

# International Study Group on the Relations Between HISTORY and PEDAGOGY of MATHEMATICS NEWSLETTER

AN AFFILIATE OF THE INTERNATIONAL COMMISSION ON MATHEMATICS INSTRUCTION

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## Calendar

1993 November 11 - 14 Santa Fe

Annual meeting of the History of Science Society. Contact the program chairs, Paul Lawrence Farber, Department of History, Oregon State University, Corvallis, OR 97331-5102, USA; fax: 503-737-2434; e-mail: farberp@ccmail.orst.edu; or Margaret J. Osler, Department of History, University of Calgary, Calgary, Alberta T2N 1N4, CANADA; fax: 403-289-8566; e-mail: mjosler@acs.ucalgary.ca.

1994 January 12 - 15 Cincinnati

Annual meeting of the American Mathematical Society and the Mathematical Association of America. There will be a special session in the history of mathematics organized by Victor Katz and Tom Archibald.

1994 January 17 - 19 Paris

Colloquium on Reforms in the Teaching of Science and Mathematics. (See inside for more details.)

1994 February 18 - 23 San Francisco

Annual meeting of the AAAS, including Section A, Mathematics.

1994 March 28 - 31      Winchester, England

HIMED 94 (History in Mathematics Education) to be held at King Alfred's College in Winchester. (See insider for more details.)

1994 April 13 - 16      Indianapolis

Annual meeting of the Americas Section of HPM in connection with the annual meeting of the National Council of Teachers of Mathematics. (See inside for more details.)

1994 April 24 - 30      Oberwolfach, Germany

31st Oberwolfach meeting on the History of Mathematics. For information, contact Eberhard Knobloch, Institut für Philosophie und Wissenschaftsgeschichte, Technische Universität Berlin, Berlin, GERMANY, Tel: (030) 401-6473; or Mathematisches Forschungsinstitut Oberwolfach Geschäftsstelle, Alberstrasse 24, W-7800 Freiburg im Breisgau, GERMANY.

1994 May 14 - 18      Nancy

Congres International Henri Poincaré will be held at the Archives of the Centre d'Etudes et de Recherche Henri-Poincaré.

1994 June 8 - 10      Calgary

Annual Meeting of the Canadian Society for History and Philosophy of Mathematics. (See inside for more details.)

1994 July 25 - 27      Blumenau, Brazil

International meeting of HPM. (See inside for more details.)

1994 August 3 - 11      Zurich

International Congress of Mathematicians. It is hoped that there will be HPM participation at the Congress. Details will be announced at a later date. For information on the Congress, contact ICM 94, International Congress of Mathematicians, ETH Zentrum, CH-8092 Zurich, SWITZERLAND.

1994 August 15 - 17      Minneapolis

Mathfest, including the summer meetings of the Mathematical Association of America and the American Mathematical Society. For details, contact Hope Daly, AMS, P.O. Box 6887, Providence, RI 02940.

1994 August 16 - 20      Shanghai

ICMI-China Regional Conference on Mathematics Education. The conference theme is Teacher Preparation in Mathematics. For information, contact Zhang Dian-zhou and Zhang Zhen-ya, Department of Mathematics, East China Normal University, Shanghai 200061, PEOPLE'S REPUBLIC OF CHINA; tel: 86-021-257-1095; fax: 86-021-257-8367.

## **From the Chair**

John Fauvel

This summer has seen a number of important conferences discussing HPM concerns. Most outstanding was the Montpellier event in July, the first European Summer University on the History of Mathematics. I pay tribute to Evelyne Barbin and her colleagues who worked hard to create such a successful and unprecedented venture, which has set a high standard for the organisers of the projected next European Summer University -- in Braga, Portugal in August, 1996. Such events as these only happen

- through superb and painstaking organisational efforts over a long time,
- through the detailed work of every contributor in distant classrooms, libraries and studies which they generously bring to share; and
- through the equally crucial endeavours of everyone to attend -- people flew to Montpellier specially for

this week from South America, from North America, from Africa; and flew or drove or took the train from many places throughout Europe. Everyone I spoke to on the opening day had got up at an unearthly hour to set out on sometimes nerve-wracking journeys of many hours.

A report on the Montpellier meeting appears elsewhere in this *Newsletter*. An especially gratifying aspect of the meeting was the enthusiasm and support of the Montpellier civic authorities. Indeed, Evelyne Barbin and I received medals, copies of the 14th century great seal of Montpellier, on behalf of the HPM community, to mark the holding of this HPM meeting at Montpellier. I hope it is of interest to share with you the speech I made in response.

M. le Maire,

It was with great pleasure that the International Study Group on the History and Pedagogy of Mathematics learned that Montpellier was to be the site for our first European Summer University; as the historic meeting-place of European and Mediterranean civilisations, Montpellier is the most appropriate place one can imagine for a meeting using history to inspire the pedagogy of the future.

The extraordinarily impressive architectural dream which is being realised here, in pursuit of the Montpellier-Languedoc-Roussillon Technopole, is vivid testimony to the way that the future is best created by those rooted in a knowledge of the past.

This afternoon I spent some time in your historic Jardin des Plantes, the oldest in France if not in Europe, which this year celebrates its 400th anniversary. It is a wonderful place of peace and beauty offering many mathematical reflections, from the numbered stones, *pierres provenant de l'ancien jardin medical de Richer de Belleval fin 16ième siecle*, to the parallel lines and other geometrical patterns which structure the rich micro-landscape. I hope that a mathematics trail may be created in your Jardin des Plantes, which would enable children to explore mathematics, history, and botany at the same time.

What I will take away from Montpellier above everything else is the memory of the magnificent commemorative statue in the Jardin des Plantes to one of your former students at the University of Montpellier, François Rabelais, whose spirit I hope will continue to imbue our activities. The legend on this statue reads "vivez joyeux", which exactly encapsulates what we are trying to achieve for mathematical teaching in the HPM movement.

