## International Study Group on the Relations Between HISTORY and PEDAGOGY of MATHEMATICS NEWSLETTER

AN AFFILIATE OF THE INTERNATIONAL COMMISSION ON MATHEMATICS INSTRUCTION

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1995 July 8 - 13 Besançon, France Interdisciplinary Summer University on the History of Mathematics. (See *Newsletter* 34 for more details.)

1995 July 20 - 21 Edinburgh Scotland's Mathematical Heritage: Napier to Clerk Maxwell. (See inside for more details.)

1995 July 24 - 29 Bergen

PDME III, Political Dimensions of Mathematics Education Conference, will take place in Bergen, Norway. The official languages of the conference will be English and Spanish. For further information, contact Stief Mellin-Olsen, Institutt for praktisk pedagogikk, University of Bergen, N-5020 Bergen, NORWAY (Phone: 47 5 544830; Fax: 47 5 544852; email: mellin-olsen@psych.uib.no).

1995 September 8 - 9 Hyderabad, India National Symposium on Ancient Indian Astronomy & Mathematics. This symposium will consist mostly of invited talks by eminent scholars. Other contributions in the fields of Vedic Astronomy, Vedanga Jyotisha, Siddhantic Astronomy and Mathematics, and other topics may be considered for presentation. Interested scholars should send a request for participation together with an abstract to Dr. B. G. Disharth, Director, B. M. Birla Science Centre, Adarsh Nagar, Hyderabad - 500 463, INDIA (Fax: 91 40 222483; email: bgs@bschyd. uunet.in)

#### 1995 September 16 - 17 Oxford

Autumn residential meeting of the British Society for the History of Mathematics. The topic will be Mathematics in Victorian Britain. The mathematical activity in various centres will be discussed: Cambridge, Oxford, London, and in military colleges, schools and societies; as will the work of many British mathematicians of the late 19th century. For more details, contact Raymond Flood, OUDCE, 1 Wellington Square, Oxford OX1 2JA, ENGLAND.

1995 October 29 - November 3 Minneapolis Third International History, Philosophy and Science Teaching Conference. (See inside for more details.)

1996 January 10 - 13 Orlando Annual meeting of the American Mathematical Society and the Mathematical Association of America. (Details on sessions of interest to readers will be forthcoming in the next Newsletter.)

1996 April 25 - 28 San Diego Annual meeting of the National Council of Teachers of Mathematics and of the Americas Section of HPM.

1996 July 14 - 21 Seville

ICME-8. Details will be forthcoming. To receive the first announcement, send your name and address to ICME-8, Apartado de Correos 4172, E-41080 Sevilla, SPAIN.

1996 July 24 - 30 Braga, Portugal

Quadrennial International HPM meeting in connection with ICME. (See inside for more details.)

#### **From the Editor**

As Karen Dee Michalowicz notes in her article on the HPM Americas Section meeting in Boston, it is clear that this Newsletter can no longer be provided free of charge to the approximately 1500 people on the United States mailing list. The University of the District of Columbia has been extremely generous over the past seven years in providing funds for the printing and mailing of these copies. However, the financial crisis in the District of Columbia and the subsequent cuts in the University's budget mean that the University can no longer continue to do this. If the *Newsletter* is to continue, there are two possibilities. Either the readership begins to pay dues in sufficient numbers so that we can reimburse the University for the printing and mailing or someone else takes the responsibility of finding a University (or other organization) who will provide the printing and mailing. A third possibility, of course, is that someone will make a large contribution to fund the organization; if you or someone you know is willing to do this, please let the editor or the Americas section chair know. Assuming such a major donor is not found, however, if those of you who find this Newsletter valuable would send in \$10 per year to our treasurer, Sherry Cox, at 532 C Fleetwood Ct., Kingsport, TN 37660, not only can we continue funding the Newsletter, but also we can pay the other expenses of having an organization, including expenses

of the annual meetings. (Note: The above information is strictly for readers of this *Newsletter* in the United States. For now, the cost of distribution in other countries is not a problem.)

One way of cutting costs which we will be exploring is the providing of the *Newsletter* electronically to those who have facilities for receiving it. In order to do this, we are beginning a database of e-mail addresses of those who have them. Please send your name and e-mail address (by e-mail) to Karen Dee Michalowicz at karendm@aol.com. Once we have a sufficiently large number of such addresses, we can begin distribution of the *Newsletter* electronically, cutting down our need for postage and printing.

#### **History of Mathematics Conference at York University**

James Tattersall, Providence College and USMA

On April 24, 1995 a conference was held in Toronto to honor Hardy Grant, a long-time member of the Canadian Society for the History and Philosophy of Mathematics who was retiring after thirty years of service at York University.

Judy Grabiner of Pitzer College began the morning session speaking on "Maclaurin among the molasses barrels: mathematics and Society in 18th- century Britain." Grabiner discussed a number of techniques used to approximate volumes of barrels beginning with Kepler's method before focusing on Maclaurin's 1742 paper. Maclaurin's treatise contained several Archimedean-type results but was written for the customs and excise agents of Great Britain whose revenues at the time accounted for between two-thirds and three-fourths of the government's revenue. Grabiner illustrated how Maclaurin's treatise can be viewed as an intriguing venture into applied mathematics. Grabiner adeptly mixed mathematics, slavery, strong drink, smuggling, taxes, British politics, and the molasses trade in the port of Glasgow in the first half of the eighteenth century.

Grabiner was followed by Len Berggren of Simon Fraser who spoke on "Mathematics and her sisters in medieval Islam." Berggren surveyed much of the important work that has been done in the past ten years in the Islamic mathematical sciences. Berggren's talk was an enlightening tour de force detailing Muslim contributions to mathematics, astronomy, optics, and cartography. Berggren mentioned a number of historians who have made contributions to the field in the last decade and discussed in detail the relevance of their exciting discoveries and what we can learn from their stimulating work.

After an exquisite lunch offered in the Faculty Common Room at the Osgoode Hall Law School, the venue for the conference, the meeting resumed with a talk by Aditi Gowri of the University of Southern California who spoke on "Symbol and sublimation: the unconscious life of mathematics." Gowri's paper was a contribution to the relatively new field of cultural studies in mathematics. Drawing on the work of the anthropological theorist Mary Douglas, Gowri explored psychoanalytic metaphors in understanding mathematics and its production. Gowri analyzed mathematical knowledge as a multi-layered symbolic structure and illustrated how it would be viewed by Jung, Poincaré, Hadamard, and Freud. Gowri discussed the role of ethnomathematics and spoke intriguingly on the unconscious mind of the mathematician as a source of inspiration.

