecture was delivered by Abhijit Ranjekar. His theme was the famous Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, … whose connection with the arrangement of leaves on plants, of petals on flowers, shell spirals, golden numbers etc. were nicely demonstrated through illustrations.

There were many other presentations which involved historical and other material useful for classroom teaching in mathematics. Poonam Gajrani pleaded that while teaching the four fundamental arithmetical operations to students, pleasant historical examples should be taken to make the learning curious and enjoyable. For example Kaprekar's discovery of his famous constant (6174) can be easily included in the subtraction process: Take *any* 4-digit (not all same) number say 5364. Arrange the digits in descending order; reverse the digits and subtract the new number from the above (formed by descending order); repeat the process with the remainder number:

6543 – 3456 = 3087
8730 – 0378 = 8352
8532 – 2358 = 6174
7641 – 1467 = 6174 (same remainder)

Thus we always get Kaprekar Constant 6174 which he discovered in 1946.

Queries are welcome.

Notices

Charming Ancient Babylonians

Introduction

I teach maths to boys and girls, who attend a high school course in which they are taught how to become either accountants or foreign office employees. This normally means that they have no wish to become great scientists and that they find it really difficult to see why they should swallow all those signs and procedures, let alone all that heavy thinking. They often do it only because they have no choice.

The students I'll be talking about are attending their first year in two different classes of our school and are normally just 14 years old, which, in spite of their desires, is very close to childhood. It is obvious that they still like to play the game "Let's pretend we were …", because they keep pretending that they are self-sufficient, pretend they don't care for knowledge; and often only pretend they are happy.

Well, I find that this fact is a wonderful opportunity to get round the passion problem.

Genesis

In late November 2004, during a meeting among teachers, my history colleague and friend (Laura Roverato), said that during the following couple of months she would be working on the civilisations of Mesopotamia and Egypt. She also proposed, to those who found it convenient, a teamwork aiming to convince the students that a connection between different subjects is often possible and interesting.

I immediately loved her idea because it was very promising in terms of fun and because it offered me an extra opportunity to present the charming and adventurous side of mathematics through the historical approach. So I enrolled, with reference to Babylonian mathematics.

To begin with, we agreed that the teamwork would consist in asking the students to refer on what they had been told by the partner teacher on a given subject. This was intended to prevent from repetitions and therefore, favour better investigations of the aspects we liked best. The final output would probably be a series of wall posters.

I knew the hard-working students would appreciate it, but I wasn't at all sure that the others (the majority) would share our enthusiasm. That means that I had to find a good trick to engage them.

First came the idea to present the research in the form of a bet. I imagined I would announce to the classes: "The history teacher and I have a challenge and need a jury to see in which subject the ancient Babylonians turn out to be most charming: can you face the responsibility of being such a jury?". All thrilled, and before anything else was done, I looked for information on the web and asked for help from "*Historia Mathematica*", a forum which I like and in which I am listed. As I had hoped, a few members of the list immediately offered me both very useful references and very nice team support. But I had got myself into a fine mess: how could I possibly find a way to tell Laura about a bet of which the whole world knew before her? I should have spoken to her first: now it was too late. Besides, I considered, there was something disturbing in the idea of a challenge: finding a winner would automatically draw the attention to the loser. Since the idea was mine, it wouldn't have mattered if I got beaten: I had looked for it. But what if Laura was defeated by the splendid references I could count on and which I had immediately started to study hard?

Wouldn't it be much nicer if we all could turn out to be winners?

And wouldn't it also be more effective if the students had an active role rather than a passive one? After all it's their motivation we wanted to stimulate.

That made it. I spoke to Laura and, in a couple of days, we had figured out that all we needed was a game in which our students could *pretend they were* successful grown-ups. Successful in a way that only knowledge could provide. So we decided to announce:

The First International Seminar On Babylonian Mathematics At The Virtual-University Of Cittadella

Organization

It was Dec. 5th by then, and we wanted the seminar to take place within the first school term, that is before the end of January. So we picked January 18th, which was a perfect date because then Laura would be teaching in one class at exactly the same time I would be teaching in the other. This fact granted us the possibility to organise things without having to ask anybody else, neither for permission nor for time. All we had to do was to book the school Aula Magna right away, so that we could hold the seminar there.

We wanted everything to look as solemn as possible and therefore gave the announcement to the students through a formal invitation card, which we had previously produced, in which we asked them to seriously consider the idea of giving a lecture on Babylonian mathematics (see its shape below).

After the students started asking, we explained that:

- I would be the chairman, introducing the subject and the lecturers;
- Laura would be the special guest, making sure that what was said kept close to the civilisation features;
- The enrolling students would play the part of university researchers, coming from the most prestigious mathematical departments in the world (with a label carrying their names and those of their colleges pinched on their chests);
- Everyone else would act as a journalist of the scientific press, attending the seminar to write an article for their editors, which Laura and I would oversee.