I toast the Eurocity of Montpellier, the concept Montpellier-Languedoc-Roussillon Technopole, and the spirit of François Rabelais: *vivez joyeux*.

Another meeting with much of interest to HPM was the International Congress on the History of Science, held in August in Zaragoza, Spain. A report on this, too, appears later in the *Newsletter*. For me its rich, spontaneous, international character, as well as its didactic relevance and concern, was encapsulated in this little incident: in one session, a *Mexican* started to give a talk in *Spanish* about changing the language of primary mathematics teaching, in a region of Mexico, from Spanish to the local *native American* language. Several members of the audience began to leave, unable to cope with talks in Spanish (despite the meeting being held in Spain); so the chair (who was from *Mozambique*) appealed to someone to act as translator. Only one brave *Frenchman* volunteered, and translated from Spanish into *English* paragraph by paragraph. The result was very fruitful and interesting. I have experienced on other occasions, too, the great benefits of a talk being given in two languages, which gives the audience a far better opportunity to absorb and think about its content because it is going, in effect, at half the rate.

At the end of a summer of conferences in different places, it is natural to dwell on the pleasures and benefits of meeting a great variety of people, friends old and new, learning fresh things, sharing the results of one's own work, developing plans and projects for the future. There are several advantages of this for an individual. But publicly, at least, its purpose is to strengthen and refertilize, to develop and improve the historical dimension of mathematics education in classrooms throughout the world. Everyone who is privileged to travel and join in such an international meeting has a responsibility to share the results with those who stayed behind, keeping the home fires burning. As far as I can tell, this is indeed happening, and

thus the HPM movement goes forward from strength to strength.

## **From the Editor**

Victor J. Katz

To add to John's observation that the HPM movement continues to grow and prosper, I want to acknowledge the awards recently presented to several mathematicians in North America who use the history of mathematics in their teaching. Thus Fred Rickey, HPM Americas Section chair, was awarded one of the first MAA awards for distinguished university teaching of mathematics. The Allendoerfer Award of the MAA for expository articles in *Mathematics Magazine* was presented to Israel Kleiner (York University, Canada) for his article "Rigor and Proof in Mathematics: An Historical Perspective," while the George Pólya Award of the MAA for expository articles in the *College Mathematics Journal* went to William Dunham (Muhlenberg College) for "Euler and the Fundamental Theorem of Algebra." Finally, David Pengelly (New Mexico State University), whose article about his course on Great Theorems appeared in *Newsletter* 28, received the 1993 Section Award for Distinguished Teaching from the Southwestern Section of the MAA. Does using history make you a better teacher and expositor? Or do better teachers and expositors use history? In either case, HPM is now having an influence in mathematics teaching and that influence continues to grow.

## **European Summer University in Montpellier**

Dédé de Haan, Freudenthal Instituut, Tiberdreek 4, 3561 GG Utrecht, THE NETHERLANDS

In my job as a maths-teacher at a secondary school in the Netherlands, I try to put as much history of mathematics as possible into my lessons. I do this not only to bring variation into my classes, but also to show the students that there is more to mathematics than formulas, that sometime somewhere someone had a problem he wanted to solve, and how solutions to this problem evolved into the very precise and rigorous laws we now have in mathematics. For example, I show them the development of the logarithm, which started with the question, "Is there an easier way to find the product of two very big numbers?" Napier said, "Yes, there is!" and came up with the idea of a logarithm.

I think mathematics will become a more lively subject to learn (and to teach) as soon as students realise that there is more to it than what their schoolbooks say. There are other possibilities to make your lessons more interesting of course, including applications (such as in cryptography) or new discoveries (such as the proof of Fermat's Last Theorem) which show students that even today mathematics is still changing. But I have to keep in mind the age of my students; whatever interesting things I treat in my lessons are only means to achieve a better understanding of the mathematics which is in their books. That means that every subject whose history I use needs to be simplified enough to be understood by secondary school students.

During my own study of mathematics at the University of Groningen, I became very interested in the history of mathematics, and I developed a unit on the invention of the logarithm in which many events were simplified for the sake of understanding. This unit, which was developed under the supervision of Dr. Jan van Maanen, was tested in a classroom of 16 year olds and was received so enthusiastically that I wanted to do more of these kinds of things in my own position as a teacher. However, in the Netherlands there are no usable textbooks in which history is the starting point to teach the mathematics which has been developed, so you need to write your own books. Because this takes a lot of time and study, I became very interested in what other people are doing in this field. I hoped I would be able to use their ideas in my own classroom or I could learn where to find material or perhaps find out which countries had good textbooks. Thus there were plenty of reasons to be very interested in what the European Summer University on History and Epistemology in Mathematics Education had to offer. It was Jan van Maanen who suggested that I go there; he knows me very well and thought it could be very useful for me. So I applied for a bursary from the European Council, received it, and went.

I arrived Sunday night, 18 July and registered on Monday. Monday was a day of plenary lectures: Christian Houzel gave a two-hour lecture on Mediterranean mathematics from the second millenium B.C. to the 17th century. I thought it was a very general story; my general impression was that it was a nice lecture with which to start the week, but it contained nothing new. Any university student who had had a course in the

history of mathematics could have told the same story. There was another thing that bothered me a little; Houzel gave the impression that Mediterranean mathematics (Egypt, Babylonia, Greece, Alexandria, Arabia, Europe) was the mathematics most influential in the mathematics we teach nowadays, that it was more influential than, for example, Asian mathematics. I tend to disagree on that, and so did other people. It therefore stimulated a nice discussion in the auditorium.

