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HPM Advisory Board

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Calendar

Meetings with HPM components are highlighted.

- 1991 August 3-7 Coral Gables
Eighth Interamerican Conference on Mathematics Education. Contact Patrick Scott, Programas Latinoamericanos de Educación, Faculty of Education, University of New Mexico, Albuquerque, NM 87131, U.S.A.
- 1991 September 11-14..... Cambridge
Annual Residential Meeting of the British Society for History of Mathematics. The theme of the meeting will be Renaissance Mathematics, and an international team of speakers has been arranged. Contact the Secretary, J. Helen Gardner, 25 Hollow Croft Road, Willenhall, West Midlands WV12 5YS, U.K. for information on the meeting and on membership.
- 1991 October 2-4.....Clayton, Victoria
Second Australian History of Mathematics Conference. Papers are invited on any aspect of the history of mathematics, especially including history and pedagogy of mathematics. Contact Professor J. N. Crossley, Department of Mathematics, Monash University, Clayton, Victoria 3168, AUSTRALIA.
- 1991 December 16-19.....Maputo
Eighth Symposium of the Southern Africa Mathematical Sciences Association. Contact SAMSA 8 Organizing Committee, Abdulcarimo Ismael, Department of Mathematics, Higher Pedagogical Institute, P.O. Box 2923, Maputo, MOZAMBIQUE. (See inside for more details.)
- 1992 January 8-11.....Baltimore
Annual meeting of the Mathematical Association of America and the American Mathematical Society. There will be a special A.M.S. session in the History of Mathematics organized by Victor J. Katz, David Rowe, and Florence Fasanelli. Contact A.M.S., P.O. Box 6248, Providence, RI 02940, U.S.A.
- 1992 January 12-16 Mexico City
Third Latin American Congress on the History of Science and Technology (III CLAHCT). The general theme of this meeting will be America in the formation of a new world: 500 years of scientific exchanges. Contact Comité Organizador III CLAHCT, Apartado postal 21-873, 04000 México, D.F., MEXICO.
- 1992 April 1-4.....Nashville
Annual meeting of the National Council of Teachers of Mathematics and the Americas Section of HPM. Contact NCTM, 1906 Association Drive, Reston, VA 22091, U.S.A. More details on the HPM meeting will be in the next Newsletter.
- 1992 May 11-15.....Kingston
Second International History, Philosophy, and Science Teaching Conference. Contact Professor Skip Hills, Faculty of Education,

Queen's University, Kingston, Ontario K7L 3N6, CANADA. (See inside for more details.)

- 1992 May 28-30 Charlottetown
Annual meeting of the Canadian Society for History and Philosophy of Mathematics. There will be a special session dealing with mathematics in the Americas. Contact Professor M. Malik, Department of Mathematics, Concordia University, 7141 Sherbrooke St. Ouest, Montreal, Quebec H4B 1R6, CANADA.
- 1992 August 12-14 Toronto
International meeting of HPM preceding ICME-7. (See inside for more details.)
- 1992 August 17-23 Quebec
Seventh International Congress on Mathematical Education (ICME-7). Contact Congrès ICME-7 Congress, Université Laval, Québec, QC, G1K 7P4, CANADA or via fax to (418) 656-2000. (See inside for more details.)

HPM in Toronto

The quadrennial international HPM meeting will take place from August 12 to 14, 1992 at Victoria College of the University of Toronto. The scientific program will consist of approximately nine plenary talks of one hour each and several sessions of shorter talks. Among the speakers who have already agreed to give hour talks are Evelyn Barbin (France), William Dunham (United States), Israel Kleiner (Canada), and Ubiratan D'Ambrosio (Brazil). About 20 speakers have so far agreed to give shorter talks of about fifteen minutes in length. The hour talks and the shorter talks will all provide details on how the history of mathematics has been used in the classroom. It is expected that there will be indications of the use of history at the primary level as well as the secondary and university levels.

There is room for many more speakers. Those of you in North America who have not been able

to attend previous quadrennial meetings are especially invited to attend and participate. Please volunteer to speak. Send your name, topic, and a brief summary to Florençe Fasanelli at the address on the first page. More details of the program, including the planned excursion and meals, as well as full details of the registration procedure and the cost of the meeting will be available in the next issue of the Newsletter.