The conference concluded with a presentation by Gregory Moore of McMaster University on "The emergence of abstract vector spaces and modules, 1888-1936." Moore discussed the evolution of the notion of a vector space as first axiomatized by Peano. Moore investigated the origin of many of the terms used in vector analysis and discussed Emmy Noether's formulation of the notion of a module over a ring, a generalization of a vector space over a field. Moore discussed many of the factors that led to the notion of a vector space being rediscovered repeatedly and how these factors kept the concept from taking hold in 1888 when it was first formulated.

After the conference there was a wine and cheese reception in the Law School Common Room. The attenders owe a debt of gratitude to the organizers Israel Kleiner, T. MacHenry, P. Rajagopal, and Abe Shenitzer, for arranging such a valuable and insightful conference to honor Hardy Grant who has been an inspiration to so many.

#### Annual Conference of the Association of Mathematics Teachers of India

#### R. C. Gupta

The 29th Annual Conference of the Association of Mathematics Teachers of India was held from December 30, 1994 to January 1, 1995 at Kancheepuram, a famous templetown in South India. It was organized under the auspices of the Sri Chandrasekharendra Saraswathi Vishwa Maha Vidyalaya, a deemed University.

A dominant part of the deliberations in this conference was its historical component, about which we report here. In fact, the chosen theme for the meeting itself was "Mathematical Traditions in India," which was well reflected in the proceedings of the conference. A small exhibition on "Glimpses of Indian Mathematics" was arranged on the occasion.

The inaugural talk was given by His Holiness Swami Jayendra Sarasvati, the present (69th) revered Sankaracharya of the local Kanchi Kamkot Pitha (traditional pontificate). He surprised the audience by speaking in Hindi and gave a broad survey of the usefulness of mathematics in all practical problems of life through the ages.

The Presidential Address was delivered by R. C. Gupta. The topic was "Mathematics as a way of life," in which he showed how certain characteristics of mathematics help individuals to cultivate good mental and other qualities. "Mathematical knowledge adds vigor to the mind, and frees it from prejudice, credulity and superstition." Gupta tried to make a point that devotion to the true spirit of mathematics leads to higher values of life such as truth, goodness, and beauty.

There followed a talk by M. S. Rangachari (Ramanujan Institute, Madras) on "Development of Analysis in India." This was mainly about the derivations of the series for sin x, cos x, tan<sup>-1</sup>x, and  $\pi$  as found in the works of the Late Aryabhata School about 250 years earlier than the corresponding appearances in Europe.

In the post-lunch session, the historical lectures included the talk "On Indian Trigonometry" by R. C. Gupta and a paper on "How Omar Khayyam solved cubic equations" by Miss Vidya (Bharata Engineering College, Madras). Gupta described the significant achievements in the field of trigonometry in India up to the twelfth century. Miss Vidya dealt with the geometrical solutions of Khayyam, which were based on the intersection of rectangular hyperbolas with other known curves, but she could not explain how the hyperbola itself would be constructed. There was also a talk on Napoleon's Theorem (which states that the centroids of the escribed equilateral triangles on the sides of any triangle form an equilateral triangle) by S. Muralidharan (Tata Consultancy, Madras). A magic square dance was performed by the students of Sankara Senior Secondary School during the cultural program in the evening.

The second day's program started with a lecture on "Ramanujan's Mathematics" by R. Balasubramaniam (Matscience, Madras). He discussed Ramanujan's treatment of partitions and of "squaring the circle," for which the latter's construction was based on  $\pi = 355/113$ . Then there was a short talk on Bhaskara II's method of finding the square root of a +  $\sqrt{b}$  by Venkatesa Murthy (Bangalore).

At noon, the first endowment lecture on the history of mathematics was delivered by M. D. Srinivas (University of Madras). He spoke on the "Indian approach to mathematics." While presenting the ancient Indian style of exposing mathematics, he maintained that most of the Sanskrit texts and commentaries were written for pedagogical purposes. There followed the P. L. Bhatnagar Memorial Lecture by K. R. Parthasarathy, who spoke on Graph Theory.

On the last day, the following three papers were read in the morning session: "On Various Means" by K. Shanthi (dealing with seven means from Greek sources); "On the history of  $\pi$  - a curtain raiser" by K. Giridharan (a brief survey); and "*Tantra Samgraha* of Nilakantha" by V. S. Narasimhan (some astronomical problems from the text of A.D. 1500).

The A. Narasinga Rao Memorial Lecture was delivered by Miss Vijayalakshmi. She spoke on "Kolam and Mathematics." Kolam (also called Rangoli, Alpana, etc.) are the artistic geometrical diagrams which have a long ancient tradition and are still drawn. Much mathematics is involved in this ethno-art.

Before breaking for lunch, the audience was introduced to the contents of the book *Indian Mathematics and Astronomy: Some Landmarks* by its author, S. Balachandra Rao.

The Valedictory Function was held in the presence of His Holiness Vijayendra Sarasvati, and the Valedictory Address was given by G. Balasubramanian (Central Board of Secondary Education).

The Anvikshki Research and Study Centre, Bangalore offered to host the next (30th) conference of AMTI. Proceedings of the 29th Conference are to be published in the *Mathematics Teacher (India)*, the official journal of the AMTI.

#### Scotland's Mathematical Heritage: Napier to Clerk Maxwell

Scots have made a contribution to world mathematics which is out of all proportion to the country's population. In this meeting, to be held on July 20 and July 21, 1995, three societies, the Edinburgh Mathematical Society, the British Society for the History of Mathematics, and the Royal Society of Edinburgh, are coming together to raise awareness of the enormous contributions of Scottish mathematicians. The talks, given by leading experts in the field, will feature many of the major figures and others less known and set them in the context of their times. Although scholarly, the talks will be accessible to all with a general mathematical knowledge: familiarity with the history of mathematics is not assumed, and it is hoped that students and members of the public, as well as professional mathematicians, will find much to engage and interest them.