We also told them that, during the lecture, pictures would be taken both of the lecturers and of the scholarly audience and that this pictures, together with a short report, would be send to real university scholars, who knew about our experiment and wished to be informed of the accomplishments.

To begin with, they all wanted to act as journalists. Maybe they felt shy because people of another class would listen; or maybe they were less naïve than we had expected and looked for a price easier to touch than glory. So Laura and I decided that we would encourage them by spending a couple of hours each teaching them how to make a research both in a library and on the web.

But, eventually, restraint was stronger than ambition and later we even had to promise a good mark to those who showed off.

Does anybody know if this can be considered valid?

Accomplishments

Finally the day came and we all gathered in the Aula Magna for the event.

The lecturers, a little bit tense when they came to the speakers' table, took their notes with them but only rarely looked at them, which meant they pretty well knew what they wanted to say.

The journalists, all with their block notes and pens, sat in their chairs smiling but in respecting silence.

I must admit that the contents of the

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indirizzi di studio I.G.E.A. – E.R.I.C.A. – ITER – CINQUE – GEOMETRI P.N.I

La Signoria Vostra è invitata a partecipare al I° Simposio di Storia della Matematica dal titolo

LA MATEMATICA NELL'ANTICA MESOPOTAMIA

Che si terrà il giorno 18 Gennaio 2005 nell'Aula Magna di questo Istituto alle ore 09.00

Sarà gradito un suo intervento, di circa 7 minuti, su un aspetto del tema che vorrà indicarci entro il 10 gennaio per dar modo al Comitato Organizzatore di stilare il programma.

(copy of the invitation card)

seminar were not too profound and also that we could not convince too many students to act as lecturers (although all of them wanted their names pinched on the chest). Still, I believe, something was gained: some kind of awareness on the fantasy needed to invent mathematics, for instance; some kind of pride at referring new ideas to a listening audience, for those who dared; some faint irritation for not showing off for those who didn't dare (who knows?).

I also must add that the journalists' work was a really good surprise: there were no two similar reports and, although, of course, some were dull, in many others the students explained also things that had not been mentioned during the seminar. Their tidy articles (often enriched by drawings of Babylonian digits or by pictures of clay tablets taken from Internet) showed that they had done a good deal of research and that they had put some passion into it.

I like to think that this was their way of saying that they had enjoyed the experiment.

Acknowledgements

I wish to express my gratitude to the "real scholars". Let alone the very interesting references they gave me, and which I am still studying (for future improvements), I owe them the special push that comes from a worldwide fan club. So many, many thanks to:
Tony Mann from the University of Greenwich-UK, Greg Frederickson from Purdue University-USA, Jochen Ziegenbalg from the University of Karlsruhe-DE, Elaine Cohen from New Mexico State University-USA, Phill Schultz from the University of Western Australia-AU, Robert H. van Gent from Utrecht University-NL and Fulvia Furinghetti from the University of Genova-IT.

I'm particularly obliged towards Fulvia Furinghetti, who even proposed that I should write this article, giving me one of the biggest emotions in my teaching career.

Many thanks also to Daniela Bellabarba, a dear colleague in charge of many aspects of the school organization who has been capable of managing a serious emergency without depriving us of the Aula Magna on the day we needed it.

But above all I want to thank Laura and my students, who allowed me to play my favourite game - "Let's pretend we were…" - and did all that was necessary for me to thoroughly enjoy it.

Francesca Bevilacqua
Italy

More on Counting Tables

It seems that there are a few remaining counting tables in museums in northern Europe, in addition to the splendid one in Strasbourg illustrated in HPM 57. The table shown here is in the Historical Museum in Basel, Switzerland and is a simpler one than the Strasbourg table. The lines and lettering are inlaid with wood of a lighter colour and the table shows obvious signs of considerable use, with some repair work to the surface. The table has a raised lip all round to prevent counters, or jetons, from slipping off the table, and provision for two drawers for holding the jetons (one drawer is missing). The surface of the table is marked with three fields for calculating.

The divisions of the counting fields are marked in d. (denarius, penny), sh. (shilling), lib. (pound), X (ten), C. (hundred) and M (thousand). While the top of the table is rather plain, the base has carved ornate gothic leaf patterns.

Counting Table: Historical Museum, Basel

Table for calculating, showing d., sh., lib., X, C, M

Since beginning my enquiries I have discovered that a lawyer and numismatist from Lausanne, Colin Martin (d. 1995), had begun a survey of all remaining counting tables, which are much more numerous than the few reported by Karl Menninger. This inventory has been extended and published by Alain Schärlig, also of Lausanne, in his *Compter avec des Jetons* (subtitled 'Calculating and Accounting Tables from the Middle Ages to the Revolution'). The book, with 288 pages and 220 illustrations, provides a comprehensive description of some thirty tables and also eight counting cloths that are known still to exist, some in museums, some in private hands.

