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## Message from our Chair

### HPM 2004

I'm writing this message having the  
homepage of the HPM Satellite meeting of  
2004 on the background of my screen.



Each image reminds me of paths in the  
preparation of the meeting. In the late 2000  
John Fauvel and I received the first offers for  
organising the 2004 meeting, just after the

meeting in Taipei. We both agreed that  
Uppsala would be a good choice and the  
preparation work started. Unfortunately John  
passed away and now he can not see how his  
project was realised.

I found a really collaborative partner in the  
department of mathematics of the Uppsala  
group and after my first visit to Uppsala I was  
very enthusiastic about the project. Now we  
are ready and the conference program is  
scheduled. The logo of the conference  
reminds me the glorious past of the place  
where we will meet: it represents the bowl on  
the top of the Gustavianum Museum, where  
memories of the glorious scientists of the past  
(Carl von Linné for one) are kept.

The program of the conference encompasses  
many different types of activities: just like the  
tradition of the past conferences. The  
conference will be held jointly with the  
Fourth European Summer University History  
and Epistemology.

We invite all the HPM members to contribute  
and to participate. It will be nice to meet  
together. For further information look at the  
website

<http://www-conference.slu.se/hpm/index.html>

Fulvia Furinghetti

## Editorial

We have a big 24 page issue this time with  
two major articles and lots of reading material  
to digest. Thank you to all who sent in  
contributions – I look forward to meeting  
readers in Copenhagen and/or Uppsala!

Peter Ransom  
Romsey, UK

# Power of Innovation: A Historical View<sup>1</sup>

## I. Introduction

Innovation of science and technology can happen in many ways. Take nanometer science and technology for example. For now it seems to be a good time for scientists to pay due attention to the scale of  $10^{-9}$  meter on which, by applying the current scientific theory, new discoveries related especially with materials have been made. Tracing back to the history of science, a parallel can be identified with 18<sup>th</sup> century science when Newtonian mechanics had been applied to as many fields as it could. This may well explain why historian/philosopher Thomas Kuhn uses the concept “normal science” to characterize science of the century in Europe. Central to the “normal science” period, as he argues, is puzzle-solving. (cf. Kuhn, 1962) Therefore, if science moves forward in this context, then “innovation” seems better than “progress” in explaining its change or growth.

Concerning mathematics itself, the “innovation” is a term by which we can understand even better how the discipline develops or evolves. In the history of mathematics, people never tear down the old building but add the new story to the old structure (cf. Dauben, 1994). Scientific development in the Western world has made several breakthroughs after, say, Copernicus’ revolution in astronomy (1543) and Newton’s revolution in physics (1687). By contrast, “mathematics has a mechanism of reinterpretation that resolves threats of competition. For example, the discoveries of non-Euclidean geometry did not force mathematicians to choose between these and Euclidean geometry. Instead mathematicians reinterpreted geometry so that both the Euclidean and non-Euclidean varieties could

be included in a systematic relation.”

(Tymoczko ed. 1986, p. 215)

Given that “innovation” makes more sense in the history of mathematics than that of science, how can mathematical knowledge be innovated through the reinterpretation to do better service to the people in the new era of knowledge management? In this process how can we learn from the history of mathematics? These are the main issues to which I am going to address in this article.

## II. An Episode

To me knowledge service is an alien term against my professional expertise, I was not sure if I could “manage” the right stuff for the conference on knowledge service. In preparing this talk however, I came across Peter Drucker’s books on management, largely at Dr. Fang-Mei Lin’s recommendation. She was teaching sociology in campus but now serves as the Commissioner of the National Youth Commission, the Executive Yuan of the central government, Taiwan. This episode is quite instructive for it may well reflect how a “bare” scholar can learn from the management and adapt herself very successfully to the governmental administration.

In my observation, Fang-Mei Lin’s success in the administration is all due to her professional expertise in the academic fields like literature, sociology and journalism.<sup>2</sup> No wonder she would remind me of Peter Drucker’s comments on the new printing revolution in terms of the internet environment. Indeed, the new technology provides very convenient and people-affordable access to information database. Yet it is the knowledge core content itself makes a difference in creating something new, although the production may wear a fancy IT (information technology) outlook. In this sense, it is the printing media products make

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<sup>1</sup> The early version of this article was presented to International Conference for Knowledge Services 2003, March 5-6, 2003, Taipei: Grand Hotel.

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<sup>2</sup> General comment on her success is due to the fact that she leads her staff to innovate the administrative strategy in a way similar to supervisor versus graduate student.

use of the new technology but not the way around. The true demand of the knowledge-based economic era is, therefore, how to innovate knowledge more accessible for ready use in people ordinary life.

I do not intend, at this point, to ignore how fascinating the internet system that offers a high speed of access. However, as one readily gets to some internet homepages related to mathematics education hosted by high school teachers who are skilful at the new technology, there is usually nothing major or substantial from which you can really learn. The situation looks just like that you drive very fast on a freeway yet nothing in the scenery is attractive. In order to deal with the myth that infrastructure provided by the internet system means everything; perhaps it is time for us to pay more attention to the knowledge proper. And that is why I am pleased to be here for a talk to the audience with whom I am yet to get familiar.

In what follows I am going to talk first about the power of symbolism, then turn to the power of analogy. I will refer basically to examples/demonstrations from the history of mathematics. Yet, in order to justify what I am going to argue for, I will go on to illustrate how the history can be innovated to benefit the mathematics education.

### III. Power of Symbolism

In addition to being the key to a method of demonstration and an invaluable pedagogical aid, symbolic notation also proved to be the key to a method of discovery. Let us take for example Newton's concept of "infinite" polynomial. When Newton (1642-1727) tried to expand  $\frac{1}{1-x^2}$  into a power series, he simply used the long division to get the desired series:  $1+x^2+x^4+\dots$ . Similarly, he used the method of extraction for square root to get the power series of  $\sqrt{1-x^2}$ :  $1-\frac{1}{2}x^2-\frac{1}{8}x^4+\dots$ . In these two cases, Newton also took the advantage of the "analogy" between the

polynomial and the power series, a topic that we will leave for Section III.

Apparently in a similar vein, Leibniz, Newton's contemporary and protagonist in study of mathematics and science, derived the formula for the differential of product of two functions,  $d(xy) = xdy + ydx$ , in the following manner:

$$\begin{aligned} d(xy) &= (x+dx)(y+dy) - xy \\ &= xy + xdy + ydx + dxdy - xy \\ &= xdy + ydx + dxdy \end{aligned}$$

in which  $dx$  is the differential of  $x$  etc. Now, as argued Leibniz, since "the quantity  $dxdy \dots$  is infinitely small in comparison with rest, and hence can be discarded." Therefore, we get the desired formula.

In the eighteenth century, Euler the master of the century "elevated symbolic manipulation to an art". His method of expanding  $\cos x$  into a power series was as "real" as incredible! First of all, he used the De Moivre's formula:

$$(\cos z + i \sin z)^n = \cos nz + i \sin nz$$

in which  $i = \sqrt{-1}$  is the imaginary number, to conclude:

$$\cos nz = \frac{(\cos z + i \sin z)^n + (\cos z - i \sin z)^n}{2}$$

By expanding the right side, he now had the following:

$$\begin{aligned} \cos nz &= (\cos z)^n - \frac{n(n-1)}{1 \cdot 2} (\cos z)^{n-2} (\sin z)^2 \\ &+ \frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4} (\cos z)^{n-4} (\sin z)^4 - \dots \end{aligned}$$

Again letting  $z$  be infinitely small,  $n$  infinitely large, and  $nz = v$  finite, it follows from  $\sin z = z$  and  $\cos z = 1$  that

$$\cos v = 1 - \frac{v^2}{1 \cdot 2} + \frac{v^4}{1 \cdot 2 \cdot 3 \cdot 4} - \dots$$

By trusting the power of manipulating symbolism, the great mathematicians Newton, Leibniz and Euler all "discovered" amazing and important formulas which proved to be crucial in the development of 18<sup>th</sup> century mathematical analysis. Perhaps these three great mathematicians were lucky since they did not need to care about the concepts of

uniform convergence and absolute convergence, which are now very much emphasised in modern textbooks on calculus or advanced calculus. So here is the moral: Follow the intuition, it pays.

