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HPM Advisory Board

Florence D. Fasanelli, CHAIR
SUMMA
The Mathematical Association of America
1529 18th St. N.W.
Washington, D.C. 20036 USA

Victor J. Katz, EDITOR
Department of Mathematics
University of the District of Columbia
4200 Connecticut Ave. N.W.
Washington, D.C. 20008 USA

Evelyne Barbin FRANCE; Ubiratan D'Ambrosio BRAZIL; Ahmed Djebbre ALGERIA; John Fauvel UK; Paulus Gerdes MOZAMBIQUE; Robert Hayes AUSTRALIA; Nikos Kostas GREECE; Ryosuke Nagaoke JAPAN; V. Frederick Rickey AMERICAS SECTION CHAIR; David Wheeler CANADA; Hans Wussing GERMANY.

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1R6); Mexico: Alejandro Garciadiego (UNAM - contact at José M. Velasco 71, Del. Benito Juárez, 03900 Mexico, D.F.); South America: Ubiratan D'Ambrosio (Universidade Estadual de Campinas, CP 5063, 13081 Campinas SP, Brazil); Australia: Robert L Hayes (Mathematics Department, Hawthorn Institute of Education, Private Bag 12, Hawthorn, Victoria 3122); New Zealand: Andy Begg (SMER Centre, University of Waikato, Private Bag 3105, Hamilton); United Kingdom: John Fauvel (Mathematics Faculty, Open University, Milton Keynes MK7 6AA); France: Evelyne Barbin (IREM Paris Nord, Université Paris XIII, Avenue Jean-Baptiste Clément, 93430 Villetaneuse Cedex); Germany and Eastern Europe: Roland Stowasser (Technische Universität Berlin, StraÙe des 17. Juni 136, 1000 Berlin 12, Germany); Belgium and the Netherlands: Jan van Maanen (Rijksuniversiteit Utrecht, Mathematisch Instituut, Budapestlaan 6, Postbus 80.010, 3508 TA Utrecht, Netherlands) Scandinavia: Bengt Johansson (Institutionen för ämnesdidaktik, Frölundagatan 118, Box 1010, S-431 26 Mölndal, Sweden); Spain and Portugal: Joao Pedro Ponte (Departamento de Educacao, Faculdade de Ciencias, Universidade de Lisboa, Av 24 de Julho, Lisboa, Portugal); Italy: Luciana Bazzini (Dipartimento di Matematica, Università di Pavia, Strada Nuova 65, 27100 Pavia); Greece, Turkey and the Balkans: Nikos Kostas (Department of Mathematics, Faculty of Sciences, Aristotle University of Thessa-

loniki, 54006 Thessaloniki, Greece); Israel: Anna Sfard (The Science Teaching Centre, The Hebrew University of Jerusalem, Givat Ram, 91904 Jerusalem); South Asia: R. C. Gupta (Department of Mathematics, Birla Institute of Technology, P. O. Mesra, Ranchi-835 215, India); East Asia: Gloria D. Benigno (Bukidnon State College, 8700 Malaybalay, Bukidnon, Philippines); Africa: J. O. C. Ezeilo (National Mathematical Centre, Private Mail Bag 118, Abuja, Nigeria); Elsewhere: U.S. Editorial Office. Send requests and address changes to the distributor for your geographical area.

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Calendar

- 1991 December 16-19.....Maputo
Eighth Symposium of the Southern Africa Mathematical Sciences Association. Contact SAMSA 8 Organizing Committee, Abdulcarimo Ismael, Department of Mathematics, Higher Pedagogical Institute, P.O. Box 2923, Maputo, MOZAMBIQUE.
- 1992 January 8-11.....Baltimore
Annual meeting of the Mathematical Association of America and the American Mathematical Society. There will be a special A.M.S. session in the History of Mathematics organized by Victor J. Katz, David Rowe, and Florence Fasanelli. Contact A.M.S., P.O. Box 6248, Providence, RI 02940, U.S.A. (See inside for more details.)
- 1992 January 12-16 Mexico City
Third Latin American Congress on the History of Science and Technology (III CLAHCT). The general theme of this meeting will be America in the formation of a new world: 500 years of scientific exchanges. Contact Comité Organizador III CLAHCT, Apartado postal 21-873, 04000 México, D.F., MEXICO.
- 1992 March 6-7 Norman
Symposium on Tradition, Transmission, and Transformation: Ancient Mathematics in its Islamic and Occidental Context. This symposium on the history of medieval Islamic and Occidental mathematics will be held at the University of Oklahoma. Contact Steven J. Livesey, Department of the History of Science, University of Oklahoma, 601 Elm, Room 622, Norman, OK 73019, U.S.A. (405) 325-2213; fax (405) 325-5068; e-mail aa9214@uoklnvsa.bitnet.
- 1992 April 1-4.....Nashville
Annual meeting of the National Council of Teachers of Mathematics and the Americas Section of HPM. Contact NCTM, 1906 Association Drive, Reston, VA 22091, U.S.A. (See inside for more details.)
- 1992 April 11-12.....Nottingham
HIMED 92 - An International Conference on the Uses of History in Mathematics Education, sponsored by the British Society for the History of Mathematics. Contact Costel Harnasz, 217 Burton Road, West Didsbury, Manchester M20 8NA, U.K. (See inside for more details.)
- 1992 May 11-15.....Kingston
Second International History, Philosophy, and Science Teaching Conference. Contact Professor Skip Hills, Faculty of Education, Queen's University, Kingston, Ontario K7L 3N6, CANADA. (See inside for more details.)
- 1992 May 22-23Brest
Ninth Inter-I.R.E.M Colloquium on Epistemology and History of Mathematics. The topic this year is the History of the Infinite. Contact Evelyne Barbin at the address on the front page. (See inside for more details.)