HPM will also participate in ICME-7 itself in Quebec. We have been allotted four 90-minute sessions. There will be three themes of the sessions:

- History of Mathematics and Pedagogical Problems
- History of Mathematics as a Cultural Approach to Solving Problems
- Historical Problems in the Classroom

Each session will consist of three speakers and one respondent. The speakers will deal with one of the themes and, it is planned that each day's talks will include one each at the elementary, secondary, and university level. The speakers who have so far been invited to participate in Quebec include V. Frederick Rickey (U.S.A.), Michèle Gregoire (France), and Maguy Schneider (Belgium).

Put Toronto and Quebec on your calendar for August, 1992. There is much to see and do in both Toronto and Quebec, so bring your families and enjoy an exciting and stimulating twelve days.

ICME-7

ICME-7, the Seventh International Congress on Mathematical Education, will be held at Université Laval in the city of Québec, Canada, from August 17 to 23, 1992. The Congress will be organized into Plenary Sessions, Working Groups, Lectures, Topic Groups, Study Groups, a Mini-conference on calculators and computers, National Presentations, Projects, and Poster Sessions. The working groups, each to meet for

four 90-minute sessions, will consider such topics as "Students' difficulties in calculus," "Multicultural and multilingual classrooms," "The role of geometry in general education," "The place of algebra in secondary and tertiary education," "Technology in the service of the mathematics curriculum," and "Mathematics education with reduced resources." About forty lectures will be scheduled on topics related to teaching, curriculum, teacher education, educational research, history and sociology of mathematics, theories of mathematics education, and recent developments in mathematics. The topic groups, which will each meet for two 90-minute sessions, include "Ethnomathematics and mathematics education," "Mathematics competitions," "Indigenous peoples and mathematics education," "Mathematics and Ibero-American art," "TV in the mathematics classroom," and "Philosophy of mathematics education." The study groups, each of which will organize four 90-minute sessions, include, besides HPM, the International Group for the Psychology of Mathematics Education (PME), and the International Organization of Women and Mathematics Education (IOWME). More details are now available in the Second Announcement, which also includes a registration form and a schedule of fees. The announcement is available by writing to ICME-7 at the address given in the Calendar.

Eighth Symposium of SAMSA

The Eighth Symposium of the Southern Africa Mathematical Sciences Association will be held at the Higher Pedagogical Institute and Eduardo Mondlane University in Maputo, Mozambique from December 16 to 19, 1991. The main theme is the Education of Mathematics Teachers in the SADCC Region. Papers are invited on all aspects of this theme including

- Pre-service education, in mathematics, didactics, history of mathematics, history of mathematics education, mathematics education and society, applications of mathematics, ethnomathematics.

- In-service education
- Levels and types of education, including primary, secondary, technical/vocational, university, remedial, special needs, literacy campaigns, adult education
- The education of teacher educators
- Research and the education of mathematics teachers, in mathematics, mathematics education, ethnomathematics, didactics, history of mathematics
- Production of didactical material for the education of mathematics teachers
- History of the education of mathematics teachers in the SADCC region

Applications to present a paper should be sent to the SAMSA 8 Organizing Committee at the address in the Calendar by August 31, 1991.

Second International History, Philosophy, and Science Teaching Conference

This second conference follows the very successful first conference held at Florida State University in November, 1989. The local organizing committee wishes the conference to have a somewhat sharper focus than the first one and to have papers that advance the questions raised there, rather than simply restate the issues. Some initial themes or topics that the organizers would like considered include Conceptual Change, Feminism, Constructivism, Curriculum Reform, Science Literacy, Exemplary Practice including Elementary Science, HPS and STS Connections, The Environment, and Values. Accounts of the application of history and philosophy of science in the science classroom are welcome as are research papers on the issue. Papers should be between 4000 and 6000 words and should be submitted to the conference secretary on disc along with 3 copies by January 12, 1992. The title and a 200 word synopsis should be submitted by October 12, 1991. All conference papers will be

printed in the *Proceedings* that will be published in advance of the conference. For further details, contact Professor Skip Hills at the address in the Calendar.