How can this “lesson” be put into the textbook of modern calculus? In his calculus textbook, Professor Liang-Chi Tsao, now the deputy director of the College Entrance Examination Centre, Taiwan, recommends his students to expand  $\tan x$  in terms of  $\sin x$  and  $\cos x$  by the method of long division:

$$\tan x = \frac{\sin x}{\cos x} = \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots}{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots} = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots$$

Prof. Tsao, who got a Ph.D. from the University of Chicago, majoring in number theory, recently retired from National Taiwan University. In addition to his research in pure mathematics, he has been writing popular mathematics for over thirty years. This may well explain why he was so enthusiastic about introducing “historical” method or “genetic” approach to his students by apparently urging them to neglect mathematical rigor temporarily.

#### IV. Power of Analogy

“Analogy” in ancient Greek sense is a very interesting term (cf. Polya, 1986a, 1986b). Put it in a modern way, in the proportion “ $a : b :: c : d$ ” the notation “ $::$ ” was read to be “analogous to”. This interpretation is also in accordance with Greek notion of the ratio “ $::$ ” in which “ratio” means “logos” or “expressible”. For example, Euclid’s illustration for his own “formula” for the circle measurement - “Circles are to one another as the squares on their diameters” - can be understood in the following way: circle  $S_1$  to circle  $S_2$  is “analogous to” square  $D_1$  to square  $D_2$ . Therefore, we can explain “ $S_1 : S_2$ ” in terms of “ $D_1 : D_2$ ”.

Thus, if something beautiful in ancient Greece means “well-proportioned”, then it is

no wonder why the concept of “analogy” can be closely related to art in their society.

In the history of Chinese mathematics, we can also find something related to analogous thinking, namely, the thirteenth-century mathematician Yang Hui used the analogy to deal with the pile summation problem. He “visualized” the problem in terms of that of area and volume. This may well explain why the term “Bi Lei” (literally comparing and categorizing) is adopted in the very title of one of his texts (cf. Lam, 1977; Wang, 2002).

Now we move to Johann Kepler, the 17<sup>th</sup> century great astronomer who is well-known for his discovery of the so-called three laws of planetary motions. Here first please look at how Kepler made sense of Archimedes’s

formula for the circle measurement:  $\frac{1}{2}Cr$ ,

where  $C$  and  $r$  respectively are circumference and radius of a circle.<sup>3</sup> In fact, Archimedes’s formula is as follows: “The area of any circle is equal to the area of a right triangle in which one of the legs is equal to the radius and the other to the circumference.” So Kepler first sliced a sector of the circle and then stretched to become a piece of the right-angled triangle. Analogously, Kepler also tried to convince that the formula for the volume of a sphere is

also sensible:  $\frac{4}{3}\pi r^3$ , where  $r$  is the radius of

the sphere. His approach is to cut the sphere into pieces that are congruent “pyramid” with “square” base. Each pyramid has the volume of  $\frac{1}{3}rm(B_k)$ , where  $m(B_k)$  is the area of the square base of the pyramid with height  $r$  (the same as the radius). Now sum up every pieces of the pyramid, we obtain the result:

$$\sum_k \frac{1}{3}rm(B_k) = \frac{1}{3}r \sum_k m(B_k) = \frac{1}{3}rS = \frac{1}{3}r \cdot 4\pi r^2 = \frac{4}{3}\pi r^3,$$

<sup>3</sup> One of my favorite graphical illustrations for the circle measurement is provided by Abraham bar Hiyya ha-Nasi (?-1136), a 12<sup>th</sup> century Jew mathematician worked in Spain. Cf. Grattan-Guinness (1997), p. 123.

where  $S = 4\pi r^2$  is the surface area of the sphere. The last is also due to Archimedes whose discovery of the formula is as illuminating as that of the volume formula of the sphere. I will come back for the topics in the next section.

To conclude this section, it still deserves to pay attention again to Euler for his discovery

of the summation of the series  $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

also illustrate quite well how he get the “Eureka” by analogy. Here we re-write his procedure as follows. Given the algebraic equation of degree  $n$ ,

$$a_0 + a_1x + a_2x^2 + \dots + a_nx^n = 0$$

has non-zero roots  $\alpha_1, \alpha_2, \dots, \alpha_n$ . The polynomial on the left hand side can be represented as a product of  $n$  linear factors:

$$\begin{aligned} & a_0 + a_1x + a_2x^2 + \dots + a_nx^n \\ &= a_n(x - \alpha_1)(x - \alpha_2)\dots(x - \alpha_n) \end{aligned}$$

By comparing the terms with the same power of  $x$  on both sides of the identity, one derives the relations between the roots and the coefficients of an equation, the simplest of which is for the term  $x^{n-1}$ :

$$a_{n-1} = -(\alpha_1 + \alpha_2 + \dots + \alpha_n).$$

There is another way of presenting the decomposition in linear factors. If none of the  $\alpha_1, \alpha_2, \dots, \alpha_n$  is equal to 0, or if  $a_0$  is non-zero, we have also

$$\begin{aligned} & a_0 + a_1x + a_2x^2 + \dots + a_nx^n \\ &= a_0\left(1 - \frac{x}{\alpha_1}\right)\left(1 - \frac{x}{\alpha_2}\right)\dots\left(1 - \frac{x}{\alpha_n}\right) \end{aligned}$$

and

$$a_1 = -a_0\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \dots + \frac{1}{\alpha_n}\right).$$

Now consider the (trigonometric) equation:  
 $\sin x = 0$

or

$$\frac{x}{1} - \frac{x^3}{1 \cdot 2 \cdot 3} + \frac{x^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} - \dots = 0.$$

Since the “polynomial” has “infinite degree”, now wonder, says Euler, that there is infinitely many roots

$$0, \pi, -\pi, 2\pi, -2\pi, 3\pi, -3\pi, \dots$$

Euler discards the root 0. He then divides the left side of the equation by  $x$ , then

$$\frac{\sin x}{x} = 1 - \frac{x^2}{2 \cdot 3} + \frac{x^4}{2 \cdot 3 \cdot 4 \cdot 5} - \dots = 0$$

has the roots  $\pi, -\pi, 2\pi, -2\pi, 3\pi, -3\pi, \dots$ .

Therefore,

$$\begin{aligned} \frac{\sin x}{x} &= 1 - \frac{x^2}{2 \cdot 3} + \frac{x^4}{2 \cdot 3 \cdot 4 \cdot 5} - \dots \\ &= \left(1 - \frac{x^2}{\pi^2}\right)\left(1 - \frac{x^2}{4\pi^2}\right)\left(1 - \frac{x^2}{9\pi^2}\right)\dots \end{aligned}$$

By comparing the coefficients of  $x^2$ , we have

$$\begin{aligned} \frac{1}{2 \cdot 3} &= \frac{1}{\pi^2} + \frac{1}{4\pi^2} + \frac{1}{9\pi^2} + \dots \\ \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} &= \frac{\pi^2}{6}. \end{aligned}$$

“Analogously,” Euler also obtains formula, among others, like the following:

$$1 + \frac{1}{4^2} + \frac{1}{9^2} + \frac{1}{16^2} + \dots = \frac{\pi^4}{90}.$$

## V. History in Mathematics Education: How innovative can it be?

In this section, I want to share some illustrations from the history of mathematics, which may serve to enlighten both students in the classroom and the general public. First of all, it is to discuss just how we can learn from a contrast of Euclid and Liu Hui (third century AD) in their dealing with the circle measurement. In Liu Hui’s case, a claim to transform the circle into a rectangle whose sides are the radius (half diameter) and the half the circumference of the circle is the key strategy in proving the area formula for the circle: Half the diameter times half the circumference. Indeed, Liu Hui’s idea comes from his “new” perspective in regarding the half diameter and half circumference as two sides of some rectangle. (cf. Horng, 2000)

In this connection, it is hardly to believe that editors of elementary school mathematics textbook in Taiwan would be shy away from

that similar to Liu Hui's treatment.<sup>4</sup> Perhaps it is all due to their being stick to the time-honoured formula  $\pi r^2$ . The last formula made its first appearance in the 9<sup>th</sup> century when Arabic mathematician Benu Musa wrote his commentary to Archimedes's *Measurement of a Circle*.