After a great lunch with a Mediterranean wind blowing so hard that completely filled wineglasses were blown off the tables (to prove Houzel's statement that products of the Mediterranean are indeed very powerful?), another two-hour lecture was given by Jens Høystrup on "The algebra of land measuring" from Naram-Sim to Luca Pacioli, a sub-scientific tradition active over four millennia. He told about "the career of a particular mathematical problem -- to find the side of a square from the sum of its four sides and its area -- from its first appearance in an Old Babylonian text until it surfaced for the last time in the same unmistakable form during the Renaissance in the works of Luca Pacioli and Pedro Nunez. The problem turns out to belong to a non-scholarly tradition carried by practical geometers, together with other simple quasi-algebraic 'recreational' problems dealing with the sides, diagonals and areas of squares and rectangles. This 'mensuration algebra' was absorbed into and interacted with a sequence of literate mathematical cultures: the Old Babylonian scribal tradition, early Greek 'metric' geometry, and Islamic al-jabr." The aim of the lecture and the article Høystrup wrote about this subject were to explore how these interactions inform us about the early history of algebraic thinking. I was very glad to receive the article about this subject. I read it carefully. It is a very nice subject to use in the classroom, and Høystrup has done more investigation on the influence of the different mathematical cultures on each other than the regular textbooks offer.

I had high expectations from the panel on the place of the history of mathematics in mathematics teaching and curriculum, organized by Jan van Maanen, with members from Great Britain, Germany, Italy, Greece, Denmark, France, Portugal, and the Netherlands. But it seems that everywhere in Europe, with the exception of Denmark, the history of mathematics appears in the curriculum as an addition or appendix and not as a substantial and essential component of mathematics education. Material is being produced (in France by the I.R.E.M.), but it depends on the teacher if it will be used or not. Of course, these talks were too general to receive a good insight into what was really happening in the different countries, but at this point I really formed the impression that this initiative of a European Summer University could be very helpful for all of the attendees.

On Tuesday, there was a two-hour plenary lecture in the morning; what I found more interesting were the lectures about different themes and the workshops. I went to a 1½ hour lecture by Marta Menghini from Italy about the Euclidean method in teaching geometry. She told about the situation in Italy at the end of the 19th century; in 1867 Euclid's *Elements* started to be taught in schools. This influenced two new trends that had to do with the problem of how one should define the equality of geometrical figures. Ricardo de Paolis developed new axioms about "rigid body motion," while Giuseppe Veronese avoided "rigid body motions." He talked about theoretic geometry and practical geometry; only the latter needs an extra postulate about "moving bodies." I thought this lecture was interesting, because I did not know much about the history of modern Italian mathematics. However, it did not supply me with information I could use in the classroom. Neither did the Bolzano workshop led by Steve Russ from Great Britain. But this workshop gave me a lot of information about Bolzano and his way of thinking. Steve Russ also provided us with a lot of original material, and that is something worthy too. However, if I ever want to use Bolzano's material in the classroom, there is a lot of simplification I have to apply to it myself. But at least I will not have to search for the material. But still... The lecture about Sophie Germain's contributions to number theory, by Reinhard Laubenbacher (U.S.A.) was very interesting, especially with regard to the contributions she made to the proof of Fermat's Last Theorem and to the fact that she is a woman -- a very neglected segment of the people who made contributions to the development of mathematics.

On Wednesday, there were again different lectures, and the workshops of Tuesday were continued. Harm Jan Smid gave a lecture on "A Conflict at the Leyden Gymnasium in the 19th Century." Smid is from the Netherlands and told a very interesting story about the way mathematics was taught in school in the 19th century: Jaco de Gelder had great influence in the realization of a mathematics curriculum for all classes in grammar school. By now I realized that I had to readjust my expectations of this week. No one was going to offer me material to use in the classroom. There were a lot of people who shared their knowledge about

certain people or certain developments in the history of mathematics with the attendees, but that was it. This was no real disappointment. I am not only interested in the history of mathematics I can use in the classroom; I am also very interested in the history of mathematics itself. Besides, I got to know a lot of interesting people who were very sociable too.

So it was a very big surprise that Thursday brought me exactly what I thought the whole week would bring me: Marjolein Kool (The Netherlands) had a workshop on the use of historical arithmetic books in teaching mathematics to low-attainers; she provided me with material I can use in the classroom right away. It is very interesting to see that the problems posed in today's textbooks are exactly the same as the ones posed in 16th century arithmetic books. Marjolein also took us outdoors to measure the height of a tree with the aid of a mirror. This was a very interesting workshop, with good material to bring home with me. So was the workshop by Peter Ransom, of Great Britain. He let us make our own sundials and let us measure with the help of the cross-staff, which gave very accurate results. His workshop was called "Navigation + Surveying: Teaching Geometry Through the Use of Old Instruments." It was the most interesting and most successful workshop I attended. He provided a lot of material, and he was enthusiastic and could tell about experiences he already had with using this in the classroom. That was exactly what I was looking for.

I have to mention here that there were many more very nice workshops on Thursday, unfortunately all at the same time. For example Jan van Maanen gave a workshop on Huygens, and Greisy Winicky (Israel) presented one on using mathematical problems with historical backgrounds in junior high school. I am very sorry I could not attend all these workshops.

Friday, the last day of the Summer University, was a day like Monday: two plenary two-hour lectures and a panel on the place of the history of mathematics in initial and in in-service teacher training. The first lecture was about theoretical teaching and practical teaching in mathematics in the 19th century. Comparisons between the mathematical cultures in England and France seemed very interesting beforehand, and it was, at first. But the lecture took too long to keep me interested. Ubi D'Ambrosio gave an interesting lecture on ethnomathematics in the history of ideas, something I have never dealt with before. It really woke me up again. The last session, the panel, was also interesting; it was about teacher training and the use of history of mathematics in the European countries. On Thursday, I had learned from a lecture by Gertrudes Amaro what the situation was in Portugal from that perspective: students must make their own lesson plans which contain historical episodes. In this panel we were told about the situation in France (all about I.R.E.M., which has taken care of a great deal of history of mathematics in teacher training), in Belgium (where courses in history of mathematics and science are given but at some universities they are optional and at others they are compulsory), in Denmark (where history of mathematics is very important, especially in teacher training), and in Great Britain (where there is very good material nowadays including courses from the Open University and refresher courses from HIMED but where the use of history still depends on individuals who are enthusiastic enough. In Germany, history of mathematics is no longer compulsory, while it was in the G.D.R. In the Netherlands, it depends on the city and the individual who is your teacher; there are no official prescriptions. It was an interesting end to a very busy week.