The meeting will be held in the rooms of the Royal Society of Edinburgh, 22 George Street, Edinburgh, and advance registration is essential. It will begin at 2:00 pm on

Thursday, July 20 and will end in the late afternoon of Friday, July 21. A reception will be held in Napier's former home, Merchiston Tower. The following have provisionally accepted invitations to speak: J. Barrow-Green (Open University), J. Fauvel (Open University), D. Forfar (J. C. Maxwell Foundation), J. Grabiner (Pitzer College), I. Grattan-Guinness (University of Middlesex), A. Malet (Barcelona), R. Rankin (University of Glasgow), S. Russ (University of Warwick), C. Smith (University of Kent), I. Tweddle (University of Strathclyde), D. T. Whiteside (University of Cambridge).

For information on registration, contact Professor A. D. D. Craik, Mathematical Institute, North Haugh, University of St. Andrews, St. Andrews, Fife KY16 9SS, SCOTLAND (email: addc@st-andrews.ac.uk).

### História e Educação Matemática: Deuxième Université d'Été Européen sur Histoire et Épistémologie dan l'Éducation Mathématique; ICME-8 Satellite Meeting of the International Study Group on the Relations Between History and Pedagogy of Mathematics

#### General Information

From 24 to 30 July 1996 the city of Braga, in northern Portugal, will host a major international conference on mathematics education. The main purpose of this conference is to bring together mathematics teachers and educational researchers from all over the world, to share their insights and experiences in using history of mathematics in the mathematics classroom. This meeting is timed to follow the International Congress on Mathematics Education, ICME-8, which is being held in Seville, Spain, July 14-21, 1996, so that many possible international participants will already be in Europe. Thanks to the generosity of the University of Minho, the costs are set at a very low level to try to ensure that no participant is unable to attend on financial grounds. In addition, it is hoped that further funding can be found to support the attendance of teachers from the Third World.

Braga-96 has two particular dimensions:

• It is the second European Summer University - the first was organized by the Instituts de Recherche sur l'Enseignement des Mathématiques (IREM) and held in Montpellier, France, in 1993 - which is a movement to bring together teachers from many countries to develop their knowledge and share their experiences of history and epistemology in mathematics education.

• It is the principal quadrennial meeting of the International Study Group on the Relations Between History and Pedagogy of Mathematics. The previous one was held in Toronto, Canada, in 1992. This study group is devoted to understanding and promoting the use of history of mathematics in mathematics teaching.

The HEM Braga 96 meeting is organized by the Portuguese Associação de Professores de Matemática (APM) and by the Department of Mathematics of the University of Minho and is supported by the Portuguese Society of Mathematics and by the Rectory of the University of Minho.

#### Activities and Themes

The main activities of the meeting are first, a series of introductory lectures on the history of mathematics in which leading scholars and historians paint a picture of the development of mathematics down through the ages - or some themes in this story -

especially designed for busy mathematics teachers from around the world who have not had much time or opportunity to study the history of mathematics themselves. And second, a series of practical workshops on the use of history in the mathematics classroom - enseignement des mathématiques - in which teachers and researchers with experience in using history share their knowledge and expertise of classroom activities with other teachers. These sessions complement the introductory history lecture series by showing what can be achieved for pupils in the classroom by a historically informed and enriched teacher. They will include workshops using mathematical texts from the past. A third major component of the activities are short talks by participants, on themes that they are keen to explore and share with others. These might be in connection with poster sessions, or might be free-standing, and are a very good way for delegates to find others with common interests and enthusiasms. In addition to the introductory lecture-course on the development of mathematics, Braga-96 will have its own particular themes. A few themes will be selected by the Programme Committee and it is expected that they will play major roles in focusing some of the activities. Some of the proposed themes that are being discussed by the P.C. are the following: Navigational mathematics and the age of discoveries, Mathematical cultures all over the world, Mathematical proofs in history, Mathematics and technics, Mathematics and Art, Mathematical modelling, Mathematics and reality, Epistemological obstacles. Themes are not a straightjacket, and contributions will be welcomed which do not fall readily into a theme; but it is hoped that having a finite number of themes will enable a focusing of participants' energies and attentions. This will lead to solid progress, both for teachers who seek helpful experiences to take back to their schools with them, and also for the researchers and educationists who will return home inspired by the feedback from teachers around the world.

#### Programme Committee

Ubiratan d'Ambrosio (Brazil), Evelyne Barbin (France) (joint chair), Florence Fasanelli (USA), George Booker (Australia), Jean Doyen (Belgium), Maria Fernanda Estrada (Portugal), John Fauvel (UK) (joint chair), Fulvia Furinghetti (Italy), Paulus Gerdes (Mozambique), Jesus Hernandez (Spain), Victor Katz (USA), Jan van Maanen (Netherlands), Carlos Sá (Portugal), Luis Saraiva (Portugal), Gert Schubring (Germany), Man-Keung Siu (Hong Kong), Eduardo Veloso (Portugal).

#### Language Issues

We expect hundreds of teachers from all over the world, speaking many languages. The official languages will be English, French and Portuguese, but speakers in other languages will be allowed if they have a translator/summarizer in French or English and transparencies in a second "official" language; speakers in one of the "official" languages must also have transparencies in another of those languages. All speakers will be instructed to speak slowly and simply. Transparencies and texts for workshops must be sent in advance to be translated into Portuguese (final dates will be settled in the second announcement).

#### Social Programme

A social programme, including a conference dinner and an excursion to the nearby Natural Parque of Gerez, is being prepared by the local organization. It is hoped and planned that these two main social events can be budgeted for so as to make them available to everyone. A special programme will be devised for accompanying persons.

#### **Practical Information**

The meeting will be held at the University of Minho, in Braga, from July 24 to July 30, 1996, with registration beginning Tuesday, July 23, in the afternoon. ICME-8, in Seville, will close July 21 at noon, so ICME participants will have plenty of time to go from Seville to Braga. Bus transportation from Seville to Lisbon will be available on the afternoon of July 21. There are frequent trains from Lisbon to Braga.

Meals and the main accommodation will be at the University of Minho facilities and campus residences and will be included in the registration fees. Accommodations will also be available at hotels.

We are trying to get funds to support a few participants from developing countries.

Proceedings will be published and given to everyone registered for the meeting.