HPM in New Orleans

The Americas Section of HPM had its annual meeting on April 18 and 20 in New Orleans in conjunction with the annual meeting of the National Council of Teachers of Mathematics. Israel Kleiner (York University, North York, Ontario) spoke on "Paradoxes in the History of Mathematics and their Use in the Teaching of Mathematics." The paradoxes he discussed included the incommensurability of the diagonal and the side of a square, the idea of a negative number, the concept of a complex number, logarithms of negative and complex numbers, and various paradoxes connected with the development of the ideas of real numbers and functions. For each paradox, he considered examples of how the historical problems connected with its development could be used in teaching today.

William Dunham (Hanover College, Hanover, IN) detailed the work of "The Bernoullis on the Harmonic Series." He noted that Nicole Oresme by 1350 had proved that the harmonic series $\sum_{k=1}^{\infty} \frac{1}{k}$ diverged. But both Jakob and Johann Bernoulli attacked this problem in several ways, each coming up with ideas for the proof that are useful in other contexts. Frank Swetz (Pennsylvania State University, Middletown, PA) discussed "The Sea Island Classic." This work develops in detail the mathematical methods of surveying used by the Chinese, methods which include the apparent use of similar triangles. Florence Fasanelli (MAA, Washington, DC) spoke on "Benjamin Banneker and his Mathematics." She discussed the biography of Banneker, the eighteenth century black mathematician who worked on the original survey of the boundaries of the District of Columbia. In particular, she showed an algebra text similar to one he must have learned from and pointed out the various algebraic techniques he had mastered

on his own, techniques which finally allowed him to prepare a well regarded almanac.

Victor Katz (University of the District of Columbia, Washington, DC) spoke on "Themes from Islamic Mathematics." He discussed several ideas of precalculus and calculus in the mathematics of Islam, ideas which could well be incorporated into the appropriate courses. These ideas included methods for solving spherical triangles in the work of al-Biruni, the use of induction in developing formulas for the sums of integral powers for use in computing volumes in the work of ibn al-Haytham, and the analysis of solution of cubic equations by what appears to be calculus techniques in the work of Sharaf al-Din al-Tusi. Finally, Frederick Rickey (Bowling Green State University, Bowling Green, OH) discussed "A Selection of Historical Ideas for Your Algebra Classroom." These ideas included material from, among other sources, the *Rhind Papyrus*, the *Whetstone of Witte* by Robert Recorde, and especially various ideas connected with the Fibonacci sequence of Leonardo of Pisa.

As usual, the HPM meeting contained lively discussion with interesting people. For various reasons, however, including the fact that the U.S. Postal Service took an extraordinarily long time in delivering the March Newsletter, the attendance was not as large as hoped. Plan now to attend the next meeting in Nashville!

CSHPM in Kingston

The annual meeting of the Canadian Society for the History and Philosophy of Mathematics took place in Kingston, Ontario from May 27 to 29, 1991. There were a wide variety of talks on many historical subjects. Those most of interest to readers of this Newsletter were probably the talks on the special theme of Women in Mathematics. Ann Hibner Koblitz (Hartwick College) gave a general introduction to the subject with "Women in Mathematics: Historical and Cross-Cultural Perspectives." More specific subjects were dealt with by Israel Kleiner (York University), "Emmy Noether: Highlights of her Life

and Work;" James Tattersall (Providence College, Providence, RI), "Women and Mathematics at Cambridge;" and Sharon Kunoff (Long Island University, Greenvale, NY), "Women in Mathematics: Is History Being Rewritten?"

The next annual meeting will be May 28-30, 1992 in Charlottetown, Prince Edward Island. All those interested in the history and philosophy of mathematics are invited to join the society and attend the meetings. For information on membership contact the secretary at the address given in the Calendar.

Working Group on History in Mathematics Education

The British Society for the History of Mathematics has set up a Working Group on History in Mathematics Education, consisting of Society members interested in pursuing various approaches to promoting the use of history. Connected with the Working Group are a set of action groups, each consisting of teachers exploring and promoting some area of common concern. These groups include ones dealing with the history of mathematics and the primary curriculum (convenor, Helen Gardner, Bentley Drive Primary School, Walsall), history and the secondary level (convenor, Paul Thornley), and history in relation to the initial training of teachers (convenor, Neil Bibby, University of Exeter). Neil Bibby is also the guest editor of the special March 1992 issue of the *Maths Gazette*, which will be devoted to history in mathematics education. The Working Group encourages others to join and contribute their ideas. Contact the Chair of the Working Group, John Fauvel, at the address on the first page or the Secretary, Helen Gardner.