- 1992 May 28-30 Charlottetown
Annual meeting of the Canadian Society for History and Philosophy of Mathematics. There will be a special session dealing with ethnomathematics. Contact Professor M. Malik, Department of Mathematics, Concordia University, 7141 Sherbrooke St. Ouest, Montreal, Quebec H4B 1R6, CANADA. (See inside for more details.)
- 1992 June 22-26 Meadville, Pa
A series of lectures on the History of the Calculus will be given for the Allegheny Mountain Section of the MAA by V. Frederick Rickey. For more details, contact Professor Richard McDermot, Mathematics Department, Allegheny College, Meadville, PA, 16335, U.S.A.
- 1992 August 12-14 Toronto
International meeting of HPM preceding ICME-7. (See inside for more details.)
- 1992 August 17-23 Quebec
Seventh International Congress on Mathematical Education (ICME-7). Contact Congrès ICME-7 Congress, Université Laval, Québec, QC, G1K 7P4, CANADA or via fax to (418) 656-2000. (See inside for more details.)
- 1992 September 10-13 Cambridge
Annual Residential Meeting of the British Society for the History of Mathematics. The theme of the meeting will be European mathematics. Contact the Secretary, J. Helen Gardner, 25 Hollow Croft Road, Willenhall, West Midlands WV12 5YS, U.K. for information on the meeting and on membership.
- 1993 August 22-29 Zaragoza
Nineteenth International Congress of History of Science. The Congress will consist of Symposia, which will address themes of special interest, Scientific Sections devoted to the various branches and periods of the history of science and technology, and Poster

Sessions. Contact the Congress Office, Facultad de Ciencias (Matemáticas), Ciudad Universitaria, E-50009 Zaragoza, SPAIN; phone (76) 357-180; fax (76) 565-852; e-mail ichs@cc.unizar.es.

HPM in Toronto

The quadrennial international HPM meeting will take place from August 12 to 14, 1992 at Victoria College of the University of Toronto. The scientific program will consist of approximately nine plenary talks of one hour each and several sessions of shorter talks. So far, some forty people have agreed to give talks:

- Hans-Georg Steiner (Germany) *On the history of the concepts of implicit definition and structure, and implications for the teaching of mathematics*
- Jean-Paul Guichard (France) *Using the Geometry of Chuquet and the Zeticorum of Viète to teach algebra*
- Maryvonne Hallez (France) *On the history of mathematics in Antiquity and the Middle Ages with pupils*
- Michell Guillemot (France) *About Egyptian mathematics*
- Frederic Metin (France) *Pupils face a problem of the twelfth century: the method of false position*
- Henri Plane (France) *The Chapter Mathematics in the Etimologie of Isadore of Seville*
- Jacques Borowczyk (France) *On the intermediate value theorem*
- Anne Boye (France) *Interdisciplinary approach of the history of mathematics for the classroom*
- Rudolf Bkouche (France) *The place of movement in the geometry of the nineteenth century*

- Michale Otto (France) *The idea of complementarity and the history of geometry*
 - Ubiratan D'Ambrosio (Brazil) *Historiographical approach to the history of mathematics*
 - Kevin Lees (Australia) *Sophie Germain's 'Shortest Path' problem*
 - Jean-Pierre Legoff (France) *Arts and sciences from the perspective of the painters to projective geometry*
 - Robert Kanigel (USA) *Mathematical creativity: the life of Ramanujan*
 - Gloria Benigno (Philippines) *History of mathematics in the Philippine schools: its status and extent*
 - Anna Sfard (Israel) *Anomaly as a catalyst of mathematical invention: a historical and psychological outlook*
 - Luciana Bazzini (Italy) *Analogical reasoning and pedagogy of mathematics*
 - Otto Bekken (Norway) *Abel and uniform convergence*
 - Victor J. Katz (USA) *Medieval Jewish mathematicians and their work*
 - Frank Swetz (USA) *The enigmas of Chinese mathematics*
 - Torkil Heide (Denmark) *History of mathematics and the teacher*
 - Jan van Maanen (The Netherlands) *New maths may profit from old methods*
 - Evelynne Barbin (France) *The historical studies of the French I.R.E.M. Institutes*
 - Karen Dee Michalowicz (USA) *Using the history of mathematics with students aged 9-14*
 - Man-Keung Siu (Hong Kong) *Integration in finite terms, from Liouville's work to the calculus classroom today*
 - Don Faust (Malaysia) *The still open question of radix sixty: evidence from the Middle East, India, China, and South America*
 - Peter Baro (Czechoslovakia) *Influence of wine barrels on the teaching of mathematics*
 - Marie-Francoise Jozeau (France) *The measure of the terrestrial meridian in the classroom*
 - Kevin Reed (Australia) *Using other number systems to foster the understanding of our own*
 - Don Crossfield (USA) *The use of Euler in the classroom as found in Introduction to Analysis of the Infinite*
 - Shmuel Avital (Israel) *History of mathematics in the high school via historical problems*
 - Jim Kiernan (USA) *The problem of points*
 - J. O. C. Ezeilo (Nigeria) *Some of the problems facing the discipline of mathematics in Nigeria*
 - Beatrice Lumpkin (USA) *False position: African (Egyptian) origin of the method and its use by Benjamin Banneker*
- Several others have agreed to speak, although they have not yet determined the titles of their talks. These include Barnabas Hughes (USA), V. Frederick Rickey (USA), Helena Pycior (USA), Donovan van Osdol (USA), William Dunham (USA), R. C. Gupta (India), and James Tattersall (USA). All of the talks will provide details on how the history of mathematics has been used in the classroom. It is expected that there will be indications of the use of history at the primary level as well as the secondary and university levels.
- There is still room for more speakers. Those of you in North America who have not been able to