Braga, in the province of Minho

The province of Minho is situated at the north-western corner of Portugal. The river Minho, which has given its name to the region, is the natural border between the provinces of Minho in Portugal and Galicia in Spain. The population is slightly over one million. Braga, the administrative center of the region, is an old imperial Roman city - it counts more than 2250 years in the annals of its history - and one of the oldest Christian towns. The climate in the region is mild and temperate; spring and autumn are very mild and pleasant; in winter the temperature rarely drops to 0°C and summers are not excessively hot.

Please register your interest and request a second announcement of the meeting by sending a note with your name, address, phone, fax and e-mail to Eduardo Veloso, APM, Escola Superior de Educação de Lisboa, Rua Carolina Michaelis de Vasconcelos, 1500 Lisboa, PORTUGAL. You may also send the information by fax to 351 1 7166424 or by e-mail to veloso@telepac.pt. Please specify the language of your preference - English, French or Portuguese - for the second announcement. Over 400 people have already indicated interest; this is a meeting you should not miss.

#### Third International History, Philosophy and Science Teaching Conference

The Third International History, Philosophy and Science Teaching Conference will be held at the University of Minnesota, Minneapolis, from October 29 to November 2, 1995. It follows the very successful First conference held at Tallahassee in 1989 and Second conference held at Kingston in 1992. The conference will be held in association with the U.S. History of Science Society's annual conference, occurring October 26-29, 1995.

The purpose of the Conference is to promote:

• The dissemination of lessons, units of work, and programs in science and mathematics, at all levels, that have successfully utilized history, philosophy, and sociology.

• Improvement of science and mathematics education by utilizing the history, philosophy, and sociology of science and mathematics.

• Collaborations among communities of scientists, mathematicians, historians, philosophers, cognitive psychologists, sociologists, science and mathematics educators, museum professionals, and school and college teachers.

• The inclusion of appropriate history, philosophy, and sociology of science and mathematics courses in science and mathematics teacher-education programs.

• Discussion of purposes of science and mathematics education, and their contribution to the intellectual and ethical development of individuals and cultures.

Details about conference registration fees, housing, and social events will be available in July. The approximate fee will be \$200. For more information, contact Professor Fred Finley, IHST Conference Chair, Department of Curriculum and Instruction, College of Education, 370 Peik Hall, 159 Pillsbury Drive S.E., University of Minnesota, Minneapolis, MN 55455, U.S.A. (Fax: 612-624-8277; e-mail: finleyfn@vx.cis. umn.edu.)

#### **Americas Section of HPM Meeting in Boston**

#### Karen Dee Michalowicz

The Americas Section of HPM met in conjunction with the National Council of Teachers of Mathematics (NCTM) Annual Meeting in Boston on April 7, 1995.

Beatrice Lumpkin (author and researcher in Egyptian Mathematics) presented a talk on "The Concept of Zero in Ancient Egypt." She shared some "rather surprising information" that some have known since the beginning of this century. Beatrice presented arguments that the concept of zero was indeed present in Egyptian mathematics.

Charles Jones (Ball State University) spoke on "Insights into Pedagogy from History: Support for New Learning." Charles addressed from a theoretical point of view questions about how history can aid pedagogy and why we should believe it does. He described ways in which cognitive science research and ideas from complexity theory might provide a framework for research and development in history and pedagogy of mathematics. Charles' ultimate goal is to define a coherent theory on which to base a research program.

Barnabas Hughes (California State University, Northridge) intrigued the audience with his talk on "The Sieve of Eratosthenes Corrected." Barnabas told the group that the Sieve of Eratosthenes, pictured in many text books and even math history books, was not a table of all the numbers from 1-100. Eratosthenes' Sieve contained only the odd numbers.

Jaime Carvalho e Silva (Universidade de Coimbra, Portugal) gave a talk entitled "History of Mathematics in the Textbooks of Jose Sebastiao e Silva (I914-I972)." Jaime told us that the Portuguese mathematician Jose Sebastiao e Silva who was well known for his contribution to the axiomatic theory of distributions was also the author of two secondary school textbooks on Algebra and Analytical Geometry. He analyzed the frequent references to mathematics history found in the texts and the implications for today's classroom.

George Booker (Griffith University, Australia) presented "Historical Problems in the Teaching of Fractions." He suggested that a look at the fractions from antiquity might help teachers to understand how children deal with fractions. George cited examples of Egyptian and Indian use of fractions. He also said that fractions originated in measurement and the need for accurate "barter." Sarah Burns, Jennifer Drew, Mike Dugger, Richard Hall, Robin Price, Christine Robinette, Erica Schmitt (undergraduates, Ball State University) presented "Historically-Based Modules for the Mathematics Classroom: Projects from a History of Mathematics Course for Pre-service Teachers." The modules presented historically based units on Area, Calculating and Calculating Aids, and on Fractions.

Following the program George Booker invited everyone to attend the International Study Group on The Relations Between the History and Pedagogy of Mathematics -Cairns Meeting 1995 in Cairns, Queensland, Australia, June 30th through July 4th.

The Business Meeting was chaired by the Americas Chair, Charles Jones (Ball State University). The meeting included the Treasurers Report, the Minutes of the 1994 Annual Meeting, a discussion on the progress of Affiliation and the By-laws, and much discussion on the need of members of the Americas Section of HPM to pay their yearly \$10 membership dues. Victor Katz, our newsletter editor, advised the group that the University of the District of Columbia was suffering financial cuts, as were many universities, and may not be able to contribute what they have in the past toward newsletter publication and mailing. In the future it will be necessary to collect dues to defray newsletter costs. The Treasurer, Sherry Cox requests that you mail her your dues. Checks should be made out to HPM and sent to Sherry at 532 C Fleetwood Ct., Kingsport, TN, 37660. The *Newsletter* is an important part of our HPM organization. We must pay our dues to continue its distribution!!

#### **CSHPM** in Montreal

The Canadian Society for the History and Philosophy of Mathematics held its annual meeting from June 3 to June 5 in Montreal at the Université du Quebec à Montreal. There was a special session on the History of Mathematics around 1900 organized by Thomas Archibald.