Les grands problèmes de l'histoire des mathématiques

Evelyne Barbin

The Commission Inter-I.R.E.M Epistemologie will publish a new book with the above title in

time for ICME-7. The work, of about 300 pages, will be addressed to students at all levels and will include a lively and narrative history of various important problems in the history of mathematics. Each chapter will be devoted to one of these "grand problems" and will be illustrated by old texts explained in their historical context and accompanied by questions for the reader. Each chapter will also contain a bibliography. The tentative titles for the chapters, along with their authors, follow:

- "Le droit et le courbe," E. Barbin and G. Itard
- "Dépasser les cas de figures," H. Plane
- "Comment mesurer la pyramide?" M. Gregoire
- "La courbe brachystochrone," J. L. Chabert
- "Le cinquième postulat," J. L. Chabert
- "L'univers," A. Belet and M. Belet
- "L'irrationalité," D. Daumas
- "Quelle réalité pour les imaginaires?" J. P. Friedelmeyer
- "Mais où est donc passé la troisième dimension?" J. P. Le Goff and D. Bessot
- "L'infini," M. Guillemot
- "Multiplier ou additionner?" F. Lalande
- "Les équations algébriques," G. Bonnefoy
- "Pourquoi la règle et le compas?" R. Bkouche and J. Delattre
- "La géométrie et le mouvement," R. Bkouche
- "Les nombres parfaits," J. Sip and M. Crubellier
- "La division du cercle," H. Plane and M. Lacombe
- "Les nombres premiers," F. Jaboeuf

Second International Colloquium on the History and Philosophy of Mathematics

Alejandro R. Garciadiego

The Second International Colloquium on the History and Philosophy of Mathematics was held at the Mexican National University (UNAM), December 10-13, 1990. The major speakers at the colloquium were Joseph Dauben (CUNY, U.S.A.), "The Gou-Gu (Pythagorean) Theorem in Ancient China" and "Abraham Robinson (1918-1974): The Man and his Mathematics," Wilbur Knorr (Stanford University, U.S.A.), "On Hearts and Ivy Leaves: What the Ancient Greeks did with Curves" and "From Exhaustion to Cuts: Early Stages of the Greek Theory of Proportion," Janis Langins (University of Toronto, CANADA), "The Codification and Mathematization of French Engineering in the Eighteenth Century: The Case of Bernard Forest de Bêlidor," and "From Bêlidor to Navier via the École Polytechnique: The Consolidation of Mathematical Engineering Science in France," Ivor Grattan-Guinness (Middlesex Polytechnic, U.K.), "The Development of the Calculus and the Foundations of Analysis in Euler and Cauchy," "Boole and his semi-follower Jevons," and "Peirce between Logic and Mathematics," and Craig Fraser (University of Toronto, CANADA), "Differential Equations in Lagrange's Theory of Perturbations" and "Isoperimetrical Problems in Lagrange's Variational Calculus." There was also an open session of three hours where colleagues and students offered comments and raised questions to all the speakers. The editors of *Mathesis* will publish Spanish translations of all the papers delivered in forthcoming issues in 1991.

A New Mathematics Education Data Base: MATHDI

Hans-Georg Steiner

MATHDI (*mathematical didactics*) refers to all publications reviewed in the International Documentation Journal *Zentralblatt für Didak-*

tik der Mathematik (ZDM) since 1977. This comprises 45,000 books and papers (from about 400 Journals) with an updating rate of about 700 items bimonthly. Accessible via STN International (The Scientific and Technical Information Network), MATHDI is the most comprehensive and up-to-date computerized information service in mathematics and computer science education. It enables STN online users to search for words from a title, from an abstract, or from a classification text or a set of key words, as well as for authors, journals, books, proceedings. For further information (also about ZDM) please contact: STN International, c/o FIZ Karlsruhe, P. O. Box 24 65, D-7500 Karlsruhe 1, GERMANY. Phone: (+49) 72 47/8 08 - 5 55; FAX: (+49) 72 49/8 08 - 6 66.

A Multicultural Mathematics Curriculum

Beatrice Lumpkin

Illinois has joined a growing number of states that have mandated the inclusion of multicultural components throughout the public school curriculum, including mathematics. In response, the Bureau of Mathematics of the Chicago Board of Education Curriculum Department has made substantial additions to curriculum materials, beginning with the systemwide objectives.