As mentioned before, I expected more days to be like Thursday, to hear from experiences other people had while using history of mathematics in the classroom and to collect material they used. However, I made a lot of contacts with other people and exchanged a lot of ideas. I know now that there is much material I can use, produced in other countries. That was something I did not know before, and I now know how to get a hand on these materials. This was my first Summer University too, and I found out that it is a very good means to exchange ideas. I plan to attend more in the future and maybe to give a contribution myself. I am planning to use material from, for example, Peter Ransom in the classroom, and I have told my colleagues about this Summer University.

### **Another View of the First European Summer University**

Peter Ransom, Prudhoe County High School, Moor Road, Prudhoe, Northumberland NE42 5LJ, U.K.

On disembarking from the plane at Montpellier, we (John Fauvel, David Nelson, and Myself) wondered what had been put into the food (or was it the wine) that had suddenly given the rest of the passengers the ability

to speak fluent French. The temperature had also risen quite considerably, demanding a change to the type of clothing rarely used in the North-east.

Registration was like Babel. We waited for friends from different countries, many of whom have made valuable contributions to HIMED conferences in the UK, to register, and then we sought refreshment. Swapping travel stories and catching up with each other's news was an excellent way to start this conference, and this continued during the walk back to the accommodation at the City University.

Monday saw the official opening of the programme, and included two plenary lectures and a panel on the place of history of mathematics in mathematics teaching and the curriculum. Because a more detailed report on the conference appears elsewhere in the *Newsletter*, I will tend to concentrate on the fringe activities.

Much discussion took place at lunch times and in the evenings. I spent time with Reinhard Laubenbacher and Mike Siddoway learning about how they had used mathematicians' own works (in editions as original as possible) with students in the USA to introduce both a deeper understanding of mathematics as well as the history. Patrick Gray helped me find and measure some local sundials, and we came across a bust of Cardan in the Faculty of Medicine.

On Tuesday evening, after being welcomed by the Mayor, M. Georges Freche, and seeing John Fauvel and Evelyne Barbin receive their medals of Montpellier, we were treated to a dinner and dance (some superb rock and rolling) at the Chateau Grammont. This helped break a bit more ice, and the bendy bus seemed a bit bendier to some people on the way back!

The lecture and workshops on Tuesday, Wednesday, and Thursday varied quite considerably. Since these were organised into seven themes, one had the opportunity to attend a wide variety of presentations. My choices included Jaroslav Folta, "Mathematical textbooks in vernacular languages: the case of Czech textbooks and their development in the 16th century;" Reinhard Laubenbacher, "Great problems of mathematics" and "Sophie Germain's contributions to number theory;" Leo Rogers, "Is it possible to reconstruct mathematical knowledge in history?" Salimata Doumbia, "L'expérience en Côte d'Ivoire de l'étude de jeux traditionnels africains et de leur mathématisation;" and Peter Bero, "Volume calculations in a manner of the 16th century."

After the close of the conference on Friday, a group of us went to visit "Grand Siècle," the art exhibition on display in Montpellier, before enjoying another meal with international company. Saturday was the day of the post-conference trip "Autour des gorges de l'Hérault." I met some more interesting people who promised me pictures of foreign sundials. (Everyone at the conference was interesting; those who showed enthusiasm for helio-centered time keeping just seemed especially interesting!) Rod Haggerty helped me with another local dial hunt on the Sunday morning before returning to the airport.

My sincere thanks to Evelyne and the organisers of the conference. The French I.R.E.M. is a wonderful institution which should be taken on board in this country. Conference organisation in one language is not an easy task, but when you have to cope with a global crowd, the problems can become magnified many times. However, the entente cordiale was superb with helpful, friendly, understanding organisers. The conference was very worthwhile, and I for one hope that it will continue and look forward to meeting many more old and new friends at future summer universities.

## **A Tale of Two Cities**

Man-Keung Siu, Department of Mathematics, University of Hong Kong, HONG KONG

Dublin is as different from Zaragoza as a Western European capital with its Georgian terraces and famous writers is from a strategic city in the Iberian hills with its medieval cathedrals and castles. But for once they provided events of common interest to readers of this *Newsletter* in the summer of 1993. I went to attend a conference on "The Mathematical Heritage of Sir William Rowan Hamilton" held at Dublin, Ireland on August 17-20, then rushed south to attend the 19th International Congress of History of Science held at Zaragoza, Spain on August 22-29.

The Dublin conference was held at the Trinity College in commemoration of the sesquicentenary of the discovery of the quaternions by Hamilton in 1843. Not all talks given at the conference were about Hamilton or his mathematical heritage. Some could be labelled so as they involved notions that bear the name of this most celebrated Irish mathematician. The talk I liked most, which was related to both Hamilton and his mathematical heritage, was the talk "The Icosian Calculus of Today", delivered beautifully by Norman Biggs. (We even had a chance to look at the one and only original icosian game in existence, which was marketed in 1859.) On the first evening, all participants were invited to a lecture on the life and work of Hamilton, delivered by Thomas David Spearman, followed by a wine reception, both events taking place at the Royal Irish Academy. The meeting room and reading room of the Library of the Academy, where the lecture was held, are well worth a visit. In one afternoon we visited the Dunsink Observatory, situated on a hill 8 km northwest of the centre of Dublin, in which Hamilton worked as the Andrews Professor of Astronomy and the royal Astronomer of Ireland from 1827 to 1865. Naturally, we would not miss the chance of tracing his steps by taking a 45-minute walk from the Observatory, along the Royal Canal, to the Brougham Bridge. A plaque bearing the famous equations  $i^2 = j^2 = k^2 = ijk = -1$  could be seen at the side of the bridge, for Hamilton once cut those equations with his pen-knife on a stone of the bridge in a moment of excitement at his own discovery. All in all, I enjoyed the conference and the city of Dublin as well.