Joseph Dauben (CUNY Graduate Center), the Invited Speaker, considered "Cantor and the Epistemology of Set Theory." The other speakers in this special session were Erwin Kreyszig (Carleton University), "From Classical to Modern Analysis;" Craig Fraser (University of Toronto), "Mathematical Existence and the Calculus of Variation, 1900-1910;" Jim Tattersall (Providence College), "Women and the Cambridge Mathematical Tripos, 1880-1910;" Christopher Baltus (SUNY College at Oswego), "Asymptotic Series after Stieltjes: What Happened to the Continued Fractions?" Louis Charbonneau (UQAM), "Les mathématiques au service de l'ingéniérie et du commerce dans les institutions d'enseignement supérieure de Montréal et du Québec entre 1878 et 1945;" Israel Kleiner (York University), "The Genesis of the Abstract Ring Concept;" Rebecca Adams (McMaster University), "The Beginnings of General Topology;" Abe Shenitzer (York University), "Remarks on Mathematical Criticism;" Irving Anellis (Modern Logic Publishing), "Peirce Rustled, Russell Pierced: How Charles Peirce and Bertrand Russell Viewed Each Other's Work in Logic, and the Debunking of Russell's School of Falsification of the History of Logic;" and Alejandro Garciadiego (UNAM), "The History of Mathematics in Mexico."

Among the numerous additional speakers were Samuel Kutler (St. John's College), "How Ancient Greeks and Moderns View the Menelaus Theorem;" Francine Abeles (Kean College), "A Closed Form of the Euclidean Parallel Postulate;" Tom Archibald (Acadia University), "Why Do Existence Theorems Exist?" Robert Thomas (University of Manitoba), "From Ethnomathematics to Real Mathematics;" W. S. Anglin (McGill University), "Egyptian Fractions in the Twentieth Century;" Glen Van Brummelen (The King's University College), "Kushyar ibn Labban's Planetary Astronomy;" and Luis Radford (Laurentian University), "Linking Pyschology and Epistemology: How Can History of Mathematics be a Useful Tool for the Comprehension of Students' Learning Processes?"

#### **Revue d'Histoire des Mathématiques**

Les mathématiques connaissent aujourd'hui un développement particulièrement dynamique. Leurs branches se diversifient et se réorganisent; les liens avec les sciences de la nature, la technologie et les sciences sociales s'intensifient. Dans ce mouvement, les rapports au passé se recomposent: d'un côté, les théories récentes permettent de renouveler la lecture de travaux anciens, de l'autre, une meilleure connaissance de l'histoire contribue à éclairer des questions actuelles sur la nature et la place des mathématiques dans l'ensemble des savoirs et dans la société.

Au cours de ces dernières années, les recherches en historie des mathématiques se sont développées de manière significative dans de nombreux pays et notamment en France. Avec l'accroissement du nombre de séminaires, de colloques, d'éditions diverses, on assiste à une prise de conscience de l'existence d'un domaine à part entière de l'historie des sciences, avec ses objets et ses méthodes, dont la pratique exige la réunion de compétences multiples (mathématiques, historiques, épistémologiques, ...). Des progrès substantiels restent cependant à effecturer si l'on songe notamment au peu de revues spécialisées en histoire des mathématiques éditées dans le monde. Or, l'existence d'un nombre suffissant de tels périodiques, leiux par excellence d'expression de la diversité et de la cohérence d'une discipline, est un point essentiel pour le progrès de la recherche.

La création de la *Revue d'histoire des mathématiques* vise à offrir un nouveau lieu de publication pour les travaux consacrés plus particulièrement à la période allant du XVII<sup>e</sup> siècle à nos jours. On peut penser que la richesse du patrimoine constitué par les textes mathématiques en langue française depuis quatre siècles, nous conférait en la matière une responsabilité particulière. Il ne saurait cependant être question de se limiter au cadre français, ni dans les thèmes abordés, ni dans les auteurs publiés. Au contraire, nous avons voulu marquer nettement la vocation internationale de la *Revue* par la composition du comité de lecture: dix-sept pays des cinq continent y sont en effet représentés.

La *Revue d'histoire des mathématiques* est ouverte aux thèmes et aux voies d'approche les plus divers: de l'analyse conceptuelle à l'étude des interactions avec d'autres champs du savoir ou avec le contexte institutionnel et social. Les seules conditions requises sont l'originalité et la qualité des articles, garanties notamment par un système de deux rapporteurs. Une place est faite dans la *Revue* à des articles de mathématiciens contemporains dont le témoignage et le regard apportent des matériaux pour l'écriture de l'histoire des mathématiques au XX<sup>e</sup> siècle.

Image de l'histoire des mathématiques aujourd'hui, la *Revue* s'adresse non seulement aux historiens des sciences mais aussi aux mathématiciens, aux enseignants de mathématiques, aux philosophes des sciences et plus généralement à tous ceux qu'intéresse une réflexion sur les mathématiques à partir d'une connaissance approfondie de leur passé. Nous espérons que la réalisation de ce premier numéro n'apparaîtra pas trop éloignée de ces objectifs sans doute ambitieux.

La *Revue* est publié deux fois par an. Prix public Europe: 360 F; prix public hors Europe: 400 F. Pour obtenir un souscription, ecrivez à Société Mathématique de France, Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05 (tel: 1 44 27 67 97; fax: 1 40 46 90 96; email: rhm@dmi.ens.fr).

# Historical Connection in Mathematics: Resources for Using History of Mathematics in the Classroom

AIMS Education Foundation has just released Volume 3 of *Historical Connections in Mathematics* by Wilbert Reimer and Luetta Reimer. This book is a collection of resources to help teachers increase interest and deepen understanding of mathematics by placing it in historical and human context. This collection includes biographical information, famous quotations, anecdotes, and illustrations from the lives of ten mathematicians: Eratosthenes, Fibonacci, Descartes, Agnesi, Lagrange, Somerville, Dodgson, Venn, Noether, and Polya. Each chapter includes a portrait of the mathematician and four or five ready-to-use classroom activities related to the work of the mathematician. These reproducible activity sheets invite students to participate in critical thinking, pattern recognition, hands-on learning experiences, and a variety of problem-solving techniques.

*Historical Connections*, Vols. 1, 2 and 3 are available for \$14.95 (U.S.) each from AIMS Education Foundation, P.O. Box 8120, Fresno, CA 93747 (tel: 209-255-4094; fax: 209-255-6396).