In this treatise, Archimedes gives rigorous demonstration by the method of reduction to absurdity. Despite that the presentation is not adequate to high-school classroom, Archimedes left the most remarkable legacy in the history of mathematics, i.e., *The Method*, in explaining how he discovered so many formulas on area and volume. Take for example his strategy in discovering the volume formula for a sphere by means of the principle of lever is as entertaining as illuminating. Apparently due to his flexibility in thinking about and doing mathematics, Archimedes has been ranked one of the greatest mathematicians in the history of mathematics.<sup>5</sup>

Now, how mathematical knowledge in historical context can be innovated in order to put into classroom or for other purpose in the new economic era? Perhaps it deserves to cite Hans Freudenthal's concept of "guided re-

invention", a quite popular principle of mathematics education in the Netherlands. According to Dr. Barbara van Ameron, "[it] specifies that students have the opportunity to experience the development of a mathematical matter similar to its original development. To this purpose a learning route needs to map out along which students can find the intended mathematics for themselves." (Van Ameron, 2002, p. 53) In other words, it is no need to follow exactly the same ways of mathematicians in history. Rather, one should be guided by a route leading to re-invention of mathematical concepts, methods and theorems, in which simulation of their original developments are emphasized. It is in this very connection that history can play a due role in the teaching and popularization of mathematics.

## VI. Concluding Remarks

I hope the above argument and illustrations should have made it clear that innovation in mathematics can be characterized as a new narrative about old stuff. When both powers of symbolism and analogy are introduced to participate in mathematical activities, they act as roles of narrative and in turn benefit mathematics in an unexpected manner. On the other hand, mathematicians also adopt the use of analogy in their attempt to explain features of mathematical knowledge to the general public. For example, mathematics looks like a tree or mathematics can be tour-guided as a scenery map. These are, among others, favourite descriptions that have appeared in popular mathematics writing. However, a comparison of mathematics with narrative is more to the point if one wants to explore in depth the status of mathematical knowledge. In this connection, different people can tell story of mathematics in diversified ways. When a new narrative is applied due to new perspective, mathematics will be innovated in a fresh manner. This can serve to inspire people that innovation of knowledge is easily available to the general public only if we can "manage" it in a meaningful way!

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<sup>4</sup> Apparently the editors of the textbook adopt an approach similar to Liu Hui's in explaining how the circle area formula becomes the form: 3.14 times radius square, where 3.14 is an approximation of  $\pi$ . In doing so, they have to refer to the formula for the relationship between the circumference and the diameter of the circle, namely  $C = \pi \cdot D$ , where D is the diameter. Combining the latter with the circle measurement  $S = (C/2)(D/2)$  (ancient Chinese form), it follows the above mentioned "conventional" formula for the circle measurement. One should be noted, however, that by assuming the formula  $C = \pi \cdot D$  to prove the conventional formula for the circle measurement is neither convincing nor illuminating. This is because, I gather, both the formulas for the circle and its circumference lie in the same conceptual level.

<sup>5</sup> It is time-honored to rank Archimedes, Newton and Gauss to be the three greatest mathematicians ever in the history. It deserves, however, to note that historians of mathematics in the past two decades are not interested in the "historiographical" problem as to who is the greatest in the history of mathematics.

In conclusion, it deserves to cite R. S. D. Thomas's comment on what means a good idea in mathematics: "A really good idea in mathematics, like Descartes' representation of loci by equation, is not cashed out by proving it but by proving things with it; it has a revelatory power that the best stories have in their different way." (Thomas, 2002) By comparing mathematics and narrative, I gather Thomas has successfully proposed an optimal way to innovate mathematics. And to this it would be even more fruitful to add one extra dimension, namely HPM!

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## A Historical Mathematics Tour of London and Surrounding Areas

For the last ten years I have been taking student tours to England looking at the historical mathematics. The tours, designed for pre-college students, have been popular at my school and most successful. In planning the tour I investigated most the sites that a high school student would like to see in London. I then added a mathematical twist to each place we visited.

I use Casterbridge Tours, a British Tour Company for my adventure. They have been most helpful through the years. If I can provide information to anyone who wants to do something similar, please send me an e-mail – [karendm@aol.com](mailto:karendm@aol.com).

The following is my itinerary:

DAY 1 DEPARTURE in late evening.

DAY 2 ARRIVE LONDON HEATHROW - VISIT HAMPTON COURT - Hampton Court Palace. This splendid palace was built for Thomas Wolsey and then taken over by Henry VIII after the cardinal's fall from grace. We visit the State Apartments, the Great Hall and the Gardens, complete with a visit to the intriguing mathematical Maze. After an Orientation Tour of the local area we have some introductory sightseeing at the British Science Museum.



There we view the Babbage Exhibit, ancient scientific instruments and polyhedra models.

### DAY 3 DAY TRIP TO GREENWICH

We make a full day excursion to Greenwich, a place that was for centuries the home of Kings and the site of Britain's finest collection of 17th and 18th century buildings. We tour the Cutty Sark, the last of the great China clippers that once ran tea from the Orient to Europe. We visit the Royal Naval College designed by Christopher Wren, the National Maritime Museum and the Old Royal Observatory, also designed by Wren. The Observatory was constructed in 1675 by Charles II in order to perfect the arts of navigation and astronomy. Since that time, the globe's longitude and time zones have been measured from the Greenwich Meridian.



### DAY 4 LONDON

Our day begins at Westminster Abbey where English kings and queens have been crowned since 1066. We view the Science Corner to see the tombs of Newton, Darwin, and Kepler amongst others. We then walk past the Houses of Parliament and Big Ben before taking a River Trip through the heart of London to the Tower of London where Newton lived for a



time and was the Master of the Royal Mint. We have the opportunity to discover William the Conqueror's magnificent bastion, which displays almost a thousand years of English history. During our visit, we see the Crown Jewels Exhibit.

#### DAY 5 DAY TRIP TO BATH, STONEHENGE AND SALISBURY

Our visit to Bath includes a tour of the Roman Baths where we have a discussion about Roman and Greek architecture and how it is related to mathematics and the golden ratio. We will be introduced to ancient math at Stonehenge, Britain's most famous ancient monument.



The importance of mathematics and the calendar is discussed. We continue to Salisbury where we see the Magna Carta before climbing the Cathedral Tower for a fascinating architectural and mathematical insight. At Salisbury Cathedral we hunt for the mathematical tomb.