attend previous quadrennial meetings are especially invited to attend and participate. Please volunteer to speak. Send your name, topic, and a brief summary to Florence Fasanelli at the address on the first page.

Although all of the details of the meeting have not yet been worked out, the social events will include a cocktail party (sponsored by a publisher), an excursion to Niagara Falls, and a banquet. Accommodations have been reserved at Victoria College. The approximate cost is \$ 40 for a single and \$ 58 for a double (Canadian funds) plus tax. Information on registration will be available in the next Newsletter.

HPM will also participate in ICME-7 itself in Quebec. We have been allotted four 90-minute sessions. There will be three themes of the sessions:

- History of Mathematics and Pedagogical Problems
- History of Mathematics as a Cultural Approach to Solving Problems
- Historical Problems in the Classroom

Each session will consist of two speakers and one respondent. The speakers will deal with one of the themes and, it is planned that each day's contributions will include material at the elementary, secondary, and tertiary levels. The speakers, respondents, and their topics will be listed in the next issue of this Newsletter.

Put Toronto and Quebec on your calendar for August, 1992. There is much to see and do in both Toronto and Quebec, so bring your families and enjoy an exciting and stimulating twelve days.

ICME-7

ICME-7, the Seventh International Congress on Mathematical Education, will be held at Université Laval in the city of Québec, Canada, from August 17 to 23, 1992. The Congress will be organized into Plenary Sessions, Working Groups,

Lectures, Topic Groups, Study Groups, a Mini-conference on calculators and computers, National Presentations, Projects, and Poster Sessions. The working groups, each to meet for four 90-minute sessions, will consider such topics as "Students' difficulties in calculus," "Multicultural and multilingual classrooms," "The role of geometry in general education," "The place of algebra in secondary and tertiary education," "Technology in the service of the mathematics curriculum," and "Mathematics education with reduced resources."

Some of these working groups will include material on the use of history. For example, subgroup 6 of the working group on the role of geometry, entitled "Geometry through its history," is being organized by John Fauvel (United Kingdom) and Roman Duda (Poland). The first session of the subgroup will deal with geometry through historical stories. It will focus on fairly gentle interactions in which the teacher encourages children's interest through telling or showing stories about geometrical practice or theory in the past. The second session, on geometry through instruments, will focus on what can be learned from geometrical instruments, especially past surveying instruments which can easily be built and demonstrated in the classroom. Pythagoras' theorem, issues of ratio and proportion, and other obvious curricular and cross-curricular benefits will be part of the discussion. The final session of the subgroup will consider geometry through texts. It will explore the use of old geometry texts for teaching geometry in the classroom, and might well focus on the texts, methods, and approaches developed in France by members of IREMs.

The topic groups at ICME-7 will each meet for two 90-minute sessions. They include "Ethnomathematics and mathematics education," "Indigenous peoples and mathematics education," "The theory and practice of proof," "Mathematical games and puzzles," and "Mathematics and Ibero-American art." Again, there will be historical material discussed throughout. In particular, Israel Kleiner (Canada) and John Fauvel are

organising a part of the session on proof under the topic "Historical, epistemological, and practical aspects of proof," in which several speakers will give brief presentations on the cultural context of various proofs along with suggestions for using this context in teaching.

About forty lectures will be scheduled on various topics, some of which will deal with history. Finally, some of the poster sessions will probably also deal with historical material. Full registration details and a schedule of fees are available in the Second Announcement, available by writing to ICME-7 at the address given in the Calendar.

HIMED 91

John Fauvel and Costel Harnass

This year the History in Mathematics Education conference HIMED 91 was a one-day meeting largely devoted to participatory workshops. Organised by the British Society for the History of Mathematics, it took place on May 4, 1991 at the Mathematics Department of Leicester University, whose cheerful new building is approached over a specially-designed brick maze. About 50 people attended, practising teachers as well as those in advisory, training, lecturing and research roles.