#### The Relationship of Egyptian to Greek Mathematics

There has been a debate in the pages of history of science journals over the past several years on the historical relationship of Greek to Egyptian science in general and mathematics in particular. The opening salvo in this debate was the article by Martin Bernal entitled "Animadversions on the Origins of Western Science," ISIS 83 (1992), 596-607. Bernal is the author of the multi-volume work Black Athena, which claims that significant portions of classical Greek culture were strongly influenced by ancient Egyptian civilization, partly, in fact through the colonization of parts of Greece from Egypt. Bernal further asserts that although the classical Greeks themselves recognized and acknowledged this influence (the "Ancient Model"), European scholars of the eighteenth and nineteenth centuries "rewrote" history to deny African influence on the progenitors of European civilization (the "Aryan Model"). Black Athena is a massive (and still incomplete) work, which brings together evidence from linguistics, theology, philosophy, archaeology and other disciplines, but does not deal specifically with the history of science itself. Thus the article in ISIS was Bernal's attempt to summarize his views on that history in particular, in relation to his general thesis about the relationships of Greek and Egyptian civilizations.

The *ISIS* article was answered a year later by Robert Palter in his "*Black Athena*, Afro-Centrism, and the History of Science," *History of Science* 31 (1993), 227-287. Bernal responded in "Response to Robert Palter," *History of Science* 32 (1994), 445-464; and Palter answered Bernal in the same issue on pp. 464-468. A further response to Palter, by Beatrice Lumpkin, has been submitted to *History of Science*. In addition, your editor sent a written response to Palter. All of the articles mentioned are well worth reading and contain much material that can be discussed in mathematics classes. But since the letter of your editor will not be published elsewhere, and Lumpkin's response has not yet been published, I thought it useful to readers of this *Newsletter* to publish excerpts from both of these, the second naturally with the permission of the author. Naturally, to understand these responses and formulate your own opinions, readers should consult the original articles.

The arguments of Bernal and Palter can be separated into two basic claims concerning Egyptian science and its relation to Greek science. Palter summarizes these as follows: "First, Bernal maintains that there were scientific elements in Egyptian

medicine, mathematics, and astronomy long before there was any Greek science at all; and, second, he maintains that Egyptian medicine, mathematics, and astronomy critically influenced the corresponding Greek disciplines."<sup>1</sup> As far as mathematics goes, although Palter argues with Bernal on many specific points and seems to deny both of Bernal's claims, he does not give a clear and definitive response to them.

#### Letter of Katz:

One can clearly argue about the meaning of "scientific" elements in Egyptian mathematics. The Egyptians certainly knew how to solve various kinds of problems, from solving linear equations to calculating the area of - let us assume - the surface of a hemisphere. What we do not know, because the documents don't tell us, is how the Egyptian scribes found the methods they use. Presumably, if they simply "guessed" correctly, then one could say that they did not have a "scientific" mathematics. But if they had some kind of argument - and not necessarily an argument based on strict logical reasoning from explicit axioms - I think one must conclude that their mathematics had a "scientific" underpinning. I agree, of course, that there is no reason to believe that the Egyptians understood the irrationality of  $\pi$  or of  $\sqrt{2}$ . The Babylonians too did not understand that notion. And there is also no clear evidence that the ancient Egyptians knew the Pythagorean theorem.

But on the other issue of whether Egyptian mathematics influenced Greek mathematics, I think there is good evidence that we must take the answer as "yes." After all, many of the ancient Greek sources say so. Not only is Pythagoras supposed to have studied in Egypt, but also Thales, the supposed father of Greek geometry, and Oenopides. And Herodotus, Heron of Alexandria, Diodorus Siculus, Strabo, Socrates (through Plato), and Aristotle all say that geometry was first invented by the Egyptians and then passed on to the Greeks. The question always seems to be, in this regard, what we mean by "geometry." If we mean by geometry an axiomatic treatment with theorems and proofs in the style of Euclid, then it is clear that the Greeks originated this. But mathematicians have always known that, in general, one does not discover theorems by the axiomatic method. One discovers theorems by experiment, by trial and error, by induction, by thinking hard about the subject, etc. Only after the discovery is made does one worry about actually proving that what you have proposed is correct. And it seems clear that when Greek writers mention that the Egyptians invented (or discovered) geometry, it is the geometric results that they mean, not the method of proof.

As one example of this, you mention that "it seems reasonable to assume that the formula for the volume of a square pyramid must also have been known" to the Egyptians, because MMP 14 attests to the knowledge of a formula for calculating the volume of a frustum of a pyramid. But then, in denying that Egyptian results had any effect on Greek geometry, you note that Archimedes claims that Eudoxus was the first to prove this result. I will not argue the question of whether Eudoxus learned the proof in Egypt. So let us assume that he did give the first "rigorous" proof of the formula, probably through a reductio argument based on the method of exhaustion, a proof similar to that in Euclid, Book XII. But what then? The guestion still remains, how did Eudoxus know the formula to be proved? As you mention in a note, Archimedes claims that Democritus was the first to state the result. If we take that claim literally, then Archimedes is simply mistaken - assuming, as already stated, that the Egyptians knew the result. What is also possible, and even probable from what we know of Democritus, is that he gave a "proof" by a method of indivisibles which was not rigorous according to Euclidean (or Archimedean) standards. But if that were so, then we would still have to ask, how did Democritus know the result? Archimedes himself notes, in the preface to his Method that "it is easier to supply the proof [of a

geometrical result] when we have previously acquired, by the method, some knowledge of the questions than it is to find it without any previous knowledge." So if, in fact, the Egyptians knew the formula for the volume of a pyramid, and if, as seems likely from all the testimonies, the Greeks learned of this result from the Egyptians, then one must credit the Egyptians with "influencing" Greek geometry, even if the Egyptians did not have a "proof" of their result. The same is naturally true with regard to other results, including the formulas for the area of a circle and, let us assume, for the area of a hemisphere. One can also consider the basic ideas of proportionality as being present in Egyptian mathematics and could conceive of these too as "influencing" Greek understanding of that subject as well.

Conceding that the Greeks learned of various geometrical results from the Egyptians takes nothing away from Greek creativity, in particular from their invention of the axiomatic method. They were simply doing what mathematicians have always done, building on the results of their predecessors.