#### DAY 6 LONDON

We make a morning visit to St. Paul's Cathedral, Sir Christopher Wren's wonderful Renaissance masterpiece. We will make an ascent of the dome to the Whispering Gallery for an impressive view of the concourse below. The Whispering Gallery will lead us to a discussion about the mathematics of domes. We visit the memorial to Florence Nightingale, the first woman statistician, in the crept of the Cathedral, along with visiting Wren's tomb. We continue on to the British Museum where we view by the Egyptian Collection, the Elgin Marbles. At the British Museum we have a private tour of the Rhind Mathematical Papyrus and the Mathematical Leather Roll. We also see the mathematical cuneiforms. During our day we also visit the British Library where we will see the Magna Carta, Essex's death

warrant, Newton's letter to Robert Hooke on gravity, and a variety of maps. We also take the opportunity to visit a London Street Market for a true 'taste' of London life before we have dinner and attend an evening Theatre Performance.

#### DAY 7 LONDON

Our morning begins with a walk through Green Park to Buckingham Palace to watch the Changing of the Guard. We continue to Trafalgar Square and Nelson's Column before spending the afternoon at Madame Tussaud's Waxwork's Museum. Madame Tussaud (1761-1850) perfected her craft during the French Revolution by taking death masks of guillotine victims before fleeing Paris and arriving in Britain with her macabre collection. Today they vary from the breathtakingly lifelike to the barely recognizable, cover celebrities from pop stars to the Pope. We also visit the Sherlock Holmes Museum. During our final evening in London, we have a session of logic and deductive reasoning at the Sherlock Holmes Pub.

#### DAY 8 DEPARTURE FROM LONDON

Karen Dee Michalowicz  
The Langley School, McLean, USA  
George Mason University, Fairfax, USA

A map of Scandinavia and surrounding regions, including Norway, Sweden, Finland, and Denmark. Key cities like Oslo, Helsinki, Uppsala, and Stockholm are marked. A blue line indicates a path or route across the region. An inset map shows the location of the conference in Sweden.

[www-conference.slu.se/hpm/index.html](http://www-conference.slu.se/hpm/index.html)

**HPM 2004**  
Miss it – miss out!

## Work in progress

We encourage young researchers in fields related to HPM to send us a brief description of their work in progress and a brief description of their dissertation

## Reviews

In you would like to be involved in reviewing books or magazines for this section, please send your contact details and area(s) of interest to the editor who will forward books or magazines for review as and when they become available.

The views expressed in this section are the views of the reviewers and may not necessarily be those of the HPM Advisory Board.

*Study The Masters. The Abel-Fauvel Conference*, O. Bekken and R. Mosvold (eds), NCM, Göteborg, 310 pages.

In June 2002 Bengt Johansson and Otto Bekken organised a meeting in Norway to celebrate the 200th birthday of Niels Henrik Abel, a meeting which is also to be considered as a “pre-conference” to the tenth ICME and to the HPM congress in 2004. This meeting appears as a cornerstone to all who spend great energy in stressing the essential role of the History of mathematics in the teaching of the subject, and as a consequence, the remembrance of John Fauvel comes up naturally. The proceedings of this meeting are now available: *Study The Masters. The Abel-Fauvel Conference*, edited by Otto Bekken and Reinar Mosvold.

Niels Henrik Abel wrote that in order to understand mathematics reading masters was more important than reading pupils, whence Otto Bekken’s claim at the opening of the conference: “Read the masters!”. Many chapters of this book are incitation to read the great masters of History like Euclid,

Descartes, Galois, Abel, but also many masters in the background. Who has ever heard of Friedrich Arentz or John Blagrove? The former was a Norwegian mathematician, the latter an English one. This book stresses how much the inheritance of the various national mathematical cultures is a gorgeous resource of practical interest for teaching mathematics nowadays.

This book is made up of five different parts. The first part focuses on the relevance of Abel’s work in the whole of mathematics, and the second concerns Fauvel’s work on the necessity of History of mathematics in the teaching of mathematics itself. Nine papers on Scandinavian mathematics form the third part of the book, while the fourth part sets forth two excellent reasons to reinstall Euclid in the teaching of mathematics. The fifth and final section puts forward excellent ideas from the European seventeenth century.

Otto Bekken claimed “Read the masters!”, and we add “Do read this book before our Uppsala meeting!”

Evelyne Barbin  
Nantes, France

In the last HPM Newsletter there appeared a review of Costa, C., *José Vincente Gonçalves: Matemático... porque professor!* (“*José Vincente Gonçalves: mathematician... what a teacher!*”), (PhD thesis), Universidade de Trás-os-Montes e Alto Douro, Portugal, 2000 I can now acknowledge the full name of the reviewer: Eduardo Fernandes (Portugal). My apologies again to Eduardo for the omission.

If you wish for a book to be reviewed, please send it to the editor who will arrange for it to be reviewed.

## Have you read these?

- Radford, L.: 2003, 'On the epistemological limits of language: Mathematical knowledge and social practice during the renaissance', *Educational studies in mathematics*, v.52, 123-150.

This article tries to answer the following important questions of mathematics education "Can we ascribe to language and to the discursive activity the force of creating the theoretical objects of the world of individuals?" The author suggests that all effort to understand the conceptual reality and the production of knowledge cannot restrict themselves to language and the discursive activity, but that they need to include the social practices that underlie them. It is remarkable that this point is illustrated through the analysis of the relationship between mathematical knowledge and the social practice of the Renaissance. The study is largely based on treatise of the Italian Abacist school.

- The book *History in mathematics education: the ICMI Study* (J. Fauvel, & J. Van Maanen (editors), Kluwer, Dordrecht-Boston-London) is reviewed by Bob Burn in the journal *Educational studies in Mathematics* (v.52, 211-214, 2003).

The author examines the main issues of the different chapters. In particular, he focuses on the notion that *ontogeny recapitulates phylogeny*, a theoretical perspective which emerges in many contributions. Also the author stress the presence in the book of surprising 'historical events' which are carried out in classroom, such as a performance based on mathematics life, history as the unifying theme of the whole curriculum, geometrical patterns in Mozambican artefacts.

- In the September 2002 the *International conference on mathematics education into 21st century* was organised by the Department of Mathematics of the University of Palermo. The proceedings are published in <http://dipmat.math.unipa.it/~grim/jourdain.htm>.

Among the various articles you may find articles concerning the relation of history with mathematics education, see for example

Spagnolo, F.: 2000, 'The role of the history of mathematics in research in mathematics education', *International conference on mathematics education into 21st century*.

- In the website <http://dipmat.math.unipa.it/~grim/articles.htm> you may find the following article of the subject which concerns HPM

- Spagnolo F., 'History and Ethno-Mathematics in the interpretation of the process of learning/teaching', 13 ICMI Comparative Study Conference, University of Hong Kong, 20-25 October 2002.

- *Des agrimensores romains aux arpenteurs du XVIème siècle*, catalogue of the exhibition organised on the occasion of the 125th anniversary of l'Union des Géomètres-Experts immobiliers de Bruxelles in the Chapelle Nassau of the Bibliothèque Royale de Belgique (22 November-21 December 2001). Introduction, communications and comments by G. Chouquer, J. De Graeve, M. de Waha, C. Lavigne, P. Godding & J. Mosselmans. Published by Union des Géomètres-Experts immobiliers de Brussels UGEB-ULEB, Bruxelles.

In 1976, on the occasion of the centenary of the foundation of the Society, the surveyor-experts of Brussels organised an exhibition concerning their profession from the sixteenth century until Napoleon. Luckily an analogous initiative was carried out in the occasion of the 150 years of the Society. The book we are considering is the catalogue of the exhibition held in 2001. The matter is organised in seven sections, which contain an introduction to the subject of the section and an annotated list of the exhibited pieces:

- A. History of the profession and of the land law in Brussels
- B. The Roman land-surveyors

- C. The transmission of knowledge. In this section we see that among the exhibited pieces there are manuscripts and the first printed version of Euclid's *Element*, as well as other interesting authors (Gerbert d'Aurillac, John Hollywood, Vitruvius...).
- D. Astrolabia
- E. Books of land-surveying of the sixteenth century
- F. Books of instruments for land-surveying
- G. The instruments for land-surveying.