Robin Wilson (Open University) started the proceedings with a talk on "Stamping through mathematics", a philatelic tour of the history of mathematics which supplied some memorable vignettes as well as a bounty of more dire puns than most teachers would dare to exhaust in a term. The day's other talk was by Mike Dampier (Leicester University), who provided a welcome after-lunch session on some of the historical mathematics which he had found particularly useful with students.

Most of the day was taken up with six parallel workshop sessions, held in a relaxed manner, which meant that people could either concentrate on one, or float from one to another, as their needs and curiosity dictated. The workshops were on the whole explaining what had been achieved in the action groups set up fol-

lowing the previous HIMED meeting a year earlier; although some were still finding their feet or showing more promise than achievement, others had provided a welcome focus for a group to make consistent progress over the intervening months.

Numeracy and basic skills was coordinated by David Kaye (Paddington College) and Helen Gardner (Bentley Drive Primary School, Walsall). David is particularly interested in exploring the potential of a historical input to numeracy schemes for adults. In connection with basic skills, Helen — who has led the primary action group, one of the more active over the previous year — brought along an impressive display of work she has done with her primary pupils, taking as the starting point quotations and puzzles from days gone by.

GCSE and National Curriculum (secondary) was led by Paul Thornley (Hereford) and Peter Ransom (Prudhoe County High School, Northumberland). Peter brought along some fascinating instruments which he had recently made — a quadrant, a cross-staff and other old surveying instruments — in order to show what could be achieved in a classroom with very simple constructions, exploring practical geometry in a way that middle-school pupils found engaging and engrossing. The instruments and their classroom use proved so popular that it is hoped a session at ICME-7 will explore this theme further.

'A' level topics, co-ordinated by Mike Dampier and Alison Walsh (Middlesex Polytechnic), divided along pure and applied lines. Extracts were made available from texts that related to the individual topics under discussion.

Initial teacher training had as its co-ordinators Neil Bibby (Exeter University) and Ruth Eagle (Keele University). They asked participants to plan a teaching approach which could offer students something active to do, which might also bring them to savour some of the excitement of the development of mathematical ideas.

Posters and things was led by Steve Russ (Warwick University) and John Fauvel (Open University). A wide range of existing posters on

historical themes was brought along, and a professional graphics designer was on hand to advise on poster design and production. It was a most welcome innovative move to invite him, and it should have benefits in the future as the action group's work progresses. Several people had suggestions for poster topics: women and mathematics, historical mathematics on stamps, and themes such as π and counting all had potential.

Use of computers was organised by Steve Russ and Costel Harnasz (Burnage High School, Manchester). Steve showed the Hypercard system on Apple Macintosh machines, while Costel demonstrated a similar database called Genesis, which operates on the Archimedes computer found in many British schools. Costel used for his material some pages of the heavily illustrated new booklet *Mathematical tradition in the north of England* (by Peter and Ruth Wallis, Peter Ransom, and John Fauvel), in effect transferring it to an electronic form in which pupils could ask questions and move in an exploratory way between images and information.

HIMED 91 set out to provide a strong focus on practical classroom activities, exploring several aspects of historical resources for teachers. The British Society for the History of Mathematics is grateful to its hard-working organisers, Helen Gardner, David Kaye and Steve Russ. Attenders valued this rather different form of conference, and are looking forward to next year's HIMED 92, in Nottingham, where workshop and talk sessions will be combined with further fresh ideas and approaches for history in the mathematics classroom.

History at the 1991 Mathematical Association Meeting

John Fauvel

This year's annual meeting of the Mathematical Association, held from 3-6 April in Newcastle upon Tyne, continued the recent practice of putting on several talks with a historical content - the 1988 joint meeting between the MA and

BSHM seems to be continuing to bear fruit.

The most noticeable historical event for delegates was the exhibition on *Mathematical tradition in the North of England*, mounted by BSHM members Peter and Ruth Wallis, Peter Ransom, and John Fauvel. Each delegate received a copy of the 66-page illustrated booklet accompanying the exhibition, which helped to set the entire meeting within a historical framework of mathematical teaching and activity in the north of England. Peter Wallis ran two sessions showing and discussing textbooks, exercise books, and the other original texts from which the exhibition was constructed. John Hersee, the MA treasurer, attended the second of these sessions with examples from his own collection of old mathematical exercise books.

Several other BSHM members contributed to the conference. Membership Secretary Helen Gardner gave two sessions, *A mathematical history tour* and *Maths from the past*, both aimed at the difficult but rewarding 9-11 age-group. Tom Brissenden, who is presently translating the historic memoirs of the life of *Evariste Galois*, spoke about Galois' life and work. John Fauvel gave two talks, one on *Teaching logarithms - can history help?* and one called *Are textbooks value free?*, a survey of explicitly value-laden practices in past mathematical texts.

Deservedly the most well-attended historical talks at the conference were two by Nick McKinnon, editor of the *Maths Gazette*, whose content did not disappoint the expectations aroused by their titles. *Babylon the Great, Mother of Harlots and All the Abominations of the Earth* was an account of the GCSE cross-curricular project work which Nick has been doing with his pupils at Winchester College on Babylonian mathematical texts and artifacts. One pupil, for example, made a very fine clay copy of YBC 7289, the tablet which gives a remarkable sexagesimal approximation to $\sqrt{2}$. In *Mahdava 1: Leibniz 0* Nick discussed, with classroom hand-outs, the work of Leibniz and Newton on the series $\frac{4}{3} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \dots$, whether Taylor's series should really be called Gregory's series or Mah-

dava's series, and other topics in this area which he has explored with his pupils via (photocopies of) the original manuscripts — lucky pupils!