I left Dublin before dawn on August 21 and arrived at Zaragoza late in the afternoon. My first impression of the place (and hence indirectly -- but also mistakenly -- of the conference that was to come) was not exactly good. The airport, one small building by the side of a large open field, might have had something to do with it. Clearly, its builders did not envisage a horde of visitors who come and go at the same time for an international gathering. (On the morning of August 30, when most participants left Zaragoza, we stood in line for check-in for almost an hour.) The weather might have had something to do with it too, for it was not the most pleasant pastime to stand under a burning sun at a temperature above 30° C to wait for a bus which should have arrived three quarters of an hour earlier, according to the announced schedule, and worse yet, then to be told that the bus may or may not come, announced schedule notwithstanding! One can imagine that many participants got annoyed when they were dispatched from one queue to another queue during registration upon arrival at the conference site straight from the airport, tired and impatient. However, things got better and better as the conference unfolded, including even the weather --- from the third day on, after a brief hailstorm one afternoon, the temperature dropped to a comfortable range, the sunshine was still plentiful but no longer burning, and the air was frequently breezy. More importantly, the interesting and rich academic program and the nice gathering of friends, old and new, were more than enough to content the participants. Besides, the warm hospitality of the host organizers and Spanish splendor of the city enhanced this good feeling.

There were altogether 60 symposia on various topics in the history of science, 10 different scientific sections with short papers, a couple of round table discussions, 10 plenary lectures, and a book exhibition. Of the 60 symposia, I counted 14 on the history of mathematics and 5 more directly connected with the history of mathematics. One of the 10 scientific sections was on the history of mathematics. Perhaps that is just a true reflection of the major role played by mathematics in the history of science.

I felt like a little boy placed before a huge display of candies and cakes, so much I would wish to taste but only that much I could possibly take for physical constraint! Mainly I attended two symposia --- in "Mathematics in Asia" I served as a commentator at the last session on developments in the 18th and 19th centuries, and in "Asian Response to the Dissemination of European Science" I presented a paper titled "Success and Failure of Xu Guang-qi: Response to the First Dissemination of European Science in Ming China." Whenever the time schedule allowed, I jumped in and out of a number of symposia that caught my fancy, such as "Algebra and Geometry Around 1600," "The Historical Role of Algebra and Discrete Methods in Infinitesimal Calculus," "Ethnomathematics, Ethnoscience and the Recovery of World History of Science," "Analysis and Synthesis in Mathematics," "Historiography of the History of Mathematics," and "The Romano College: Transmission of Science Through the Jesuits." I would have wished to attend a few others too, like "Some Aspects of Mathematics in the 20th Century," "Arts and Mathematical Science," "The Theory of Parallels up to the End of the 20th Century," and "Formation of Mathematical Schools in the 19th and 20th Centuries," but could not because of time clashes. From the titles of these symposia mentioned above, the reader can see the wide coverage in the history of mathematics at the Congress.



In the scientific section on mathematics, the short papers were divided into 3 sessions on ancient and medieval mathematics and 4 sessions on mathematics thereafter. I could only listen to a few of those talks, and I found them interesting as well. One highlight, for participants interested in the history of mathematics, was the meeting of the International Commission on the History of Mathematics during which the Kenneth O. May Medal was awarded to Christoph Scriba and Hans Wussing. Members may also wish to know that John Fauvel, the HPM Chair, was elected to the Council. On Friday evening, a reception by the International Commission on the History of Mathematics was held at the Council Hall of Zaragoza to celebrate the 20th anniversary of the journal *Historia Mathematica*. Unfortunately, I missed that happy occasion, but, along with others who were present, I wished even more prosperous years to come for the journal. When I left Zaragoza for home, I brought back more papers, more books, more knowledge as a result of listening to the talks and discussions, and best of all the friendship of more acquaintances, old and new.

#### **IV Coloquio Internacional de Filosofía e Historia de las Matemáticas; I Coloquio Colombiano de Historia, Filosofía y Pedagogía de las Matemáticas**

The First Colombian Colloquium on History, Philosophy and Pedagogy of Mathematics along with the Fourth International Colloquium on Philosophy and History of Mathematics was held at the Universidad Nacional de Colombia in Bogotá from July 26 to August 6, 1993. It was sponsored jointly by the Sociedad Colombiana de Matemáticas and the Asociación para la Historia, Filosofía y Pedagogía de las Ciencias Matemáticas. The International Organizing Committee included Ivor Grattan-Guinness (U.K.), Javier de Lorenzo (Spain), Alejandro Garcíadiego (Mexico), and Ubiratan D'Ambrosio (Brazil), while the local organizing committee was headed by Carlos Vasco (Dept. de Matemáticas y Estadística, Avenida 32 #15-31, Bogotá, 1, D.C., COLOMBIA). Coordination was provided by Clara Helena Sánchez, also of the Mathematics Department.