We are delighted to note that these professional workers have such a strong interest for the history of their profession and the documents (books, instruments) that accompanied its development. For mathematicians the book it is a further evidence of the strict links between their discipline and the land-surveying. The documents exhibited could provide good materials for mathematics teaching in classroom.

We hope to be able to establish contacts and fruitful collaboration between our community (HPM) and the surveyors' society.

Fulvia Furinghetti  
Università di Genova, Italy

## L'Enseignement Mathématique

The Proceedings of the meeting for celebrating the centenary of the journal *L'Enseignement Mathématique* have been recently published. In the following we report on the meeting as appeared in *L'Enseignement Mathématique* (t.49, 2001, p.181–183), and the presentation of the Proceedings written by Bernard Hodgson, present General-Secretary of ICMI.

***An International Symposium Organised Jointly by the University of Geneva and the International Commission on Mathematical Instruction (Geneva, 20–22 October 2000) by Geoffrey HOWSON (Former ICMI Secretary from 1983 to 1990)***

It was in 1899 that Henri Fehr (Geneva) and Charles-Ange Laisant (Paris) founded the international journal *L'Enseignement Mathématique*. An event to mark that centenary was delayed by a year in order that the celebrations should also serve as a contribution to the World Mathematical Year 2000. The form that the celebratory symposium took was a historical survey of developments in mathematics education at key periods in the 20th century followed by a short discussion of the manner in which mathematics education might meet the demands of societies today and in the near future.

The meeting was jointly organised by the University of Geneva (the home of *L'Enseignement Mathématique*) and ICMI because of the long-standing relation ICMI has had with the journal. At the time that Fehr and Laisant launched it there were already a few periodicals devoted to mathematics education — but these were all national and normally the organs of national teacher organisations. The new journal was the first to seek an international audience and early in its life it began to carry articles describing current teaching practices in different countries. This led the US teacher educator, D.E. Smith, to suggest, in a paper published in its pages in 1905, that an international commission should be established to enquire into mathematics teaching in countries worldwide. Smith's suggestion was formally submitted to the International Congress of Mathematicians held in Rome in 1908 where it was accepted and resulted in the founding of the *Commission internationale de L'Enseignement Mathématique* (CIEM), the body from which ICMI developed. Fehr was appointed Secretary General of the Commission (a post he held until his death in 1954) and *L'Enseignement Mathématique* became the new commission's official organ. Since that time there has always been a link between the two bodies and it is in this periodical that the history of ICMI can best be traced. In that journal and in its indices we can, for instance, follow the development of ICMI's 'enquiries' and 'studies'. (One in the

index for 1959 I found particularly intriguing: “Appendice au questionnaire préparant l’enquête sur la pénurie des professeurs”. No doubt ‘pénurie’ should be translated as ‘scarcity’ — a suitable topic for a present-day ICMI study — but the faux-ami ‘penury’ would prove equally appropriate!)

Three periods, the beginning of the century, the years of ‘modern math’, i.e. 1950-70, and the present, and three themes, geometry, analysis, and applications of mathematics: mathematics as a service subject, were chosen for study. Thus for example, one session was devoted to thirty-minute talks on geometry in each of these periods, a reaction and a plenary discussion. In the final session there were talks on international activity, current periodicals in mathematics education, and mathematics education within society. The meeting began with an account of the foundation of *L’Enseignement Mathématique*.

There was much of interest in all of these talks and as these will be published sometime in 2001, readers will be able to study these for themselves. Here I shall confine myself to some impressions gained from the meeting as a whole. How have things changed since *L’Enseignement Mathématique* was first published? Perhaps the most obvious change is that the periodical first appeared at a time when French was still the language of diplomacy and the idea of publishing an international journal with all its articles in French was conceivable, even if at that time limiting (and a policy later changed). One result was that the circulation — and the range of authors to be found in *L’Enseignement Mathématique* (apart from in some of the ICMI sections) — is very heavily biased towards a few European countries. Moreover, ICMI, in its early days, was led by university mathematicians with an interest in what was happening in schools, and it is the writings of such authors that are best represented in the pages of the journal, rather than those of schoolteachers, teacher trainers, or what came to be known as mathematics educators. Indeed, there have been periods when the periodical’s papers were much more

concerned with mathematics than with its teaching. As a result any survey based on papers to be found in *L’Enseignement Mathématique*, is likely to reflect not what was actually happening in schools but what some influential mathematicians thought might with advantage happen in them. It is important though to remember that in the 1950s and 60s one could find in its and other journals’ pages articles on school mathematics written by distinguished mathematicians such as Artin, Cartan, Dieudonné, Freudenthal, Leray, M.H. Stone, Thom and Whitney. Many of the papers were, in the words of Freudenthal, the equivalent of publishing theorems without proofs, for ideas were never worked out in a form that could be used in classrooms (or would fit naturally within a school curriculum). Nevertheless, the concern of the mathematicians and the professional strength of a considerable proportion of schoolteachers in that period did lead to some extremely good and impressive mathematical writing — even if on many occasions this also demonstrated either a lack of pedagogical understanding or ill-founded optimism. It was right, as was exemplified at the seminar that the needs of university mathematicians should cease to be paramount — especially as secondary and higher education expanded to encompass a greater proportion of the population. Yet there was little to be heard at the symposium about what mathematics we should be teaching now and in the future. There were valuable comments on how technology could be employed to improve the teaching of ‘old’ mathematics, but no concrete suggestions were made on what form ‘new math’ might take in this new century. One wonders from what quarter such ideas are likely now to come.

The symposium demonstrated then how over the century the emphasis shifted from discussions of the mathematics to be taught to an élite, to the needs of a wider range of students and of society. Little was said though about the needs of, and for, teachers. Their problems of scarcity (and social status) have not lessened — indeed these would seem to increase year by year. We now have to tackle

problems not only concerning the mathematics to be taught, the needs of the learner, the professional concerns of individual teachers, but also the well-being of educational systems. The last was not a great concern of educators from the developed countries during the 20th century — one suspects that it will concern us much more in its successor.

Yet, the symposium indicated the great advances that have been made during the last century: of how mathematics has been brought to so much greater a proportion of adolescents world-wide. It reminded us of the way in which two generations had tried to make enormous changes in the content of school mathematics and methods of teaching it. It gave us an opportunity to see where these earlier efforts had not been wholly successful and challenged us to determine why. With such an understanding we should be better equipped to tackle both the problems that now face us and those which will arise in the future.

**ONE HUNDRED YEARS OF L'ENSEIGNEMENT MATHÉMATIQUE: MOMENTS OF MATHEMATICS EDUCATION IN THE TWENTIETH CENTURY**

Proceedings of the EM-ICMI Symposium (Geneva, 20-22 October 2000)

Edited by D. Coray, F. Furinghetti, H. Gispert, B.R. Hodgson, G. Schubring, (ISBN 2-940264-06-6) softbound; 336 pages, 2003; 63 CHF (*L'Enseignement Mathématique*, Monograph no. 39)

These Proceedings are organised similarly to the programme of the symposium, namely around three mathematical themes (geometry, analysis, applications of mathematics) which are considered from an educational point of view at three different periods of the 20<sup>th</sup> century (1900, 1950, 2000). The Table of Contents of the book appears at the bottom of this message.

More information about the book can be obtained on the website of L'Enseignement Mathématique:

<http://www.unige.ch/math/EnsMath/>

Those interested in ordering the book should contact directly L'Enseignement Mathématique either by e-mail (EnsMath@math.unige.ch) or by fax (+41 22 309 14 99), following the instructions which can be found on the website of L'EM. Payment can be made via a credit card. The price of the book is 63 Swiss francs (CHF), which includes shipping.