#### Response of Lumpkin:

So far as a mathematics teacher can contribute to the Palter-Bernal discussion of Egyptian mathematics and science in *History of Science*, I welcome this opportunity to participate. First, I do accept the ground which Bernal staked out for me, an "insistence on the profound influence of racism within conventional scholarship."<sup>2</sup> All of us who were raised and educated in the United States, where legal apartheid was only recently set aside and de facto apartheid continues, are subject to the pervasive influence of racism. To overcome prejudice takes a conscious effort, rather than a denial that racism is still with us in the United States and elsewhere. ...

Palter begins his discussion of Egyptian mathematics with the Egyptian numerals. "The important point here is what the Egyptians did not do: they did not use a so-called place-value notation."<sup>3</sup> In contrast. Carl Bover wrote about what the Equptians *did* do. which he considered a decisive contribution to the development of numerals. He credited the Egyptians with introducing the idea of cipherization with their use of hieratic numerals. Mathematical and other papyri are written in a cursive form of the hieroglyphs that we now call hieratic. Although hieratic numerals were developed from hieroglyphic numerals, they used a different concept to represent numbers. Hieroglyphic numerals were grouped and repeated as needed to represent a value. A number such as 19,607 written in hieroglyphs would require 23 symbols, but in hieratic would need only 4 symbols. The hieratic numerals used single symbols or ciphers, as Boyer explained, "for each of the first nine integral multiples of integral powers of ten." He called the hieratic numeral system, "decimal cash-register cipherization," referring to old-style cash registers which sent up a flag for each decimal place. Bover added that, "The introduction by the Egyptians of the idea of cipherization constitutes a decisive step in the development of numeration."<sup>4</sup>...

Palter questions my evaluation of Egyptian unit fractions, especially a claim that scientists continued to use them for thousands of years.<sup>5</sup> For those times, the development of unit fractions in ancient Egypt was a big step beyond restriction to integers only. No doubt Egyptian scribes prided themselves on their astounding ability to perform complex operations. The ability to take a value of 16 + 1/56 + 1/679 + 1/776, find 2/3 of it as 10 + 2/3 + 1/84 + 1/1358 + 1/4074 + 1/1164, then add 1/2 of it and 1/7 of it and show that it all adds up to 37 is no mean feat.<sup>6</sup> Why unit fractions persisted so long, I do not know. For example, in Leonardo of Pisa's *Liber abbaci* (1202), tables are provided for conversion from common fractions to unit fractions. As Boyer said, "Fibonacci evidently was fond of unit fractions -- or he thought is readers were ... "<sup>7</sup> ...

[According to Gillings], "it is not proper or fitting that we of the twentieth century should compare too critically the Egyptian methods with those of the Greeks or any other nation of later emergence, who, as it were, stood on their shoulders. We tend to forget they were a people who had no plus, minus, multiplication or division signs, no equals or square-root signs, no zero and no decimal point, no coinage, no indices, and no means of writing even the common fraction p/q; in fact, nothing even approaching a mathematical notation, nothing beyond a very complete knowledge of a twice-times table, and the ability to find two-thirds of any number, whether integral or fractional. With these restrictions they reached a relatively high level of mathematical sophistication."<sup>8</sup>

Gillings' overly modest statement needs some modification. ... The Ahmose papyrus does show the beginnings of the use of symbols. Egyptians used the word for heap (variously pronounced aha and hau) for the unknown quantity, much as cosa was used in 15th-16th century Italy. Some symbols were used for addition and subtraction: a pair of feet would walk a number towards another for addition. For subtraction, a pair of feet would walk the number away.<sup>9</sup>

Gillings' statement about Egyptians having no zero also needs modification. It is true that a zero placeholder was not used (or needed) in the Egyptian "cash register" decimal system. However, the Egyptians did use a zero symbol for at least two other applications of the zero concept. Dieter Arnold shows that a zero symbol was used to label the ground level reference line at the Old Kingdom Meidum pyramid. A series of horizontal leveling lines were used as construction guides for the pyramid. Lines above the zero level were labeled 1 cubit above zero, 2 cubits above zero, and so on. Lines below the zero level were labeled according to the number of cubits below zero.<sup>10</sup> The zero symbol was the trilateral hieroglyph, *nfr*.

The same *nfr* symbol was also used to express zero remainders in a monthly account sheet from the Middle Kingdom, dynasty XIII (c. 2770 BCE). It looks like a double entry account sheet with separate columns for each type of goods. At the end of the month, the account was balanced. For each item, income was added, then disbursements were totaled. Finally, total disbursement was subtracted from the total income for each column. Four columns had zero remainder, shown by the *nfr* symbol.<sup>11</sup> ...

Other important mathematical achievements of ancient Egypt are recorded in construction and artwork rather than in mathematics texts. All of these sources reveal highly developed proportional thinking, from the canon of proportions for drawing and sculpture of the human body, to proportional enlargement of drawings by placing a net of squares over the drawing and enlarging, square by square. Proportional reasoning was also used in the false position solutions of equations....

Palter sees little possibility of Egyptian influence on Hellenistic mathematics. He contrasts the "rigorous axiomatic" proofs of [the Greeks] with Egyptian methods: "It is difficult to exaggerate the difference between such proofs and anything found in Egyptian mathematics."<sup>12</sup> But there was much in the ancient Egyptian mathematics and literature that could have contributed to the development of logical, deductive methods. Debate and argument were held in high regard in the literature and culture. The story of "The Eloquent Peasant"<sup>13</sup> tells about a peasant's suit in court which he pleaded so eloquently that it was reported to the king. The king ordered the man detained (while his family was taken care of) so that they could hear the eloquent peasant day after day until they finally granted his plea and gave him a rich reward. Egyptians pleaded their own cases in court and they were a litigious society, with some cases extending for three generations. ...

Within the mathematical papyri themselves, Gillings found many examples of concrete solutions given to illustrate a general method ... "We have to accept the circumstance that the Egyptians did not think and reason as the Greeks did. If they found some exact method (however they may have discovered it), they did not ask themselves *why* it worked. They did not seek to establish its universal truth by an a priori symbolic argument that would show clearly and logically their thought processes. What they did was to explain and define in an ordered sequence the steps necessary in the proper procedure, and at the conclusion they added a verification of proof that the steps outlined did indeed lead to a correct solution of the problem. This was science as they knew it."<sup>14</sup>

References:

1. Palter, Robert, "*Black Athena*, Afro-Centrism, and the History of Science," *History of Science* 31 (1993), 227-287, pp. 227-228.