It would be interesting to know if there are any more of these tables, outside northern Europe. If you do know of any, please let me know.

Finally, I would like to thank Franz Egger of the Basel Historical Museum for his help in obtaining more information and also Alain Schärlig of the University of Lausanne for information about his forerunner in this field of enquiry.

Menninger, K. *Number Words and Number Symbols*, Dover reprint, 1992

Schärlig, A. *Compter avec des Jetons*, Presses Polytechniques et Universitaires Romandes, Lausanne, 2003

Contact: chrisweeks@eurobell.co.uk

Historical Modules for the Teaching and Learning of Mathematics

edited by Victor J. Katz and Karen Dee Michalowicz

This CD contains eleven historical modules, each being collections of lesson materials designed to demonstrate the use of the history of mathematics in the teaching of mathematics. They have been written by teams of college and high school teachers and have been field tested in a variety of situations. The materials can fit many different types of objectives and can be used in a variety of mathematics classes, from pre-algebra to calculus.

Each module consists of a number of activities, with the time frame for the activities varying from fifteen minutes to two or more weeks. Some of the activities may be used to introduce a topic. Other activities are better used as supplements, once the students have an initial familiarity with the subject matter. However these activities are used, the authors believe that the history will enable students better to understand the mathematical ideas.

Each activity contains Student Pages, designed to be duplicated and distributed to the students, and Teacher Notes for the teacher. The Student Pages are often designed as discovery lessons, with questions to guide students toward understanding the mathematical ideas. Sometimes, they are more like traditional textbooks, designed to be read in conjunction with a teacher's presentation. Written assignments are given for some of the activities. The Teacher Notes make suggestions as to when and how the activity should be used, and also contain solutions to the problems. They may also contain masters for transparencies. Websites are frequently mentioned for easy reference to relevant topics and biographies of mathematicians. Modules also include bibliographies for students and teachers

interested in further study of the mathematical content from a historical perspective.
The eleven modules are as follows:

- Archimedes: Activities from the work of Archimedes.
- Combinatorics: The elementary formulas for combinations and permutations along with an introduction to probability.
- Exponentials and Logarithms: The development of the exponential and logarithmic functions with applications.
- Functions: The general idea of a function, with illustrations from many sources.
- Geometric Proof: An historical study with numerous examples.
- Lengths, Areas, and Volumes: Activities from around the world dealing with the measurement of these quantities.
- Linear Equations: The idea of a proportion along with the solution of linear equations and systems of linear equations.
- Negative Numbers: How these quantities are used and why.
- Polynomials: Methods for solving quadratic and cubic equations, as well as more general polynomials.
- Statistics: Basic concepts of statistical reasoning, including graphs.
- Trigonometry: From the creation of a sine table to the measurement of plane and spherical triangles.

The CD requires Adobe Acrobat Reader 5.0 or later. Navigation through the files can be accomplished easily by using the Bookmarks feature Adobe Acrobat.

The CD is available from the Mathematical Association of America for US $41.95 ($33.95 for members). It can be ordered by going to www.maa.org, clicking on Bookstore, and then either on New Publications or on Classroom Resources.

Victor J. Katz, USA

* * *

Problématique – A Question on Euler

In 1745, Euler adopted Leibniz's law of continuity and called it an "incontestably certain law of nature according to which nothing happens by leaps". As a consequence, he rejected the existence of perfectly hard bodies:

"It follows therefore that a perfect hardness […] does not accord with the Laws of nature".

Study the development of the controversy about hardness and elasticity of bodies in the context of philosophy, mechanics, and mathematics and explain Euler's position.

See as a first reference Wilson Scott's book: *The conflict between atomism and conservation theory: 1644-1860*, London : Macdonald Elsevier , 1970

Gert Schubring, Germany

Complementary to the comment above on Euler's rejection of the concept of a "hard" (rigid) body are the following observations:

As it stands, Euler's point of view seems to be compatible with much later developments in Einstein's Special Theory of Relativity in the early 20th century (obviously, Euler could not have foreseen these developments which took place in an entirely different conceptual framework!):

The concept of a rigid body (in its classical sense; a body such that the distance between any two of its points remains constant in time) is incompatible with Einstein's Special Theory of Relativity, because in such a body, signals (e.g. sound waves) would be transmitted instantaneously, hence at a speed greater than that of light (see e.g. J.L. Synge, *Relativity: The special theory* North Holland 1962, p.36; W. Rindler *Essential Relativity*, Springer 1977, 2nd edition, p.36). Already in 1907, just two years after his seminal paper that laid the foundations of special relativity theory, Einstein pointed out that " … to date both the dynamics and the kinematics of the rigid body …. must be considered unknown" (see Einstein's biography by A. Pais, *Subtle is the Lord … The science and the life of Albert Einstein*, Oxford University Press 1982, p.154). In fact in 1909 M. Born gave a definition of what is called a "*rigid motion*", but P. Ehrenfest in the same year showed that a body in such a motion cannot be set in rotation. The next year F. Noether and independently G. Herglotz showed that a body in rigid motion has only 3 degrees of freedom, hence that it is specified by the motion of only one of its points (in contrast to the classical case, in which there are 6 degrees of freedom; cf. Euler's theorem in classical mechanics that the motion of a rigid body with one point fixed is a rotation). Finally, M. von Laue in 1911 showed that a body in special relativity has an infinite number of degrees of freedom. For more details on the original references, see the classical review paper by W. Pauli in the *Mathematical Encyclopedia* in 1921, reprinted as W. Pauli, *Theory of Relativity*, Pergamon Press 1958/ Dover Publications 1981, §45.