From the Summer, 1991 Newsletter of the British Society for the History of Mathematics

Annual Mathematics Meetings in Baltimore

There will again be a special session in the History of Mathematics at the annual meeting of the American Mathematical Society to be held in Baltimore from January 8 to 11, 1992. The special session will convene on Friday afternoon, January 10, and on Saturday morning and afternoon, January 11. The speakers will include Frank Swetz (Pennsylvania State University), *What's New in the History of Mathematics*; Ubiratan D'Ambrosio (Universidade Estadual de Campinas, Brazil), *The Life and Work of Joaquim Gomes de Sousa*; Paul Wolfson (West Chester University), *Cayley's Derivation of Euclidean from Projective Geometry*; Erik Sageng (St. John's College), *The Foundations of the Method of Fluxions in 18th Century Britain*; Suzanne Hensel (Pace University), *Conflicts in Mathematics and Mathematics Education in German Technical Colleges around 1900*; Robert Kanigel (Baltimore), *Ramanujan and Hardy in England*; Michael Mahoney (Princeton University), *Computers, Mathematics, and Theoretical Computer Science*; Charles Ford (St. Louis University), *Father Pavel Florensky and Modern Soviet Mathematics*; Boris Schein (University of Arkansas), *The Theorem on Representation of Semigroups by Transformations*; Boris Rosenfeld (Pennsylvania State University), *Affine and Projective Transformations in the Medieval East*; Thomas Archibald (Acadia University, Canada), *Emile Picard, the Method of Successive Approximation, and the Development of an International Style in Mathematics*; and Della Dumbaugh Fenster (University of Virginia), *Women in the American Mathematical Research Community, 1891-1906*.

Other aspects of the Mathematics Meetings

which may be of interest to readers include the MAA minicourse *Using history in teaching calculus*, given by V. Frederick Rickey (Wednesday, 8:00-10:00 a.m. and Thursday, 2:15-4:15 p.m.), the panel discussion on ICME-7, led by Eileen L. Poiani (Wednesday, 8:00-9:20 a.m.), and the panel discussion on Preparing Teachers of Mathematics, sponsored by the Committee on the Mathematical Education of Teachers (James R. C. Leitzel, Chair) and the Committee on the Teaching of Undergraduate Mathematics (Don R. Lick, Chair) (Thursday, 2:15-4:10 p.m.).

Annual Meeting of HPM Americas Section in Nashville

As usual, the annual meeting of the Americas Section of HPM will be held at the time of the annual meeting of the National Council of Teachers of Mathematics, this coming year in Nashville from April 1 to April 4. HPM will meet on Thursday evening, April 2 and on Saturday afternoon, April 4. Presentations on any aspect of the use of history in teaching mathematics are welcome. Abstracts should be sent to Erica Voolich, 244 Summer St., Somerville, MA 02143 by February 1. Part of the HPM session will be devoted to an open discussion of general ideas for using history in teaching, particularly at the middle and secondary school levels. All teachers are invited to share their favorite material.

HIMED 92

HIMED 92, an International Conference on the Uses of History in Mathematics Education, will be held at Nottingham University in Nottingham, England April 10-12, 1992, immediately preceding the joint conference of the Mathematical Association and the Association of Teachers of Mathematics. The meeting will include plenary speakers as well as several workshops. The speakers include Marjolein Kool (Netherlands), *Using History with Low Attainers*; Annie Michel-Pajus (France), *Les Grands Problèmes*; Sir Christopher Zeeman (Oxford University),

Gears from the Greeks; Pat Perkins (City of London School for Girls), *The Dilemmas of Using History*; Ubiratan D'Ambrosio (Universidade Estadual de Campinas, Brazil), *Ethnomathematics*; and Leone Burton (University of Birmingham), *Working Together*. Workshops in such areas as Probability, the Euler-Maclaurin Formula, Alluvial Deposits, Series for Pi, Roman Mosaics, Vedic Mathematics, and Mathematical Instruments through the Ages will be led by Evelyn Barbin (IREM Université Paris XIII), Peter Baxandall (Bryanstone School, Dorset), Jan Dangerfield (Penzance Sixth Form College), J. Helen Gardner (Bentley Drive Primary School, Willenhall), Torkil Heide (Royal Danish School for Educational Studies, Copenhagen), George Joseph (University of Manchester), Nick McKinnon (Winchester College), Frédérick Métin (Lycée Val de Muriguy, Reims), Peter Ransom (Prudhoe County High School, Consett), Jan van Maanen (Rijksuniversiteit Utrecht, Netherlands), John Warner (Peers School, Oxford), and Chris Weeks (College of St. Mark and St. John, Plymouth). There is also a visit scheduled to Green's Mill in Nottingham, the birthplace of George Green (1793-1841), who pioneered the mathematical description of electricity and magnetism. For more information, and a booking form, write to Costel Harnasz, 217 Burton Road, West Didsbury, Manchester M20 8NA, ENGLAND.