The first week of the conference consisted of six short courses. These included Ivor Grattan-Guinness (Middlesex Polytechnic, U.K.), "El Cálculo de Cavalieri a Cauchy;" Javier de Lorenzo (Universidad de Valladolid, Spain), "Tendencias actuales en filosofía de las matemáticas;" Luis Vega Reñón (Universidad a Distancia, Spain), "La prueba matemática en Grecia;" Florence Fasanelli (SUMMA, U.S.A.), "El uso de la historia en la pedagogía de las matemáticas;" Alejandro Garcíadiego (UNAM, Mexico), "Temas de historia de las matemáticas;" and Francisco Rodríguez Consuegra (Universidad de Barcelona, Spain), "Temas de filosofía de la matemática: los problemas básicos de la filosofía de las matemáticas."

The second week included some 45 presentations of various topics in the history and philosophy of mathematics. Among the presentations were Ubiratan D'Ambrosio (Brazil), "Limitações historiográficas no estudo da formação de escolas matemáticas: um estudo de caso, o Brasil;" Clara Lucia Higuera Acevedo (Colombia), "La Yupana: un ejemplo de lo histórico como elemento pedagógico;" Rafael Marino (U.S.A.), "Contribuciones no europeas a la matemática;" Florence Fasanelli (U.S.A.), "El uso de la historia en la pedagogía de las matemáticas;" Marina Ortiz Legarda and Hector Ramon Orobio (Colombia), "Historia y pedagogía de las matemáticas en la enseñanza básica;" and Cesar Guevara (Mexico), "Matemáticas y cosmos en la obra de Giordano Bruno."

#### **Colloque: Réformer l'Enseignement Scientifique: Histoire et Problèmes Actuels**

Deux grands moments marquent l'histoire de l'enseignement scientifique au XXe siècle: les années 1900 quand des réformes décisives voient le jour dans de nombreux pays, et les années 1960-1970, années de rénovation profonde de l'enseignement des mathématiques et de la physique.

Une réflexion parallèle sur ces réformes dont les enjeux sont plus que jamais d'actualité, semble opportune. C'est le but de ce colloque conçu comme une manifestation large et ouverte, dépassant le cercle des histoires de l'enseignement scientifique et destinée au milieu enseignant dans son ensemble.

À partir des questions que soulève actuellement l'enseignement scientifique, on tentera de susciter des débats mêlant étroitement les questions d'ordre historique, philosophique, scientifique, didactique. Ainsi les différentes sessions seront respectivement consacrées aux enjeux sociaux des réformes, aux réformes de l'enseignement face à l'évolution des sciences, à la dimension internationale des réformes et, enfin, à l'écart

entre les principes et les réalités. Cinq sessions d'une demi-journée, axées sur des thèmes spécifiques permettant de mener de front la réflexion sur les deux périodes, ont donc été prévues.

Chaque session comprendra une première partie consacrée à des conférences portant sur les réformes des années 1900 et 1960-70 et une seconde partie, sous forme de débats avec le public, abordant les problèmes actuels des réformes de l'enseignement.

Si vous êtes intéressé par les problématiques de ce colloque, qui sera à Paris de 17-19 janvier, 1994, veuillez adresser une demande de programme et de formulaire d'inscription à Éliane Delannoy, Cellule Colloques, Institut National de Recherche Pédagogique, 29, rue d'Ulm, 75230 Paris Cédex 05; tel: 1 46 34 91 11; fax: 1 43 54 32 01.

### **Americas Section of HPM Meeting in Indianapolis**

The Americas Section of HPM will hold its annual meeting in connection with the meeting of the National Council of Teachers of Mathematics in Indianapolis from April 13 - 16, 1994. An HPM session will be held on Friday, April 15 from 4:30 - 7:00 pm and a joint session will be held with the International Study Group on Ethnomathematics on Saturday, April 16 from 1:00 - 3:30 pm. The ISGEM will also hold a session on Thursday, April 14 from 4:30 - 7:00 pm. The theme of the joint session will be "Patterns in Many Cultures." For that session, each group will invite two speakers and will encourage them to take the concept of pattern and develop it for the precalculus classroom. For example, someone could deal with patterns in Native American cultures or patterns in the Islamic world. If you are interested in making a presentation at either the HPM session or the joint session, please send an abstract to Erica Voolich, 244 Summer St., Somerville, MA 02142; email: [evoolich@rcnvms.rcn.mass.edu](mailto:evoolich@rcnvms.rcn.mass.edu).

### **HIMED 94**

A residential conference on the value and use of history in mathematics education will be held at King Alfred's College, Winchester, England on March 28-31, 1994. It is the fifth annual event designed to bring together researchers and teachers at all levels of education to explore issues around the educational use of history of mathematics. Past meetings in this series have established the fruitful interplay among those interested in the history of mathematics from a variety of perspectives.

The conference is from Monday to Thursday, the week before Easter, in this very beautiful and historic cathedral city in southern England. The programme is to contain a rich variety of workshops to explore the themes of the conference in particular case studies. The following participants have already agreed to present material or lead workshops: Peter Baptist (Germany), "The development of triangle geometry in the 19th century;" Peter Bero (Slovakia), (topic to be determined); Sue Burns (U.K.), "Using the *Illustrated London News* in the mathematics classroom;" Ubiratan D'Ambrosio (Brazil), "The historical dimension of ethnomathematics as a pedagogical strategy;" Paul Ernest (U.K.), "History and philosophy of mathematics: some new approaches: what benefit for teachers?" Athanasius Gagatsis and Yannis Thomaidis (Greece), "The history of absolute value and its classroom implications;" Paul Garcia (U.K.) (topic to be determined); Helen Gardner and Irith Shillor (U.K.), "How to set up a maths trail;" Ray Godfrey (U.K.), "Mathematics in ancient Arab folklore: activity in the primary school;" Michelle Gregoire, Philippe Brin and Maryvonne Hallez (France), "The emergence of perspective in the Renaissance: cross-disciplinary activity for 16-17 year olds;" Ivan Tafteberg Jakobsen (Denmark), "Geometry and masonry: Art and science in Renaissance cathedrals;" Marjolein Kool (The Netherlands), "Archimedes, Vitruvius, Peter de Halle, and Hiero's crown;" Manfred Kronfellner (Austria), "History of mathematics in the classroom: problems and teaching methods;" Jan van Maanen (The Netherlands), "Must mathematical progress mean the loss of geometrical insight? A moral for teachers;" Anne Michel-Pajus (France), "The eventful story of divergent series;" Luis Moreno (Mexico), "Mathematics: a historical and pedagogical perspective;" Irith Shillor and Pat Perkins (U.K.), "Recreational mathematics in the classroom;" Frank Swetz (U.S.A.), "The Chinese derivation of the volume of a sphere: learning implications;" Christian Thybo (Denmark), "Written assignments on the history of mathematics: use, evaluation, pitfalls, and pleasures;" Gerry Verhaegh and Anko Haven (The Netherlands), "How can pupils' study of history of mathematics be assessed?" and Guillermina Waldegg (Mexico), "Historical studies and educational research."