C. Tzanakis, Greece

* * *

History of Mathematics, Religious Traditions and Mathematics Education.

A bibliographical survey

In the last few decades, some papers have been published that focus on the relation between mathematics and religious traditions. In most of these papers the emphasis is on the historical and educational aspects of this relation, a fact evidently compatible with the *HPM* perspective. We think that raising this issue may enrich the *Newsletter* and widen the interests of the *HPM* Group.

In this direction, Andy Norton's "trilogy" in *The Mathematics Educator* (2002 and 2003) stresses the different influence of religious beliefs in the personal attitude adopted towards mathematics by a Christian, a Jew and a Buddhist. This is a comparative, hence, in principle wide, approach. In addition, looking for correlations between religious beliefs and different attitudes towards mathematics may contribute to better understand the methodological characteristics of these different attitudes. In this way, any interrelation between religious beliefs and mathematics is seen as a factor that shapes mathematics as a cultural artefact, either in a historical, or educational perspective.

From this point of view, the (definitely incomplete) bibliographical survey below (in which major religious traditions are practically absent), is not only informative in character, but also may constitute a fruitful challenge for a deeper understanding of this issue. We invite all those who are interested in this subject and/or know about work that has been done, to supply more references that will enlarge and complete the present list and we acknowledge the immediate response by W-S. Horng, G. Schubring and M-K. Siu.

CHRISTIANITY (Roman Catholic & Evangelical tradition)

Chase, G.B./Jongsma, C.: *Bibliography of Christianity and Mathematics: 1910-1983*, Dordt College Press, 1983.
[See also http://www.messiah.edu/acdept/depthome/mathsci/acms/bibliog.htm]

de Klerk, J.H.: Can mathematics be taught in a Christian way? http://www.acmsonline.org/deKlerk99.pdf

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Kvasz, L.: Mathematics and the History of Religion, *Human Affairs,* 9, (2), 1999, pp. 110-125.

Luoma, K.: Mathematics and Religion: An Augustinian Synthesis http://www.aug.edu/dvskel/Luoma2002.htm

Nickel, James: *Mathematics: Is God Silent?,* Ross House Books, 2001.
Norton, A.: Mathematicians' Religious Affiliations and Professional Practices: The Case of Charles http://jwilson.coe.uga.edu/DEPT/TME/issues/v12n2/v12n2.html
Malet, Antoni: Isaac Barrow on the Mathematization of Nature: Theological Voluntarism and the Rise of Geometrical Optics, *Journal of the History of Ideas* 58.2 (1997) 265-287.
Shelford, April G.: Thinking Geometrically in Pierre-Daniel Huet's Demonstratio evangelica (1679), *Journal of the History of Ideas*, 63 (4), 2002, pp. 599-617.
Sriraman, Bharath: The Influence of Platonism on Mathematics Research and Theological Beliefs, *Theology and Science*, Vol. 2, No. 1, 2004, pp. 131 – 147.

Relevant to Roman Catholic tradition

Crombie, A.C.: Mathematics and Platonism in the Sixteenth-Century Italian Universities and in Jesuit Educational Policy, in Maeyama,Y. / **Saltzer, W.G**.(eds.): *ΠΡΙΣΜΑΤΑ Naturwissenschftsgeschichtiche Studien*, Franz Steiner Verlag, 1977, pp.63-94.
Dear, P.: Jesuit Mathematical Science and the Reconstitution of the Experience in the Early Seventeenth Century, *Studies in. History and Philosophy of Science*, 18 (2), 1987, pp. 133-175.
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Feingold, M. (ed.): *The New Science and Jesuit Science: Seventeenth-Century Perspectives*, Kluwer Publ., 2003.
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Homann, F.A.: Christopher Clavius and the Renaissance of Euclidean Geometry, *Arch. Hist. Soc. Iesu,* 52, 1983, pp. 233-245.
Mazzotti, M.: Maria Gaetana Agnesi: Mathematics and the Making of the Catholic Enlightenment, *Isis*, Vol. 92, No. 4. (Dec., 2001), pp. 657-683.
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[**Smolarski, D.C**. (trans.)]: Historical Documents, Part I. Sections on Mathematics from the Various Editions of the *Patio Studiorum*, *Science in Context*, 15 (3), 2002, pp. 459-464.
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Seneta, Eugene: Mathematics, religion, and Marxism in the Soviet Union in the 1930s,