2. Bernal, Martin, "Response to Palter," *History of Science* 32 (1994), 445-464, pp. 446.

3. Palter, *op. cit.*, p. 247.

4. Boyer, Carl, "Fundamental Steps in the Development of Numeration," *ISIS* 35 (1944), 153-168.

5. Palter, *op. cit.*, p. 282.

6. Chace, Arnold B., *The Rhind Mathematical Papyrus* (Reston, Va.: National Council of Teachers of Mathematics, 1979), p. 40.

7. Boyer, Carl, *A History of Mathematics* (New York: Wiley, 1968), pp. 280-281.

8. Gillings, Richard J., *Mathematics in the Time of the Pharaohs* (Cambridge: MIT Press, 1972), p. 234.

9. Chace, op. cit., p. 99.

10. Arnold, Dieter, *Building in Egypt* (New York, 1991), p. 17.

11. Scharff, Alexander, "Ein Rechnungsbuch des Königlichen Hofes aus der 13. Dynastie (Papyrus Boulaq Nr. 18)," *Zeitschrift für Ägyptische Sprache und Altertumskunde* 57 (1922), pp. 58-59.

12. Palter, op. cit., p. 251.

13. Lichtheim, Miriam, *Ancient Egyptian Literature*, vol. 1 (Berkeley: University of California Press, 1975), pp. 169-184.

14. Gillings, op. cit., p. 234.

#### **Have You Read?**

Ronald Calinger, ed.

This column welcomes references from across the history or pedagogy of

mathematics, as well as other works with sections that have potential for encouraging and motivating students to learn mathematics better or that enrich courses. Please send citations with complete bibliographic information to the section editor c/o Department of History, Catholic University of America, Washington, DC 20064, U.S.A.

Abeles, Francine F., ed., *The Mathematical Pamphlets of Charles Lutwidge Dodgson and Related Pieces* (Volume 2 of *The Pamphlets of Lewis Carroll*) (Charlottesville, Va.: University Press of Virginia, 1995). This volume concentrates on the Eminent Victorian Dodgson's career as a mathematical lecturer at Christ Church, Oxford.

Bendegem. J. P. van, "Ludwig Wittgenstein, een ingenieur ontleedt de wiskunde," ("Ludwig Wittgenstein, an Engineer Looks at Mathematics"), *Wiskunde en Onderwijs* 18 (1992), 103-114.

Chikara, Sasaki; Mitsuo, Sugiura; and Dauben Joseph, *The Intersection of History and Mathematics* (Basel: Birkhäuser, 1994).

Denkez, R., "Simon Stevin," Wiskunde en Onderwijs 19 (1993), 75-98.

King, John, *His Arithmetic Book* (1795). This is a reproduction of the arithmetic notes of John King, a 9-year old in 1795, including vulgar fractions, double fellowship, single position, partial alligation, and promiscuous questions. It can be ordered from John King, the great grandson of the author, at 24 Ailsa Avenue, Twickenham, TW1 1NG, ENGLAND (tel/fax: 081 892 9056) for £14.00 + £1.00 P&P.

Kleiner, Israel, "The Roots of Commutative Algebra in Algebraic Number Theory," *Mathematics Magazine* 68 (1995), 3-15.

Kleiner, I. and Movshovitz-Hadar, N., "The Role of Paradoxes in the Evolution of Mathematics," *The American Mathematical Monthly* 101 (1994), 963-974.

Le Ru, Veronique, *Jean Le Rond d'Alembert philosophe* (Paris: Libraire Philosophique J. Vrin, 1994).

Meli, Domenico Bertoloni, *Equivalence and Priority, Newton versus Leibniz* (Oxford: Oxford University Press, 1993).

Menger, Karl, *Reminiscences of the Vienna Circle and the Mathematical Colloquium*, ed. by Louise Golland, Brian McGuinness, and Abe Sklar (Vienna Circle Collection, Vol. 20) (Dordrecht: Kluwer Academic Publishers, 1994).

Meskens, A., Wiskunde tussen Renaissance en Barok: Aspecten van wiskundebeoefening te Antwerpen 1550-1620 (Mathematics between Renaissance and Baroque: Aspects of the practice of Mathematics at Antwerp 1550-1620) (Antwerp: Stadsbibliotheek, 1994).

Oliver, David, *The Shaggy Steed of Physics: Mathematical Beauty in the Physical World* (Berlin: Springer-Verlag, 1994).

Price, Michael H., *Mathematics for the Multitude: A History of the Mathematical Association* (Leicester: Mathematical Association, 1994).

Pulkkinen, Jarmo, *The Threat of Logical Mathematism: A Study on the Critique of Mathematical Logic in Germany at the Turn of the 20th Century* (Frankfurt am Main: Peter Lang, 1994).

Rashed, Roshdi, The Development of Arabic Mathematics: Between Arithmetic and Algebra,

trans. by A. F. W. Armstrong (Dordrecht: Kluwer Academic Publishers, 1994).

Reimer, Luetta and Wilbert, *Mathematicians Are People, Too* (vol. 2) (Palo Alto: Dale Seymour Publications, 1995). This book is a collection of 15 stories about great mathematicians. The contributions of each mathematician are presented in a dramatic style, using dialogue and descriptions to accentuate their humanity. The stories may be read aloud by teachers or read by students for pleasure or research.

Roszak, Theodore, *The Cult of Information: A Neo-Luddite Treatise on High Tech, Artificial Intelligence, and the True Art of Thinking* (Berkeley: University of California Press, 1994).

Sharratt, Michael, Galileo: Decisive Innovator (Cambridge: Blackwell, 1994).

Spalt, Detlef, "Quo Vadis -- History of Mathematics?" *The Mathematical Intelligencer* 15 (4) (1993), 4-6.

Stephenson, Bruce, *Kepler's Physical Astronomy* (Princeton: Princeton University Press, 1994).

Verbruggen, "En Eulers graf was het begin" ("Euler's Graph was the Beginning"), *Wiskunde en Onderwijs* 18 (1992), 481-500.

Woolhouse, R. S., Descartes, Spinoza, Leibniz: The Concept of Substance in Seventeenth Century Metaphysics (New York: Routledge, 1993).

Woolhouse, R. S., *G. W. Leibniz: Critical Assessments* (The Routledge Critical Assessments of Leading Philosophers), 4 Volumes (New York: Routledge, 1994).

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