**ABOUT THE BOOK:**

It was in 1899 that Henri Fehr (Geneva) and Charles-Ange Laisant (Paris) founded the international journal L'Enseignement Mathématique. The form that the celebratory symposium took was a historical survey of developments in mathematics education at key periods in the 20th century followed by a short discussion of the manner in which mathematics education might meet the demands of societies today and in the near future.

The symposium demonstrated how over the century the emphasis shifted from discussions of the mathematics to be taught to an élite, to the needs of a wider range of students and of society. It reminded us of the way in which two generations had tried to make enormous changes in the content of school mathematics and methods of teaching it. It gave us an opportunity to see where these earlier efforts had not been wholly successful and challenged us to determine why. With such an understanding we should be better equipped to tackle both the problems that now face us and those which will arise in the future.

**TABLE OF CONTENTS:**

Introduction: Daniel Coray - Bernard R. Hodgson

***L'Enseignement Mathématique: birth and stakes***

- Fulvia Furinghetti: Mathematical instruction in an international perspective: the contribution of the journal L'Enseignement Mathématique

- Gert Schubring: L'Enseignement Mathématique and the first International Commission (IMUK): the emergence of international communication and cooperation
- Gila Hanna: Journals of mathematics education, 1900-2000
- Reaction: Jean-Pierre Bourguignon

- Jeremy Kilpatrick: Scientific solidarity today and tomorrow
- Reaction: Hyman Bass

Fulvia Furinghetti  
Italy

### Geometry

- Rudolph Bkouche: La géométrie dans les premières années de la revue *L'Enseignement Mathématique*
- Geoffrey Howson: Geometry: 1950-70
- Colette Laborde: Géométrie - Période 2000 et après
- Reaction: Nicolas Rouche

### Analysis

- Jean-Pierre Kahane: L'enseignement du calcul différentiel et intégral au début du vingtième siècle
- Man-Keung Siu: Learning and teaching of analysis in the mid twentieth century: a semi-personal observation
- Lynn Steen: Analysis 2000: challenges and opportunities
- Reaction: Michèle Artigue

### Applications of mathematics: mathematics as a service subject

- Philippe Nabonnand: Les débats autour des applications des mathématiques dans les réformes de l'enseignement secondaire au début du vingtième siècle
- Hélène Gispert: Applications: les mathématiques comme discipline de service dans les années 1950-1960
- Mogens Niss: Applications of mathematics '2000'
- Reaction: Gerhard Wanner

### Perspectives for mathematics education

- Ubiratan D'Ambrosio: Stakes in mathematics education for the societies of today and tomorrow



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**HPM 2004**

**Uppsala**

July 12-17 2004



## Have you been here?

The British Society for the History of Mathematics web site at [www.dcs.warwick.ac.uk/bshm/](http://www.dcs.warwick.ac.uk/bshm/) has many links to related sites.

The Italian Society of History of Mathematics web site at [www.dm.unito.it/sism/index.html](http://www.dm.unito.it/sism/index.html)

The HPM-Americas web site is up and going. The new web site is [www.hpm-americas.org](http://www.hpm-americas.org)

The HPM satellite meeting in connection with the Copenhagen ICME-10 in 2004 is planned for Uppsala with Sten Kaijser as the local person in charge. Visit <http://www.math.uu.se/hpm/index.html> You can find out more about ICME-10 and register for the first announcement now at [www.ICME-10.dk](http://www.ICME-10.dk)

The AMUCHMA newsletter on the history of mathematics in Africa can be found at [www.math.buffalo.edu/mad/AMU/amuchma\\_online.html](http://www.math.buffalo.edu/mad/AMU/amuchma_online.html) All the earlier issues are available on the same web page.

For a history of HPM visit [http://mcs.open.ac.uk/puremaths/pmd\\_department/pmd\\_fauvel/HPM\\_%20history.htm](http://mcs.open.ac.uk/puremaths/pmd_department/pmd_fauvel/HPM_%20history.htm)

History and Epistemology for the Teaching of Mathematics has been activated at the address:

[www.syllogismos.it](http://www.syllogismos.it)

On the site it is possible to find material relating to the teaching of mathematics and some historical references which will be useful in the field of mathematics. Every/any suggestion to improve such a site, conceived mainly in terms of helping colleagues involved in education and in particular in teaching will also be welcomed.

The editor welcomes information about other sites.

## Conference reports

### **PME 27 and PMENA**

July 2003, Honolulu

The Discussion Group 7 in the PME Conference held in Honolulu (July 2003) dealt with a topic that concerns HPM. In the following we report the abstracts of the presentation, see also the proceedings of the conference: N.A. Pateman, B.J. Dougherty, J.T. Zilliox (editors), *Proceedings of the joint meeting PME 27 and PMENA* (Honolulu, CRDG College of Education, University of Hawai'i), v.1, 191.

### **Semiotic and socio-cultural evolution of mathematical concepts**

Coordinators: Adalira Sáenz-Ludlow, University of North Carolina at Charlotte, USA  
Norma Presmeg, Illinois State University, USA

The goal of the group will be to discuss epistemological and semiotic aspects in the historical development of mathematical concepts to gain insight into the teaching and learning of mathematics. The role of signs in mediating the expression of mathematical ideas and the conceptualisations of new ones has been a prevalent force and those same signs evolved as mathematical concepts went from being empirical and concrete to being generalised and abstract. The discussion will focus on the pedagogical implications of Greek thought on geometry and the evolving conceptualisation of the second-degree equation. To launch the discussion there will be presentations followed by written questions provided by the presenters for the discussion in small groups.

### **Revisiting guided reinvention**

When students are immersed in a situation similar to that termed by Lakatos *pre-Euclidean* (a situation in which the theoretical frame is not well defined and one has to look for 'convenient' axioms) they are challenged to construct a theory. The didactical suggestion implicit in Lakatos' words is that it is advisable to recover the spirit of Greek geometers. In antiquity, geometry developed in an empirical way through a naïve phase of trials and errors; it started from a body of conjectures, followed by

mental experiments of control and proving experiments (mainly analysis) without any fixed axiomatic system. These ideas suggest a way of approaching proof in the classroom. This is a kind of guided reinvention (in Freudenthal style) in which reinvention may be fostered by the use of dynamic geometric software (in our case Cabri). Through this software students may create a 'local theory' of geometry (a theory with only a few theorems and definitions) and may come to better appreciate the complete theory.

Fulvia Furinghetti & Domingo Paola  
Università di Genova, Italy

### **Semiotic Aspects in the Development of the Solution of the Second Degree Equation.**

The historical development of the solution of the second-degree equation provides an illustration that mathematical thinking can be conceived as a semiotic expression of the rationality of the cultures in which the mathematical activity took place. From the Babylonians, to the Greeks, to the Arabs, to Descartes, to Euler and Carlyle, the solution of the second-degree equation was achieved through different symbolic representations mediating particular ways of thinking that, in turn, were influenced by the socio-cultural and economic factors of the time. As Restivo (1992, p. ix) argues, "to study a mathematical form is to study a sensibility, a collective formation, a world view". An analysis of the ways of solving second-degree equations will be done using Radford's (2003) notion of Cultural Semiotic Systems and Peirce's notions of iconic, indexical, and symbolic signs.

Adalira Sáenz-Ludlow  
University of North Carolina at Charlotte, USA

### **A Retrospective Analysis of Early History of Geometry in Light of Peirce's "Commens"**

(A) More than 2000 years ago, Archimedes used a method of exhaustion to calculate the area enclosed by a parabola and the line segment perpendicular to the axis of symmetry. Why was it only in the 17<sup>th</sup> century that such methods became widespread with the advent of the calculus? (B) Hipparchus of Crete generated some excitement when he figured out that the area of his "lune" was the same as that of a right triangle whose hypotenuse was the diameter of the lune. Why was this discovery important in

the geometry of the time? (C) Why did it take two millennia for the consequences of challenging Euclid's parallel postulate to be brought to fruition in systems of spherical and hyperbolic geometries?