Historia Mathematica, 31 (3), 2004, pp. 337-367

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Lancerman, Y. Tzvi & Simonson, S. : The Hebrew Mathematical Tradition, in Selin H.(ed.): *Mathematics Across Cultures. The History of Non- Western Mathematics*, Kluwer, 2000, pp. 167-203.
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Norton, A.: Mathematicians' Religious Affiliations and Professional Practices: The Case of Joseph http://jwilson.coe.uga.edu/DEPT/TME/Issues/v12n1/v12n1.html

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Golshani, M.(ed.): *Can Science Dispense with Religion?*, Institute for Humanities and Culture Studies, Tehran, 2002.
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BUDDHISM, DAOISM

Norton, A.: Mathematicians' Religious Affiliations and Professional Practices: The Case of Bo http://jwilson.coe.uga.edu/DEPT/TME/issues/v13n1/v13n1.html
Volkov, A.: Science and Daoism: An Introduction, *Taiwanese Journal for Philosophy and History of Science*, No.8 (Vol.5, No.1), April 1996, pp. 1-58.
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Relevant publications by Gert Schubring

"Pure and Applied Mathematics in Divergent Institutional Settings in Germany: the Role and Impact of Felix Klein", *The History of Modern Mathematics. Volume II: Institutions and Applications* eds. David Rowe, John McCleary (Boston: Academic Press 1989), 171-220.

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"Recent research on institutional history of science and its application to Islamic civilization", ed. Athanasios Gagatsis, *Learning in Mathematics and Science and Educational Technology*. Vol. II Nicosia: Intercollege Press, 2001), 57-76.

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N. Kastanis, C. Tzanakis

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Announcements of events

4th Colloquium on the Didactics of Mathematics (with international participation)

Friday 22 & Saturday 23 April 2005
Rethymnon, Crete, Greece
http://www.edc.uoc.gr/4colloquium

The 5th CIBEM (Ibero-American Congress in Mathematical Education)

July 17-22, 2005
Oporto, Portugal

The four previous editions of this Congress took place in Sevilla, Spain (1990), Blumenau, Brazil (1994), Caracas, Venezuela (1998) and Cochabamba, Bolivia (2001). This time the event is an iniciative of the Associação de Professores de Matemática, in collaboration with Faculdade de Ciências and the Centro de Matemática da Universidade do Porto and will be hosted by the Departments of Mathematics. One of the Discussion Groups (GD06) in this Congress is dedicated to the History of Mathematics and will be chaired by Sérgio Nobre, from UNESP, Rio Claro, Brazil. Registrations fees are €130 until the 30th March and €200 until the 15th June. You can register on-line at http://www.mytwt.net/cibem5

Carlos Correia de Sá, Portugal

The 22nd International Congress of History of Science

24-30 July 2005

Beijing, China

The general theme is "Globalisation and Diversity". Discussions will focus on the diffusion of science and technology between different cultures in the past, and its impact on the world today, as well as its prospects for the future advance of human civilisation.

The First Circular is available from the Congress Website: http://2005bj.ihns.ac.cn

For further information, please contact to the Congress Secretariat:
Secretariat of the 22nd ICHS
Institute for History of Natural Science
Chinese Academy of Sciences
137 Chao Nei Street
Beijing 100010
CHINA
e-mail: 2005bj@ihns.ac.cn

The 1st International Conference on History of Exact Sciences along the Silk Road

July 31-August 3, 2005-02-17

Xian, China

The 1st International Conference on History of Exact Sciences along the Silk Road will be held at the Department of Mathematics, Northwest University, Xian, China from July 31 to August 3, 2005.

This series of conferences will provide a platform for those who are interested in the comparative studies on history of mathematics and astronomy in the old civilisations along the Silk Road, from China to Europe, including Japan, Korea, India, Arabic/Islamic countries, Mesopotamia, Egypt and Medieval Europe. It is expected that every 2 or 3 years a conference will be organised in different countries. Each time the conference will focus on one or two special topics, and about 10 invited speakers who come from all the fields mentioned above will offer plenary talks on the main topics.

The main topics at the 1st conference are:

1. Transmission and transformation of exact sciences
2. Mathematical methods in astronomy

At the same time, contributions related to the history of mathematics and astronomy along the Silk Road are also welcome.

For further information, please contact to Professors Qu Anjing (Executive Vice-President of the Conference) or
Yang Baoshan (Member of Local Organizing Commitee), the address of both persons is:
Center for the History of Mathematics and Sciences, Northwest University,
Xian 710069
CHINA
e-mail: <hs@nwu.edu.cn> .