HIMED 94 is organised by the British Society for the History of Mathematics. Proposals for additional talks or workshops should be sent to Mrs. Pat Perkins, 5 Tower Road, Orpington, Kent BR5 0SG, U.K. More information on HIMED 94 can be had from Irith Shillor, King Alfred's College, Winchester, Hampshire, U.K. or by email at [irith@winchester-kac.ac.uk](mailto:irith@winchester-kac.ac.uk).

### **CSHPM Meeting in Calgary**

The annual meeting of the Canadian Society for History and Philosophy of Mathematics will be held June 8-10, 1994 in Calgary in association with the annual meeting of the Canadian Learned Societies. There will be a special session dealing with the history of mathematics in North America as well as general sessions. Abstracts should be sent to the program chairman, Craig Fraser, at the Institute for the History and Philosophy of Science and Technology, Victoria College, University of Toronto, Toronto, Ontario M5S 1K7, CANADA or via email to [cfraser@epas.utoronto.ca](mailto:cfraser@epas.utoronto.ca). In addition to the scientific sessions, the Learned Societies are scheduling various social events at the meeting, including a chuckwagon breakfast and a mountain barbecue. So plan on coming to Calgary next June and enjoying western hospitality.

### **International HPM Meeting in Brazil**

After discussions at a meeting in Coimbra, Portugal on the "Primeiro Encontro Luso-Brasileiro de Historia de Matemática" during the week following the International Congress on History of Science, it was decided to hold an international meeting of HPM on July 25-27, 1994 at the University of Blumenau, in Blumenau in the State of Santa Catarina, Brazil. The meeting will be the week following the II CIBEM (Second Iberoamerican Conference on Mathematics Education), which takes place in the same city from July 18 - 22. The conference will have several invited plenary addresses, many short talks given by participants, and plenty of time to meet with colleagues from Latin America and around the world. The languages of the meeting will be Portuguese, Spanish, and English. Blumenau is a city with a population of about 200,000, in the southern state of Santa Catarina, about 650 km south of São Paulo, and is easily accessible by plane and road. It was founded by German immigrants and still has a Germanic flavor. It is also close to some excellent beaches. Although July is winter time in Brazil, the weather is mild. For further information and to receive the First Announcement, write to Ubiratan D'Ambrosio, Rua Peixoto Gomide 1771, ap. 83, 01409-002 São Paulo, SP, BRAZIL, or contact him by phone or fax at 55 11 280 0266.

### **Constitution for HPM Americas Section**

At the HPM Americas Section meeting in April, 1993 it was decided that the organization should affiliate officially with the National Council of Teachers of Mathematics. Such affiliation will enable HPM to have an NCTM delegate and to have input into information printed in programs and in the NCTM *Bulletin*. It is hoped that the affiliation will enable HPM to get its message across to more secondary school teachers. In order to affiliate with the NCTM, it is necessary to put the organization on a more formal basis by adopting a constitution. The following constitution was adopted tentatively at the 1993 HPM meeting and is presented for your approval. Any recommendations for change can be made by mail to Erica Voolich (address above) or to Karen Michalowicz (Langley School, 1411 Balls Hill Road, McLean, VA 22101; email: [karendm@aol.com](mailto:karendm@aol.com)) and will be considered at the meeting in Indianapolis.

#### Article 1. Name

- (1) This organization is the Americas Section of the International Study Group on the Relations Between History and Pedagogy of Mathematics, commonly known as HPM.
- (2) The international organization (ISGHPM) is an affiliate of the International Commission on Mathematics Instruction (ICMI) and the Americas Section is an affiliate of ISGHPM.

#### Article 2. Purpose

- (1) To encourage teachers at all levels to use history of mathematics to motivate and instruct their students.
- (2) To further a deeper and more correct understanding of the pedagogical aspects of integrating history into the teaching and learning of mathematics and the implications thereof.
- (3) To promote national and international contacts and exchanges of scientific information on the relations between history and pedagogy of mathematics.

(4) To promote and stimulate interdisciplinary research in the history and pedagogy of mathematics with the cooperation of historians, ethnographers, mathematicians, and mathematics teachers.

### Article 3. Membership

- (1) Membership is open to teachers of mathematics and other individuals involved in the furtherance of the aims of the group.
- (2) Membership dues are determined at the Annual Meeting.
- (3) Honorary membership, without payment of dues, may be granted at the Annual Meeting or by the executive board.

### Article 4. Meetings

- (1) There will be at least one Annual Meeting held each year.
- (2) The agenda for the Annual Meeting will be published in the *Newsletter* and sent to each member before the meeting. The agenda will include information about vacancies for officers and committees.
- (3) At meetings, the chair is taken by the President or Vice-President or their delegate.
- (4) Approval of a resolution requires a simple majority of the votes cast.
- (5) The secretary will prepare minutes of the Annual Meeting which, after approval by the President, will be published in the *Newsletter*.