These and other questions involving the history of geometry will be explored using Peirce's construct, *commens*, which he defined as "that mind into which the minds of utterer and interpreter have to be fused in order that any communication should take place" (Peirce, 1998, p. 478).

Norma Presmeg  
Illinois State University, USA

### **Inter-American HPM 2003 - HPM Satellite of the XI Inter-American Conference on Mathematics Education - 2003**

July 14-17 2003  
Blumenau, Brazil

The HPM-Satellite was realised from July 14-17, in the city of Blumenau - Brazil, during the working group from XI IACME "History and Philosophy of the Mathematics and of the Mathematics Education". The chairs were Angel Ruiz (University of Costa Rica) and Sergio Nobre (State University of São Paulo - Brazil). It was presented 15 short scientific communications and the following other activities:

#### **Plenary Conference:**

Perspectivas de la Educación Matemática - la influencia histórica de la reforma de las "Matemáticas Modernas", el escenario actual y las principales tendencias epistemológicas.  
Angel Ruiz (Universidad de Costa Rica)

#### **Panel of Discussion:**

The History of the Mathematics in the graduate course of Mathematics  
Participants: Lúgia Arantes Sad (UFES - Vitória - Brazil), Marcos Vieira Teixeira (Unesp - Rio Claro - Brazil) and Sergio Nobre (UNESP - Rio Claro - Brazil)

#### **Conferences:**

- History and Formation of the Teacher of Mathematics: Difficulties and

Perspectives - Romélia Mara Souto (UFSJ - Unesp - Brazil)

- Michael Crowe's Laws and the theory of Thomas Khun's scientific revolutions: a necessary confrontation for a historiography of the mathematics - João Carlos Gilli Martins (UFSM - Unesp - Brazil)
- The visual language of the mathematics - José Carlos Cifuentes (UFPR - Brazil)

Sergio Nobre (Unesp - Brazil)

### **VII Symposium of SEIEM**

September 11-13 2003

Granada

The Spanish society of research in mathematics education SEIEM (Sociedad Española de Investigación en Educación Matemática) organised its annual Symposium in Granada University (September 11-13, 2003). A section of the Symposium has been dedicated to the theme 'Historical research in mathematics education', coordinated by D. Bernardo Gómez. The following talks were delivered:

- Storia della matematica per insegnanti e studenti (Fulvia Furinghetti)
- Historia de las ideas algebraicas: componentes y preguntas desde el punto de vista de la matemática educativa (Luis Puig)
- El método de investigación histórico en la didáctica del Análisis Matemático (Teresa González & Modesto Sierra)

At the end of the presentations there was a discussion on the theme of the session. From the historical point of view the most impressive highlight of the Symposium was the night visit to La Alhambra. The moon and the Arab geometric patterns were very inspiring. The texts of the talks are published in the Proceedings. Further information on the Symposium may be found in the web page of SEIEM ([www.ugr.es/local/seiem](http://www.ugr.es/local/seiem))

### **Colloquium "François Viète, un mathématicien en son temps" ("François Viète, a mathematician at his times")**

September 19-20 2003

Nantes & Fontenay-le-Comte, France

François Viète was born in Fontenay-le-Comte in 1540 and died in Paris in 1603. The Centre d'histoire des sciences et des techniques François Viète of the University of Nantes, in collaboration with the IREM of Nantes and the town of Fontenay-le-Comte, organised a two day meeting (September 19-20, 2003) to celebrate 400 years since his death.

On the first day scholars from France, Italy, and Canada illustrated the various facets of Viète's mathematics (algebra, cryptography, geometry). The second day was meant for participants to embed themselves in the atmosphere of Viète's life. At sunrise we were brought to the natal town of the mathematician (Fontenay-le-Comte). The travel through the peaceful countryside prepared us to go back in the past and to live a day in a charming town that keeps the old-time flavour. The talks were delivered in the ancient theatre situated in rue Rabelais (the writer is another great son of the town). In these talks the civil aspects of Viète's personality (lawyer, man of the king, and politician) were illustrated. Two historians helped us to go in depth into the life of the second half of sixteenth century, including the religious fights between Catholics and Protestants that took place in the region.

During the colloquium it was remembered the importance of Viète's approach to algebra in the pedagogy of mathematics.

The proceedings of the colloquium will be published. For further information on the colloquium see the website <http://www.univ-irem.fr/index.php>

Jean-Paul Guichard has designed a nice website on Viète, see

<http://www.district-parthenay.fr/parthenay/creparth/GUICHARDJP/VIETEacueil.html>

Fulvia Furinghetti  
Genova, Italy

## Announcements of events

### ***History in the undergraduate mathematics curriculum: why and how?***

**15 November 2003**  
Oxford, UK

A one day BSHM meeting.

For further information please contact Eleanor Robson [eleanor.robson@all-souls.ox.ac.uk](mailto:eleanor.robson@all-souls.ox.ac.uk)

### ***John von Neumann Centenary Meeting*** **29 November 2003** Greenwich, UK

John von Neumann contributed to many areas of mathematics and was a pioneer of computing. A joint BSHM meeting with the School of Computing and Mathematical Sciences, University of Greenwich and the Centre for the History of the Mathematical Sciences, Open University

Organisers: Martin Campbell-Kelly, Jeremy Gray and Tony Mann ([A.Mann@gre.ac.uk](mailto:A.Mann@gre.ac.uk))

### ***American Mathematical Society*** **December 2003** Bangalore, India

There will be a session on History of Mathematics at this meeting. The coordinator for the session is Professor Gérard Emch. Contact him at the Department of mathematics, University of Florida, Little Hall, P.O.Box 118105, Gainesville FL 32611-8105, U.S.A.  
Email [gge@math.ufl.edu](mailto:gge@math.ufl.edu)

### ***Christmas Meeting and AGM 2003*** **December 13, 2003** London, UK

A BSHM meeting to be held in Birkbeck College (Gordon Square Building), London

Contacts:

Talks: Jackie Stedall, The Queen's College, Oxford OX1 4AW;

[Jackie.Stedall@queens.ox.ac.uk](mailto:Jackie.Stedall@queens.ox.ac.uk)

General Enquiries: Tony Mann, School of Computing & Mathematical Sciences, University of Greenwich, London SE 10 9LS; [A.Mann@gre.ac.uk](mailto:A.Mann@gre.ac.uk)

### ***"Mathematics in the Metropolis: A Survey of Victorian London"*** **January 19, 2004** London, UK

A BSHM special lecture and joint meeting with Gresham College

Speaker: Adrian Rice, Randolph-Macon College, Virginia, U.S.A.

Monday 19th January 2004, 6pm  
(refreshments from 5:30)

Admission free

The Victorian period was a time of massive change for London, not least in the development and availability of university-level mathematics. This talk investigates the changes that took place, highlighting some of the famous mathematicians involved, and comparing their teaching styles and the courses they offered at a wide variety of teaching establishments across the capital between 1837 and 1901.

### ***400 YEARS OF BRITISH MATHEMATICS: A one-day conference to celebrate the 60th birthday of Robin Wilson***

**January 20, 2004**  
Milton Keynes, UK

Centre for the History of the Mathematical Sciences, Open University, Milton Keynes, room CMR15 (Central meeting rooms), on Tuesday 20 January from 10 am to 5 pm

A BSHM meeting

The talks will cover the period 1600 to 1950 (approx), and the speakers will be: June Barrow-Green, Norman Biggs, Allan Chapman, Raymond Flood, Keith Hannabuss, Graham Jagger, Adrian Rice, Jackie Stedall and Jim Tattersall.