Osamu Kota, Japan

6th International Symposium on the History of Mathematics and Mathematical Education using Chinese Characters (ISHME)

August 4-7, 2005

Tokyo, Japan

The 6th ISHME will call its participants to discuss general issues related to the history of mathematics and mathematical education in East Asia. In addition to these topics, the Symposium will make the mathematics in East Asia from the 16th through the 19th centuries in global network a special subject of discussion.

The First Circular is available from the Secretariat of the ISHME6.

For further information, please contact to the Congress Secretariat:

Secretariat of the ISHME6
Prof. Kobayashi Tatsuhiko
Maebashi Institute of Technology
460-1 Kamisadori
Maebashi, Gumma, 371-0816
JAPAN
e-mail: koba@maebashi-it.ac.jp

Osamu Kota, Japan

The Euler 2005 Conference

August 7-10, 2005
Portsmouth, RI, USA

The theme of the conference will be the life and work of Leonhard Euler during the 1760s. For more information, see http://www.EulerSociety.org/

Mathematical Textbooks: History, Production and Influence

September 24-25, 2005
Oxford, UK
A joint meeting of the Oxford University Department for Continuing Education and the BSHM.
Rewley House Organiser: Raymond Flood
BSHM Organisers: Jackie Stedall, June Barrow-Green (j.e.barrow-green@open.ac.uk)
Further details will be available in due course.

The first Iranian workshop on the history of mathematics

October 12-15, 2005
Zirab, Iran

Held by the Iranian Mathematical Society and Shahid Beheshti University.The major items to be discussed are as follows:

- History of arithmetic
- History of geometry
- History of algebra
- History of astronomy
- History of optics
- History of mathematical entertainments and enigmas

The workshop will be held in Persian for a limited number of participants. For more information contact Dr. M.-GH. Vahidi at the Mathematics Faculty of Shahid Beheshti University (Tehran).

Mohammad Bagheri, Iran

Espace Mathématique Francophone : Colloque EMF 2006

May 26-31, 2006
Québec, Canada
General Theme**:** L'enseignement des mathématiques face aux défis de l'école et des communautés 26-31 May 2006**,** *Université de Sherbrooke (Québec, Canada)*
This is a meeting regularly organized on a three-year basis since 2000. The first two have taken place in Grenoble, France (2000) and Tozeur, Tunisia (2003). These meetings, recognized as regional conferences by ICMI, aim to explore the current important problems in Mathematics Education at all levels of instruction, and by taking into account the existing cultural diversity, to favour the emergence of a French-speaking Mathematics Education Community. The EMF meetings are addressed to all those involved in Mathematics Education (mathematicians, researchers in mathematics education, mathematics educators, teachers of Mathematics at all levels). The official language of the meetings is French.

There will be 8 working groups, each one on a different theme. The theme of the group no3 is *Intégration des dimensions historique et culturelle des mathématiques dans leur enseignement* coordinated by Mahdi Abdeljaouad (Tunisie), Abdellah El Idrissi (Maroc), Louise Poirier (Québec)
More information is available at the EMF 2006 web site
http://emf2006.educ.usherbrooke.ca/
Contact: emf2006@usherbrooke.ca

Anne-Michel Pajus, France

5th European Summer University on the history and epistemology in mathematics education

July, 2007
Prague, Czech Republic
For more information, see below.

ICME-11

July 6-13, 2008

Monterrey, Mexico

ICME-11 will take place in Monterrey (Mexico), at the "CINTERMEX", the convention centre of the city. There is the tradition to organize the Satellite meeting of HPM in sites 'close' to the venue of ICME: for example, in 1992 ICME was in Quebec city and the Satellite meeting was in Toronto (both in Canada), in 1996 ICME was in Sevilla (Spain) and the Satellite meeting in Braga (Portugal), in 2000 ICME was in Tokyo/Makuhari (Japan) and the Satellite meeting in Taipei (Taiwan), in 2004 ICME is in Copenhagen (Denmark) and the Satellite meeting in Uppsala (Sweden). We encourage the members of HPM to submit proposals for the organization of the Satellite meeting of ICME-11.

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5TH EUROPEAN SUMMER UNIVERSITY

On History and Epistemology in Mathematics Education

July 2007, Prague, Czech Republic

This is a preliminary announcement. More detailed information, both on the scientific program and practical matters will be available, hopefully in the next issue of the *Newsletter* (No 59, July 2005).

This is going to be the 5th meeting of this kind. The initiative of organizing a *Summer University* (SU) on the *History and Epistemology in Mathematics Education* belongs to the French Mathematics Education community, in the early 1980's. From those meetings emerged the organization of a SU on a European scale, as the *European Summer University* (ESU) *on the History and Epistemology in Mathematics Education*. Some more information on the previous ESU is given in the Appendix below.

1. Aim and main themes of the ESU

The ESU aims, not only to stress the *use* of history and epistemology in the teaching and learning of mathematics, in the sense of a technical tool for instruction, but also to reveal the following aspects of mathematics:

• Mathematics should be conceived as a living science, a science with a long history, a vivid present and an as yet unforeseen future

• This conception of mathematics should not only be the core of the teaching of mathematics, but it should also be the image of mathematics spread to the outside world.