Second International Conference on the History and Philosophy of Science in Science Teaching

This conference will meet at Queen's University in Kingston, Ontario, Canada from May 11 to 15, 1992. The Conference provides a forum for discussion of potential contributions of the history and philosophy of science to the teaching of science from the elementary school through to graduate school. This will be the second occasion on which an international gathering of philosophers, historians of science, and science educators have met with science teachers, science ad-

ministrators, and educational policy makers to investigate ways in which the history and philosophy of science has contributed and can contribute to the preparation of science teachers, the development of curricula, the enhancement of science education, and the development of a more scientifically literate community. For further details, contact Professor Skip Hills at the address in the Calendar.

8ème Colloque Inter-I.R.E.M. Epistémologie et Histoire des Mathématiques

La Commission inter-I.R.E.M. "Epistémologie et Histoire des Mathématiques" et l'Institut de Recherche sur l'Histoire des Mathématiques (I.R.E.M.) de Brest organisent, les 22 et 23 Mai 1992, le 8ème Colloque inter-I.R.E.M. consacré à l'enseignement et à l'histoire des mathématiques. Le thème de ce Colloque est l'infini.

Aujourd'hui comme hier, pour les chercheurs comme pour les enseignants, "faire des mathématiques" c'est "mieux comprendre l'infini". La question de l'infini intervient dans l'histoire des mathématiques aussi bien que dans les classes de mathématiques, comme un élément à la fois perturbateur et moteur. Perturbateur, parce que les problèmes liés à l'infini ont constitué des obstacles et sont à l'origine de nombreuses ruptures dans l'histoire des mathématiques, depuis Aristote jusqu'à Cantor. Perturbateur, parce que l'infini est incontournable dès le début de l'apprentissage des mathématiques et que les intuitions sont trompeuses. Moteur parce que la nécessité de résoudre les paradoxes et les difficultés amènent à construire de nouveaux savoirs et de nouvelles conceptions.

La conception formaliste du 20ème siècle, en assimilant l'infini à un simple signe, a pensé bannir toute idée intuitive. Mais cette conception a montré suffisamment ses limites dans l'enseignement de l'analyse, et les problèmes didactiques restent aigus: comment enseigner l'infini?

Pour aborder les différentes facettes de l'infini, l'organisation du Colloque prévoit des conférences plénières et des ateliers en parallèles. Ils traiteront de questions aussi bien épistémologiques, historiques et didactiques concernant : le discret et le continu, l'infiniment petit et l'infiniment grand, le rôle des paradoxes, les obstacles épistémologiques et didactiques liés à l'infini, les outils de l'analyse : suites, séries et limites, les problèmes de l'analyse : intégrales et dérivées, l'analyse non standard, l'actualité de l'infini : chaos et fractals, les calculatrices et l'infini, les philosophies et l'infini et l'infini dans la culture.

Pour s'inscrire, contacter Evelyne Barbin, I.R.E.M. Paris Nord, Université Paris XIII, Avenue Jean Baptiste Clément, 93430 Villetaneuse Cedex, FRANCE.

CSHPM in Charlottetown

The Annual Meeting of the Canadian Society for History and Philosophy of Mathematics will be held in Charlottetown, Prince Edward Island from May 28 to 30, 1992. Michael Closs of the University of Ottawa will be the guest speaker. His topic will be a topic in Ethnomathematics, the theme of the special session of the meeting. Abstracts of contributed papers either on that theme or on any topic of the History and Philosophy of mathematics should be sent to the program chair, Gerald Lenz, Department of Mathematics, St. John's University, Collegeville, MN 56321, U.S.A., by February 1, 1992. General information about the Canadian society can be had from the Secretary, M. A. Malik, at the address given in the calendar.

Science & Education

Kluwer Academic Publishers have commenced a new journal titled *Science & Education: Contributions from History, Philosophy and Sociology of Science and Mathematics*. It will be published quarterly beginning January 1992. The editor is Dr. Michael R. Matthews, School of Education, University of New South Wales, Kensington,

NSW, Australia. The journal is affiliated with the International History, Philosophy, and Science Teaching Group which held a very successful first conference at Tallahassee, Florida, U.S.A. in 1989 and which is holding its second conference at Kingston, Ontario, Canada in May, 1992.

Science & Education hopes to become a cross-disciplinary forum. The members of the editorial committee are drawn from philosophers of science, philosophers of education, historians of science, cognitive psychologists, scientists and mathematicians, science and mathematics teachers, and educators. (Among the members of the committee are John Fauvel, Ubiratan D'Ambrosio, and Victor J. Katz.) There are fourteen countries represented on the committee.

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Contributions: Papers, reports, letters are welcome. Papers involving history and mathematics can be sent to John Fauvel, Ubiratan d'Ambrosio, or Victor J. Katz, at the addresses on the first page. Papers can also be sent to the Editor, *Science & Education*, c/o Kluwer Academic Publishers, P.O. Box 17, 3300 AA Dordrecht, THE NETHERLANDS.