The meeting will be followed by a reception and dinner for those who want to join in (at their own expense). There is no admission charge, but since space may be limited, it would be appreciated if those wishing to attend could inform Robin Wilson (R.J.Wilson@open.ac.uk)

### ***Research in Progress 2004***

**28 February, 2004**

Oxford, UK

A BSHM meeting to be held at The Queen's College, Oxford, Saturday 28th February 2004. Further details available from the organiser: Jackie Stedall (jackie.stedall@queens.ox.ac.uk)

### ***HPM 2004 satellite conference of ICME-10***

**July 12 - 17, 2004**

Uppsala, Sweden  
(First Announcement)

We are happy to inform you that the HPM satellite conference of ICME-10 will take place on July 12 - 17, 2004 in the historic town of Uppsala, Sweden. It will be organised by the department of Mathematics at Uppsala University.

The chairman of the local organising committee is Sten Kaijser who is also the contact person in Uppsala.

A programme committee has been founded consisting of

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### ***About the conference***

HPM is the International Study Group on the Relations between History and Pedagogy of

Mathematics affiliated to ICMI. Among the activities of the group HPM there is the tradition of organising satellite meetings of the conference ICME. We list below these meetings:

1984 ICME-5 (Adelaide, Australia), satellite meeting in Sturt Campus of the University of Adelaide

1988 ICME-6 (Budapest, Hungary), satellite meeting in Florence (Italy)

1992 ICME-7 (Québec, Canada), satellite meeting in (Toronto, Canada)

1996 ICME-8 (Seville, Spain), satellite meeting in (Braga, Portugal)

2000 ICME-9 (Tokyo-Makuhari, Japan), satellite meeting in (Taipei, Taiwan).

The HPM Satellite conference is a unique occasion to attend lectures, workshops, research reports from all over the world about the use of history in mathematics education, history of mathematics, history of mathematics education. The participants to the HPM meetings are researchers in history, in mathematics education, and teachers who have experimented with the use of history in their teaching.

Books or proceedings published after the previous HPM satellite meetings:

- Calinger, R. (editor): 1996, *Vita mathematica*, MAA Notes n.40. (HPM 1992)
- Lagarto, M. J., A. Vieira & E. Veloso (editors): 1996, *Proceedings of Second European summer university and satellite meeting of ICME-8* (Braga, Portugal). (HPM 1996)
- Katz (editor): 2000, *Using history to teach mathematics: An international perspective*, Mathematical Association of America. (HPM 1996)
- Horng, W.-S. & F.-L. Lin (editors): 2000, *Proceedings of the HPM 2000 Conference History in mathematics education. Challenges for a new millennium. A satellite meeting of ICME-9*. (HPM 2000)

### **About the venue**

The city of Uppsala is one of the oldest cities in Sweden. It was once considered the capital

of Sweden and it is still the ecclesiastic capital since the residence of the archbishop of Sweden lies in Uppsala.

Uppsala has a famous university, founded 1477, which is the oldest in Scandinavia. The university has had many famous scholars and scientists of which the founder of botany, Carl von Linné is perhaps the most well known. Also some of Sweden's most prominent mathematicians during the 20<sup>th</sup> century, foremost among them Arne Beurling and Lennart Carleson, were educated and for a substantial part of their career active in Uppsala.

For further information contact Sten Kaijser <sten@math.uu.se>. There will soon be a web page under <http://www.math.uu.se/hpm>

Fulvia Furinghetti & Sten Kaijser  
Italy & Sweden

### ***First Brazilian Colloquium on the History of Mathematics and the Fourth Luso-Brasilian Meeting on the History of Mathematics***

**October 24-27, 2004**

Natal, Brazil

(First Announcement)

Joint Conference

The First Brazilian Colloquium on the History of Mathematics and the Fourth Luso-Brasilian Meeting on the History of Mathematics will be held jointly in Natal, RN (Brazil), from 24th to the 27th of October 2004. The General Coordinator of the events is Dr. John A. Fossa. A web page with more details is expected to be on line by early next year. For early registration, please contact Prof. Fossa at [fosfun@digicom.br](mailto:fosfun@digicom.br).



[www-conference.slu.se/hpm/index.html](http://www-conference.slu.se/hpm/index.html)  
HPM 2004

## **Distributors:**

If you wish to be a distributor in a new or unstaffed area please contact the editor.

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We welcome Sue Pope as the new distributor for the United Kingdom. Sue takes over from David Lingard to whom we are most grateful.

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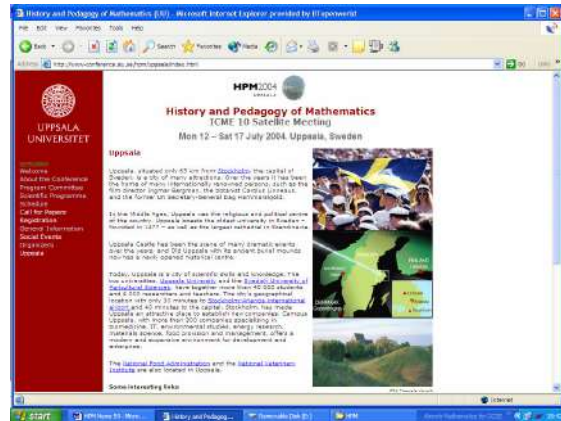
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## INDEX

|  |    |
|--|----|
| Message from our Chair   | 1  |
| Editorial  | 1  |
| Power of Innovation:<br>A Historical View                        | 2  |
| A Historical Mathematics Tour of<br>London and Surrounding Areas | 8  |
| Work in progress   | 10 |
| Reviews  | 10 |
| Have you read these?   | 11 |
| Have you been here?  | 16 |
| Conference reports   | 16 |
| Announcements of events  | 19 |
| Distributors   | 22 |
| News of distributors   | 22 |
| Index  | 24 |

The views expressed in this Newsletter may not necessarily be those of the HPM Advisory Board.

Please pass on news of the existence of this newsletter to any interested parties.

Items for the Newsletter should be sent to the editor, preferably by email.

The Newsletter appears three times a year with the following deadlines for next year.

|                       |                      |
|-----------------------|----------------------|
| Deadline for material | Sent to distributors |
| 12 February 2004      | 1 March 2004         |
| 14 June 2004          | 1 July 2004          |
| 15 October 2004       | 1 November 2004      |

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**UPPSALA UNIVERSITET**

**HPM2004**

**History and Pedagogy of Mathematics**  
ICME 10 Satellite Meeting

Mon 12 – Sat 17 July 2004, Uppsala, Sweden

**Scientific Programme**

**Invited speakers**

**1. Plenary sessions**  
The following invited speakers will give plenary lectures related to the following topics:

- André Cauty, Bourdeaux, France
- Jens Høyrup, Roskilde, Denmark
- Viktor Katz, Washington D.C., U.S.A.
- Michael Otte, Bielefeld, Germany
- Lub Raoford, Sudbury, Ontario, Canada
- Peter Ransom, Southampton, United Kingdom

The titles will be announced later.

**2. Panel Discussions**  
There will be (at least) two Panel Discussions:

- Panel 1: Proof in History and in the classroom  
Coordinator: Sten Kaijser  
Speakers: Man Keung-Siu, Anders Öberg, Guershon Harel, Christine Proust
- Panel 2: Original sources in the classroom  
Coordinator: Evelynne Barbin  
Speakers: Abdellah El Horssi, Frédéric Miletin, Otto Bekken

**3. Workshops**

**Invited work-shops themes:**  
Mathematics in literature - Anne-Michelle Fajus (+ Fulvia Fulinghetti)  
Mathematics and Music - Anne Boyd  
Mathematics in literature - Mercedes Hutter

Themes linked to ethno-mathematics  
History  
History  
Epistemology  
Mathematic Education  
Experience of teachers

Done

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