In this connection, emphasis put on historical and epistemological issues of mathematics may lead to a deeper awareness of the fact that mathematics is not only a system of well-organized finalized and polished mental products, but also a human activity, in which the processes that lead to these products are equally important with the products themselves. In particular, integration of historical and epistemological issues of mathematics in mathematics education may help to realize that

• Mathematics is the result of contributions from many different cultures;

• The philosophy of mathematics has evolved through the centuries;

• The teaching of mathematics has developed through the ages;

• Mathematics has been in constant dialogue with other sciences

• Mathematics has been a constant force of scientific, technical, artistic and social development

The ESU is neither a collection of intensive courses, nor a conference for researchers, but something in between. More specifically, it is a place where beginners, more experienced researchers and teachers from all levels of education present their teaching experience to the benefit of the participants and get a constructive feedback from them. The programme and activities of the ESU are structured around some *main themes*, which will be specified and announced shortly (for the themes of the previous ESU, see the Appendix).

2. Activities during the ESU

The ESU includes a few *plenary lectures* and *panels*. However, a major part of the ESU consists of *workshops*. The scientific program of the ESU will be structured along its *main themes*.

Plenary lectures and *workshops* should refer to these main themes.

In the *panels* the participants will work together, well before the ESU, so that there is a real discussion among them and/or with the panel coordinator during the panel session

In *workshops* the participants read and work on the basis of material distributed by the organizer of the workshop (e.g. original texts, chosen and briefly presented by the organiser, or teaching material used or proposed by the organizer etc). This means that there are many workshops in parallel, which may vary in duration (say, from 2 to 5 hours). It would be very good and stimulating if there were workshops, which elaborate on the general ideas presented in the plenary lectures.

In addition, there will be parallel sessions with *oral presentations* for participants who want to speak about their own experience, or research. This is an activity in the spirit of a conventional research conference.

Finally, it is expected that *poster sessions* and *exhibitions* of books and other didactical material will also be present in this ESU.

3. Target population

The major part of the participants is expected to be (elementary or secondary) schoolteachers, who may wish to gain new ideas on how they can integrate the history of mathematics into their teaching. However, there will be also university teachers and students, interested in the history and epistemology of mathematics and their integration into mathematics education, as well as, historians of mathematics, who may give lectures and workshops to inform others about recent developments in their domain, and mathematicians with an interest in the relation between mathematics, its history and epistemology, and its role at present and in the past.

4. Time and place

The 5th ESU will take place in July 2007 at the Faculty of Education, Charles University, in Prague, Czech Republic. The exact dates will be announced in due course.

5. The (international) *Scientific Program Committee* (SPC) and the *Local Organizing Committee* (LOC)

Neither the SPC, nor the LOC have been finalized yet, but both will be announced in due course. In general the SPC will include members of the international community (from European, or even non-European countries, as it has happened in the past) and of the country that hosts the ESU. The LOC should consist of our Czech colleagues, members of the hosting institution and/or the scientific and educational community of the Czech Republic

6. The web site

Making known the ESU in various countries (in Europe and beyond) is a major task to be realized by the SPC. To this end, a well structured and regularly updated web site will be available at http://www.pedf.cuni.cz/kmdm . This is going to be a very efficient tool to make known the ESU worldwide, to allow for online registration etc.

7. Proceedings

Publishing the Proceedings of the ESU is also a major task. In fact, Proceedings of the previous ESU have become standard references in this area (cf. the Appendix). It is important to have high quality reports and texts in the Proceedings. To this end, the members of the SPC will review the submitted proposals at the usual international standards.

APPENDIX*: *Some information on the European Summer Universities on the History and Epistemology in Mathematics Education

Brief history and statistics of the previous ESU

The initiative of organizing a *Summer Univeristy* (SU) on the *History and Epistemology in Mathematics Education* belongs to the French Mathematics Education community in the early 1980's. It was the French IREM's (*Institut des Recherches sur l' Enseignement des Mathématiques*) that organized the first interdisciplinary SU on the History of Mathematics in 1984 in Le Mans, France. It was followed by other SU in France (1986 in Toulouse, 1988 in La Rochelle, and 1990 in Lille). The next one was organized in 1993 on a European scale, and was called the 1st *European Summer University* (ESU) *on the History and Epistemology in Mathematics Education*, (a name coined since then), but many participants in it and in the subsequent ESU came outside Europe.

The previous ESU (see next table) took place in July,

- 1993, Monpellier, France

- 1996, Braga, Portugal (conjointly with the *HPM* Satellite meeting of ICME 8)
- 1999, Louvain-la-Neuve & Leuven, Belgium
- 2004, Uppsala, Sweden (conjointly with the *HPM* Satellite meeting of ICME 10)

Remarks:

(a) In the 2nd ESU there was only one *plenary lecture*, but many *introductory lectures*, which run in parallel and which were addressed to schoolteachers, providing an introduction to the topics elaborated in the workshops.