For each grade level K to 12, the state has grouped specific objectives for mathematics achievement under seven State Goals. Each group of objectives is introduced with a paragraph citing some multicultural contributions to mathematics. The content varies with grade level and subject. For example, "Measurement" objectives for Algebra are introduced by:

Students should be able to relate the origin of measurement to real-life situations. For example, the building of the African pyramids required extremely accurate measurement to construct right angles in the base so that any error would be less than one part in 27,000 or 1/27000. The unit of measure was the cubit, the length of an early pharaoh's forearm. The idea of a 24-

hour day — 12 hours of day and 12 hours of night — originated in Egypt. The Babylonians of Mesopotamia established the time measures of 60 seconds to 1 minute and 60 minutes to 1 hour. They also created the angle measures of 60 seconds to 1 minute and 60 minutes to 1 degree. Native Americans, especially the Inca, Maya, and Aztec, developed a system of measurement that was so accurate that they were able to lay out miles of direct highways across high mountains and rugged terrain. The Ashanti of Ghana used standard gold weights to calibrate their scales with the accuracy required by their extensive commerce.

For Algebra II (advanced algebra), the introduction to the specific algebra objectives includes the work of Hypatia:

Students should know that the modern algebra developed in Europe is based on the algebra that began in Africa and Asia. Indeed, the word algebra is Arabic in origin; Europe received algebra as a gift from Asia and Africa. Under the influence of the African Moors, algebra spread through Europe from Spain and Italy. Equations were first solved in Africa 4,000 years ago by using proportions. Ancient Egyptians introduced the concept of the unknown or variable, which they called *aha*, the Egyptian word meaning heap. They also used the first symbols for addition (feet walking toward a number) and subtraction (feet walking away). Africans were the first to use rectangular coordinates for their Egyptian star-clocks and for their construction plans for large temples. Babylonians (Mesopotamians) developed algorithms to find square and cube roots in the solution of equations. Geometric series, which play an important role in calculus and science, were first explored in Egypt 4,000 years ago. Hypatia, an Egyptian woman, worked with conic sections and indeterminate equations. The matrix method for the solution of systems of equations was pioneered by the Chinese 1450 years before Cramer's rule was formulated in England. Chinese mathematicians also used the so-called Horner's method for the solution of higher degree equations long before

Horner was born in England.

In addition to Measurement and Algebra, areas covered by the state objectives are Number Concepts, Quantitative Relationships, Geometric Concepts, Data Analysis and Applications. The multicultural introduction for 10th grade geometry includes some examples of ethnomathematics:

Students should examine the contributions to geometry made by people all over the world. For example, African mathematicians in ancient Egypt developed formulas for the area of a triangle, a rectangle, a trapezoid, and a circle. Their study of geometry was stimulated by the need to resurvey the fields after the annual Nile River flood had wiped out all farm boundaries. The ancient Egyptians were also the first to develop the concepts of congruency and similarity of geometric figures. The right triangle theorem was used by the Babylonians 1500 years before Pythagoras was born. Therefore, the Pythagorean theorem is a misnomer. The Egyptian formula for the area of a circle used a value of π that very closely approximated the known value of π today. The Egyptian value was 3.16 which was almost equal to the correct value of 3.14. The first known use of trigonometry was in the application of the cotangent in the construction of pyramids in Africa 4,800 years ago.

Today people throughout the world apply geometry to everyday needs. Eskimos build their dome-topped igloos along the lines of an inverted catenary for greater strength. On Mount Kenya, families lay out the circular base of their homes by using a string attached to a center pole as the radius. Mozambicans build rectangular houses by using equal-length ropes as the diagonals.

A different challenge was met for the new Algebra Framework now in preparation. Here the challenge was to integrate multicultural materials in the form of real-life examples which could relate to the student's world. Actually, many of the famous problems from the history of mathematics in Africa, Asia, and Latin America proved to be quite suitable for 9th grade algebra. The material is now in the process of field testing and

initial reactions have proved favorable.