### Article 5. Officers

- (1) The officers of the Section are the President, Vice-President, Secretary, Treasurer, Historian, and Newsletter Editor. They are elected for four-year terms during the year of the quadrennial ICME meeting.
- (2) The officers shall constitute the executive board. The board shall transact whatever business is necessary between meetings.

### Article 6. Amendments

This constitution may be amended by the approval of two-thirds of those voting at the Annual Meeting.

### Note from Barnabas Hughes

Barnabas Hughes apologizes for asking those at the HPM Conference in Toronto, August 1992, who received his handout on Maestro Dardi's solution of a cubic equation to make the following change: toward the end of line 25, replace that 1/4 that is the 1/48 with *the 1/4, that is less the root of 1/48*.

### An Indian Extension of Ptolemy's Theorem

R. C. Gupta, Professor of Mathematics, Birla Institute of Technology, P.O. Mesra, Ranchi-835215, INDIA

Let ABCD be a cyclic quadrilateral. The mathematically exact and simple relation

$$AB \cdot CD + BC \cdot DA = AC \cdot BD \quad (1)$$

is known as Ptolemy's Theorem after the famous astronomer Claudius Ptolemy (A.D. second century) who proved it in Book I of his *Almagest*. Several extensions of this theorem are known. For example, a recent extension applicable to any plane convex quadrilateral ABCD states [1]

$$(AC \cdot BD)^2 = (AB \cdot CD)^2 + (BC \cdot DA)^2 - 2AB \cdot BC \cdot CD \cdot DA \cdot \cos(A+C).$$

In this note we shall describe an elegant ancient Indian extension of Ptolemy's Theorem. First, an abstract approach is presented. Let a, b, c, d be any four positive numbers such that any three of them together are greater than the fourth. Taking all these four numbers, the following three sums of products two at a time can be formed:

$$ab + cd, bc + da, ca + bd \quad (2)$$

Here each sum contains just the (given) four numbers and nothing more and only three such sums are possible. If a, b, c, d represent certain lengths, the dimension of each sum will be  $L^2$ . So we define three

new linear positive numbers  $x, y, z$  (each having dimension  $L$ ) by the equations

$$ab + cd = yz \quad (3)$$

$$bc + da = zx \quad (4)$$

$$ca + bd = xy \quad (5)$$

If now  $a, b, c, d$  respectively were the lengths of the sides  $AB, BC, CD,$  and  $DA$  of any cyclic quadrilateral, the Indian extension consists in defining and recognizing  $x, y, z$  to be the three diagonals of the cyclic quadrilateral. Here  $x$  and  $y$  are the lengths of the usual diagonals  $AC$  and  $BD$ .

To define the third diagonal, the Indians carried out the age-old transformation which preserves the area as well as the perimeter of the plane figure. For this, any two adjacent sides of the quadrilateral were interchanged. For instance, if we interchange  $CD$  and  $DA$ , we shall get a new corner  $E$  such that

$$CE = AD = d \text{ and } AE = CD = c.$$

The new quadrilateral  $ABCE$  will be equal to  $ABCD$  in area and perimeter. Both also have a common diagonal  $AC = x$ . But for the new figure we also generally get a new diagonal  $BE$  which is defined to be the third diagonal  $z$  of the original quadrilateral  $ABCD$ . One may make a similar interchange of any two other *adjacent* sides of  $ABCD$  to affect the area and perimeter preserving transformation. However, it can be shown that only three diagonals  $x, y, z$  are possible and no more. A simple sixteenth century Indian proof of this statement as well as derivations of the equations (3), (4), and (5) have already been published [2].

A simple classroom derivation of the three basic equations may be given here. Relations (4) and (5) follow from Ptolemy's Theorem applied to the quadrilaterals  $ABCE$  and  $ABCD$  respectively. To get (3), we mark a point  $K$  on the circumcircle such that  $EK = BA = a$ . Now arc  $BAD =$  arc  $CEK$ , so that  $CK = BD = y$ . Also, arc  $BADK =$  arc  $CEKD$ . Therefore,  $BK = CD = c$ . Hence, now, we get (3) by applying Ptolemy's Theorem to the quadrilateral  $BCEK$ .

From (3) follows another important result. We write it as

$$ab/2R + cd/2R = yz/2R \quad (6)$$

where  $R$  is the circumradius of the figure  $ABCD$ . Now using the well-known theorem that in any plane triangle the product of the flank sides divided by twice the circumradius equals the altitude, the equation (6) yields

$$h_1 + h_2 = yz/2R \quad (7)$$

where  $h_1$  and  $h_2$  are the lengths of the perpendicular on  $AC$  from  $B$  and  $D$  respectively. Now (7) can be written as

$$\frac{1}{2}xh_1 + \frac{1}{2}xh_2 = xyz/4R$$

or

$$S = xyz/4R \quad (8)$$

where  $S$  is the area of the quadrilateral  $ABCD$  (equal to the sum of the areas of triangles  $ABC$  and  $ADC$ ).

The beautiful relation (8) connecting area, circumradius, and the three diagonals of any cyclic quadrilateral was known to Nārāyana Pandita (A.D. 1356) [2]. Of course, the values of  $x, y, z$  in terms of the sides  $a, b, c, d$  were easily found by solving (3), (4), (5). Those for the usual diagonals  $x$  and  $y$  were already known to Brahmagupta (A.D. 628) who also knew that [3]

$$S = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where  $s = (a+b+c+d)/2$ .

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### Have You Read?

Ronald Calinger, ed.

This column welcomes references from across the history or pedagogy of mathematics, as well as other works with sections that have potential for encouraging and motivating students to learn mathematics better or that enrich courses. Please send citations with complete bibliographic information to the section editor c/o Department of History, Catholic University of America, Washington, DC 20064, U.S.A.

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