(b) The 2nd and 4th ESU have been organized conjointly with the *HPM* Satellite Meeting of the corresponding ICME (ICME 8 and ICME 10, respectively)

(c) In most ESU, more than half of the participants were local people: Portuguese in the 2nd ESU (310); French in the 1st ESU (134). In the 3rd ESU about 40% were Belgians (64). Thus, in general, there was a strong participation from local people, mainly primary and secondary schoolteachers.

(d) In general, a key element of the program was the great number of workshops, which gave the opportunity to presenters to explain their ideas, teaching practice, share their experience with participants and distribute relevant material. The workshops were of variable duration usually, from 1 to 3 hours.

(e) Non-local participants came from many countries, either European, or from other continents, although with a few exceptions, only a small number from each country (usually less than 5, or 6).

Themes of the previous ESU

The activities and the program of each ESU were structured around some *main themes*, which were the following:

1st ESU Montpellier, France, 19-23/7/1993

-The historical construction of mathematical knowledge
-Introducing a historical perspective into the teaching of mathematics
-The relationship between mathematics education and culture
-Epistemology and its relationship to didactics and pedagogy
-History of mathematics in initial teacher training and in-service courses
-Mediterranean mathematics
-Ethnomathematics

2nd ESU Braga, Portugal, 24-30/7/1996

Main themes:
-Mathematical cultures all over the world
-Mathematics as a science
-Mathematics, arts and technics

Special topics:
-History of mathematics education
-Epistemological obstacles
-Views on Mathematics
-Mathematics for all
-Mathematical proof in history

ESU	Duration	No of participants	Number of talks, workshops etc
1st Montpellier France	19-23/7/1993, 5 working days	254 from 29 countries (17 European)	5PL, 2PN, 48WS, 37T
2nd Braga, Portugal	24-30/7/1996, 5 working days+a morning session	548 from 33 countries (14 European)	1PL, 28IL, 4PN, 33WS, 71T
3rd Louvain-la-Neuve /Leuven, Belgium	15-21/7/1999, 6 working days	159 from 22 countries (16 European)	6PL, 2PN, 37WS, 35T
4th Uppsala, Sweden	12-17/7/2004, 4 working days+ two half morning sessions	120 from 32 countries (15 European)	6PL, 2PN, 9WS, 59T

PL=Plenary lecture
PN= Panel discussion
WS=Workshop
T= Talk/ oral presentation
IL=Introductory Lecture

3rd ESU Louvain-la-Neuve /Leuven, Belgium, 15-21/7/1999

There were not any main themes specified a priori. However, themes proposed in due course included
Mathematical journals in Europe and their use in education
-The historical construction of mathematical knowledge
-The relation between mathematics and science in history; its in education
-Relations between mathematics and music up to Euler's era; their use in education
-History of mathematics education
-Mathematicians in the Low Countries
-About the 19th century geometry: the Belgian theorems; what may be the insights for the education?

4th ESU Uppsala, Sweeden 12-17/7/2004
-The history of mathematics
-Integrating the history of mathematics into the teaching of mathematics
-The role of the history of mathematics in teacher's training
-The common history of mathematics, science and technology
-Mathematics and different cultures
-The philosophy of mathematics

Proceedings
An important aspect of the ESU has been the publication of its Proceedings. In the 2nd and 4th ESU the Proceedings became available in advance and were distributed to the participants on the spot.

1st ESU: *Actes de la première Université d' Été Européenne sur l' Histoire et Épistémologie dans l' Éducation Mathématique*, F. Lalande, F. Jaboeuf, and Y. Nouazé (editors), IREM de Montpellier, Université Montpellier II, Montpelier, France, 1995 (598 pages in one volume).

2nd ESU: *Proceedings of the 2nd European Summer University on the History and Epistemology in Mathematics Education and the ICME 8 Satellite Meeting of HPM*, M.J. Lagarto, A. Viera & E. Veloso (eds), Portuguese Association of the Teachers of Mathematics & Department of Mathematics, University of Minho, Braga, Portugal, 1996 (813 pages in two volumes).

3rd ESU: *Proceedings of the 3rd European Summer University on the History and Epistemology in Mathematics Education*, P. Radelet-de-Grave & C. Brichard (editors), Université Catholique de Louvain, Leuven and Louvain-la-Neuve, Belgium, 2001 (944 pages in two volumes).

4th ESU: *Proceedings of the HPM 2004: History and Pedagogy of Mathematics ICME 10 Satellite Meeting and 4th European Summer University on the History and Epistemology in Mathematics Education*, F. Furinghetti, S. Kaijser & A. Vretblad (editors), Uppsala University, Uppsala, Sweden, 2004 (482 pages in one volume).

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