The Crest of the Peacock

Victor J. Katz

St. Martin's Press in the United States and I. B. Tauris in England have just published George Ghevarghese Joseph's new book, *The Crest of the Peacock: Non-European Roots of Mathematics*. The importance of this book lies in its contribution to the debate now taking place in many American colleges and universities over the relevance of non-Western materials for the liberal education of today's students. Inasmuch as mathematics, the basis of our modern technological civilization, is studied by virtually every student, a detailed account of its own non-Western heritage is greatly to be welcomed. For mathematics, as for many other sciences, the standard "trajectory" of its growth is that it originally developed in Greece, with perhaps a bit

of help from Egypt and Mesopotamia, and was brought to a high point in Alexandria in the centuries from 300 B.C. to A.D. 400. It was then preserved by the Arabs during the European "dark ages" until the Renaissance, when Europe recaptured its Greek heritage and out of it developed modern mathematics. Joseph's aim in his book, developed in detail in the first chapter, is to present a more accurate trajectory of mathematical development, stressing the contributions of civilizations in China, India, Western Asia, and North Africa, particularly during those "dark ages." In particular, he wants to highlight "(1) the global nature of mathematical pursuit of one kind or another; (2) the possibility of independent mathematical development within each cultural tradition; and (3) the crucial importance of diverse transmissions of mathematics across cultures, culminating in the creation of the unified discipline of modern mathematics." Joseph succeeds admirably in his first two aims, presenting a far better picture of mathematical developments around the world than can be found in any of the standard histories. He does not succeed in proving his third thesis, that modern mathematics drew from all of these various sources because the documentary evidence for transmission of mathematical ideas is lacking.

For those interested in using history in their teaching of mathematics, and particularly history from non-European sources, this book presents a wealth of information. Chapter 2 summarizes material in "protomathematics," including the Inca quipu from Peru and the numeration system of the Mayans of Central America; while chapters 3 and 4 present a brief overview of Egyptian and Babylonian mathematics. The strength of the book, however, lies in the two chapters each on Chinese and Indian mathematics, in which the author gathers together material not easily available elsewhere. In these chapters one can find material on the Chinese methods of extracting square and cube roots, the Chinese use of the "Pascal triangle" to develop the "Horner" method of solving polynomial equations, the Chinese development of the "Gau-

sian" elimination technique for solving systems of linear equations, and the ancient Chinese proof for the Pythagorean theorem. One also finds a discussion of the Indian geometry of the *Sulbasutras*, the early Jaina work on permutations and combinations, a treatment of the gradual development of the place-value system in India, and the procedures developed by Bhaskara and others for solving the "Pell" equation $ax^2 + b = y^2$, where a and b are given integers and x and y are integers to be determined. Joseph also discusses the Indian contributions to trigonometry and gives us hints of the development of the power series for the sine, cosine, and inverse tangent functions worked out probably by Madhava (14th century), series which did not appear in Western mathematics until the time of Newton.

The final chapter of *The Crest of the Peacock* is devoted to the Arabic contributions to modern mathematics. Joseph gives here a concise overview of Arabic work in algebra, geometry, and trigonometry from which one can develop ideas to share with classes.

It is clear from Joseph's book that there were wide-ranging mathematical developments in China, India, and the Arab world, developments which should be discussed with students not only in courses in the history of mathematics but also in relevant mathematics courses themselves. Many of Joseph's examples can easily be adapted into classroom lessons in such courses so that American and European students are made aware that mathematics was not and is not a "Western" subject. I highly recommend *The Crest of the Peacock* for anyone who wants a balanced picture of the worldwide growth of mathematical ideas.

Ethnomathematics: A Multicultural View of Mathematical Ideas

Victor J. Katz

In this fascinating and ground-breaking new book published by Brooks/Cole Publishing Company, Marcia Ascher introduces the mathematical ideas of people in traditional, or "small

scale," cultures often omitted from discussions of mathematics. As Ascher explores mathematical ideas involving numbers, logic, spatial configuration, and the organization of these into systems and structures, readers will gain both a broader understanding of mathematics and an appreciation for the ideas of other peoples.

Among the topics considered in the text are number words and symbols, graphs, the logic of kin relations, chance and strategy in games and puzzles, the organization and modeling of space, and symmetric strip decorations. Ascher takes her examples from such people as the Inuit, Navajo, and Iroquois of North America, the Incas of South America, the Malekula, Warlpiri, Maori, and Caroline Islanders of Oceania, and the Tshokwe, Bushoong, and Kpelle of Africa. This beautifully illustrated book contains much material which can be used in classrooms in "standard" mathematics courses, by way of illustrating that mathematical concepts have been developed in societies all over the world.

For example, one mathematical idea which appears in the Bushoong culture in Zaire and also in the Tshokwe culture of northeastern Angola is the graph theoretical idea of tracing out certain figures in a continuous curve without lifting one's finger from the sand. In western mathematics, this idea was first considered by Leonhard Euler in 1736. The Bushoong children, who first showed their diagrams to a European ethnologist in 1905, were evidently not only aware of the conditions which insured that the graph could be drawn continuously but also knew the procedure which permitted its drawing most expeditiously. For the Tshokwe, figure drawing is not a children's game, but part of a storytelling tradition among the elders. As part of their storytelling, dots are used to represent humans and rather complex curves are drawn including certain dots within the figure and excluding certain others. In fact, the procedure for drawing is to set out a rectangular grid of dots on which the curve is superimposed. Without a special study of the diagrams, it is not easy to determine which dots are inside and which are outside, but the

detailed drawing rules which the Tshokwe follow enable them to construct the curves quickly in one continuous motion.