From the Newsletter of the International Study Group on Ethnomathematics

Mary Boole - An Early Mathematics Pedagogist

Karen Dee Michalowicz

The Langley School, McLean, VA

Born in England in 1832, Mary Everest Boole lived during an era in which little credit was given to the mathematical abilities of women. Furthermore, she stood in the shadow of two famous men, her uncle, George Everest, who conquered Mt. Everest, and her husband, George Boole, the famous algebraist and logician. Biographies about great people in mathematics give nominal attention to Mary. Usually, she is identified as the niece of George Everest and the wife of George Boole. Although little mention is given to the fact, Mary Everest Boole is one of the first pedagogists of mathematics who encouraged the use of manipulatives.

The story begins almost two centuries ago when a young 18-year-old girl has as her mathematics tutor and mentor, George Boole. Mary, in turn, provides encouragement and advice to George as he revises his book, *Laws of Thought*. Five years later they marry, she, 23 and he, 40. By the time she is thirty-two, Mary is the widowed mother of five daughters, one an infant of six months.

Mary needed to support her family. In desperation, she turned to a friend at Queen's College. The college had been established to train women to become governesses. Although Queen's College did offer college level courses to women, college degrees were not offered to women in England during the middle of the 19th century. Neither were women permitted to hold positions on the faculty of a college. Instead, Mary had to take a job as librarian at the college, a "respectable" position for a woman. Additionally, Mary taught classes in mathematics for children.

As a teacher of children, Mary was "in her glory." Her ideas about the instruction of math-

ematics included using natural materials to develop the children's understanding and excitement of mathematics. She believed that as children did things physically (manipulatively), the unconscious grew as well as the understanding of many fundamental mathematical ideas. One wonders why this remarkable pedagogy disappeared for 150 years. Mary Boole's ideas sound very much like the ideas presented in the new mathematics standards.

To encourage her students to explore their environment, Mary invented curve stitching cards, called Boole cards, which became popular in England for a time. Unfortunately, as Mary's ideas faded away, so did the use of the Boole cards. Today, one can find ideas similar to Mary's in Victoria Pohl's book *How to Enrich Geometry Using String Designs*, available from NCTM.

Although Mary was not permitted to teach in the University because of her sex, she still managed to inspire students of mathematics. One can not forget that Mary herself, high in mathematical ability, had been a student of one of the most famous mathematicians of the period, her husband, George Boole. She organized "Sunday Night Conversations" during which she and the students discussed her husband's mathematics, Darwinian theory, philosophy, psychology, etc. These soirees became very popular.

Mary considered herself a mathematical psychologist. Throughout her life, she tried to understand how people learned mathematics using their conscious and unconscious mind and their physical bodies (manipulating things). She never tired of using her ideas and techniques with her students.

To find out more about Mary Everest Boole, read *Women, Numbers and Dreams* by Teri Hoch Perl and Joan M. Manning.

This is the first draft of an article written for use by middle school teachers. It will probably form the basis of a talk at the next NCTM meeting in Nashville.

Middle School Concepts and Related Historical Topics

Karen Dee Michalowicz

From the *Virginia Mathematics Teacher*, Winter, 1989: The beauty about using historical topics in mathematics is the interest it generates in the students. The uses may take many approaches. For example, you can design a time line with students which denotes important dates or events in mathematics history. You can also use a bulletin board to display pictures of famous mathematicians. This could lead to a "Mathematician of the Week." One of my favorites is to act out interesting "moments in history" (events) involving famous mathematicians. The story of young Carl Gauss is perfect for this, and students love to hear about Archimedes' bath and his famous war machines.

In order to help you use the history of mathematics in your teaching, I have taken a typical middle school curriculum and related historical mathematical topics to it. In addition, I have developed a list of references which correspond to the historical topics. While the lists are long, they are in no way complete.

There is no space here to provide the lists. However they include references for the topics Numbers and Numeration, Whole Numbers, Fractions, Decimals, Number Properties, Integers and Rational Numbers, Ratio, Proportion, Percent and Interest, Graphing and Probability, Geometry, Measurement, Time and Money, Technology, and Algebra. The full article is available from the author, who can be reached at The Langley School, 1411 Balls Hill Road, McLean, VA 22101.

Have You Read?

Ronald Calinger, ed.

This column welcomes references from across the history or pedagogy of mathematics, as well as other works with sections that have potential for encouraging and motivating students to learn mathematics better or that enrich courses. Please send citations with complete bibliographic information to the section editor

c/o Department of History, Catholic University of America, Washington, D.C. 20064, U.S.A.

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