In the South Pacific, we find the idea of tracing figures continuously in the sand also in Malekula, in the Republic of Vanuatu. The drawing of figures here is imbedded in Malekulan religious life. In fact, passage to the Land of the Dead requires being able to draw these figures accurately. The Malekulans devised standard algorithms for tracing their quite complicated figures using symmetry operations on a few basic drawings. That is, one can analyze the Malekulan figures using some of the language of modern day group theory.

Group theory is also convenient in analyzing the kin relationships in Malekula. In fact, the elders explained these relationships to an anthropologist using diagrams which can easily be transformed into a group table. The basic idea is that the society is divided up into six sections, and men of one section can only marry women of a different section, while their children belong to still another section. If a given male belongs to the section we label as e (identity), his mother will belong to section m and his father to section f . Then the mother of his father will be in section mf and the father of this mother in section fm . It turns out that the kin rules are such that all the possible "products" of m and f form the dihedral group of order 6, that is, the group of six elements generated by the elements m, f with the relations $m^3 = e, f^2 = e$, and $(mf)(mf) = e$. Marriage can only take place between A and B if B belongs to the section of the mother of the father of A , or, equivalently, if A belongs to the section of the mother of the father of B . A similar kin relationship group structure of order 8 occurs among the Warlpiri of northern Australia.

To explore in more detail these and other mathematical ideas in various cultures, you must acquire a copy of this book. Your students will thank you for introducing them to the mathematics in these farflung cultures.

A Bit of History using Fractions in 1850 BC

Karen Dee Michalowicz

It is quite remarkable that so much is known about arithmetic in ancient Egypt. The major source of our knowledge is the Rhind Mathematical Papyrus, circa 1650 BC, named after A. Henry Rhind. Sir Henry purchased the papyrus in 1858 after it had been discovered in Thebes in the ruins of a small structure. There is no other document which presents such extensive information about early Egyptian mathematics. The papyrus can be viewed in London at the British Museum as a result of a legacy of the Rhind estate.

The papyrus was written in hieratic, a cursive form of Egyptian writing. Originally, it was a single roll measuring approximately 18 feet by 13 inches. It is divided into three parts: arithmetic, geometry, and miscellaneous arithmetic problems. The information on fraction usage in ancient Egypt has been collected from the section of the papyrus on arithmetic.

In ancient Egypt there developed two kinds of numbers. The first was an ascending series of integers from 1 to approximately 1,000,000. At the same time, there was a corresponding descending series of numbers which consisted of the reciprocals of the ascending numbers. These were the unit fractions $1/2, 1/3, 1/4, 1/5$, etc. The only non-unit fraction that the Egyptians accepted was $2/3$, for which they had a special symbol and which they considered the largest fraction. Since all fractional parts of a natural number were expressed as unit fractions, it often required the complicated manipulations of numbers in order to express a computation as a unit fraction. For example, if the Egyptians arrived at a number that was not a single unit fraction, they used a combination of fractions to express it. Also, unit fractions could not be repeated. The fraction $3/4$ had to be written $1/2 + 1/4$, not $1/4 + 1/4 + 1/4$. Consider the product of 2 and $1/5$. The answer is $2/5$. However, $2/5$ is not a unit fraction. Therefore, the ancient Egyptians wrote $2/5$ as

$1/3 \ 1/15$ (no comma nor addition sign between even though addition is implied).

Obviously, it was not a simple task to express a fraction in such a way. However, considering the fact that the Egyptian numeration system only included the positive integers and their reciprocals, fractions had to be expressed in such a manner.

The following is a table of division found in the Rhind Papyrus. Perhaps your students would enjoy trying to extend the table:

- 1 divided by 10 gives $1/10$
- 2 divided by 10 gives $1/5$
- 3 divided by 10 gives $1/5 \ 1/10$
- 4 divided by 10 gives $1/3 \ 1/15$
- 5 divided by 10 gives $1/2$
- 6 divided by 10 gives ?
- 7 divided by 10 gives ?
- 8 divided by 10 gives ?
- 9 divided by 10 gives ?

For more information about ancient Egyptian mathematics, read *The Rhind Mathematical Papyrus* by Arnold Buffum Chase, available from NCTM.

Thanks to Erica D. Voolich for her editorial suggestions. Karen Michalowicz welcomes contacts from other middle school teachers who want to use history in their classes. She can be reached at The Langley School, 141 Balls Hill Road, Mclean, VA 22101, U.S.A.

Have You Read?

Ronald Calinger, ed.

This column welcomes references from across the history or pedagogy of mathematics, as well as other works with sections that have potential for encouraging and motivating students to learn mathematics better or that enrich courses. Please send citations with complete bibliographic information to the section editor c/o Department of History, Catholic University of America, Washington, D.C. 20064, U.S.A.

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