International Study Group on the Relations Between HISTORY and PEDAGOGY of MATHEMATICS NEWSLETTER

HPM Advisory Board

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Calendar

Meetings with HPM components are highlighted.

- 1991 April 3-6 ...... Newcastle upon Tyne
  Annual Conference of the Mathematical Association. There will be several talks dealing with history in mathematics. Contact Peter Ransom, 12 Annaside Mews, Leadgate, Consett, Co. Durham, DH8 6HL, U.K.

- 1991 April 8-12 .... Berlin
  International Berlin Conference on the Teaching of Mathematics, co-sponsored by Humboldt-Universität zu Berlin, Sektion Mathematik; Freie Universität Berlin, Zentralinstitut für Fachdidaktiken; Technische Universität Berlin, Fachbereich Mathematik; and Brandenburgische Landeshochschule Potsdam, Sektion Mathematik. Contact Professor Roland Stowasser at the address on the first page.

- 1991 April 17-20 .... New Orleans
  Annual Meeting of the National Council of Teachers of Mathematics and the Americas

Section of HPM. Contact NCTM, 1906 Association Drive, Reston, VA 22091, U.S.A. (See inside for more details.)

- 1991 April 22-24 .... Istanbul
  Symposium on Science Institutions in Islamic Civilization, sponsored by the Turkish Society for History of Science. Contact the Symposium Secretariat, P.O. Box 24, 80692 Besiktas, Istanbul, TURKEY. (Tel: 160 5988; Telex: 26484 isam tr; Fax (01) 158 4365).

- 1991 May 4 .... Leicester
  HIMED 91 – History in Mathematics Education Workshop, sponsored by the British Society for History of Mathematics and the Department of Mathematics, University of Leicester. Contact Steve Russ, Department of Computer Science, University of Warwick, Coventry CV4 7AL, U.K. (See inside for more details.)

- 1991 May 9-11 .... Orlando
  Conference on History, Geometry, and Pedagogy at the University of Central Florida, Orlando in honor of the 80th birthday of Howard Eves. Contact Professor Joby Anthony, Department of Mathematics, University of Central Florida, Orlando, FL 32816-6990, U.S.A. (Tel: (407) 823-2700; Fax: (407) 281-5156)

- 1991 May 11-12 .... Oxford
  800 Years of Mathematics at Oxford. This is a meeting for non-specialists. Contact the Department of Extramural Studies, University of Oxford, Beverley House, Oxford, U.K.

- 1991 May 27-29 .... Kingston
  Annual meeting of the Canadian Society for History and Philosophy of Mathematics. Contact Professor M. Malik, Department of Mathematics, Concordia University, 7141 Sherbrooke St. Ouest, Montreal, Quebec, H4B 1R6, CANADA (See inside for more details.)

- 1991 July 1-3 .... London
The Bicentennial Conference on Computing, organized by the Institute of Electrical Engineers in collaboration with the National Museum of Science and Industry. The conference will mark the 200th anniversary of the birth of Charles Babbage and will be held at Imperial College, South Kensington, London. Contact IEE Conference Services, Savoy Place, London WC2R 0BL, U.K.

- 1991 July 13-16 .... Loughborough
British Congress of Mathematics Education. One of the purposes of the meeting is to prepare for ICME7. There will be discussions of various aspects of mathematics education, including one on "Using History in Mathematics Education," convened by Neil Bibby. Contact Rita Nolder, Department of Education, Loughborough University, Loughborough, LE11 3TU, U.K. (Tel: 0509 222776)

- 1991 August 3-7 .... Coral Gables
Eighth Interamerican Conference on Mathematics Education. Contact Patrick Scott, Programas Latinoamericanos de Educación, Faculty of Education, University of New Mexico, Albuquerque, NM 87131, U.S.A.

- 1991 September 11-14..... Cambridge
Annual Residential Meeting of the British Society for History of Mathematics. The theme of the meeting will be Renaissance Mathematics, and an international team of speakers has been arranged. Contact the Secretary, J. Helen Gardner, 25 Hollow Croft Road, Willenhall, West Midlands WV12 5YS, U.K. for information on the meeting and on membership.

- 1992 January 12-16 .... Mexico City
Third Latin American Congress on the History of Science and Technology (III CLAHCT). The general theme of this meeting will be America in the formation of a new world: 500 years of scientific exchanges. Contact Comité Organizador III CLAHCT, Apartado postal 21-873, 04000 México, D.F., MEXICO.

- 1992 August 9-13 .... Toronto
International meeting of HPM preceding ICME-7. (See inside for more details.)

- 1992 August 16-23 .... Quebec
Seventh International Congress on Mathematical Education (ICME-7). Contact Congrès ICME-7 Congress, Université Laval, Québec, QC, G1K 7P4, CANADA or via fax to (418) 656-2000.) (See inside for more details on HPM's participation.)

From the Editor

I recently received from the Maryland State Department of Education the new requirements for certification to teach secondary mathematics in the state. Among the courses listed is a 3-semester hour course in the history of mathematics. (Interestingly, several of the major colleges and universities in Maryland do not at present offer the course.) The District of Columbia does not require a history course for certification, although the University of the District of Columbia, the "state" university for D.C., requires such a course for graduation in the mathematics department's mathematics education option. I and other readers would be interested in learning which states in the U.S. – and which other countries – require such a course for secondary school mathematics teachers. If readers would let me know, I will publish the results in future issues of this *Newsletter*.

On another matter, Florence Fasanelli, the chair of H.P.M., has been compiling a list of new mathematics texts with historical notes. These include

- James Franklin and Albert Daoud, *Introduction to Proofs in Mathematics* (Sydney: Prentice Hall of Australia, 1990)

- Susanna Epps, *Discrete Mathematics* (Belmont, Ca.: Wadsworth). This book has extensive notes and pictures.

- Karl J. Smith, *Precalculus Mathematics* (Pacific Grove, Ca.: Brooks/Cole). Includes notes and quotes of mathematicians.

Finally, all readers of this *Newsletter* should think about contributing to it. Please consider writing short articles on how you use the history of mathematics in your classes as well as book reviews and reports of interesting meetings. In other words, please share your ideas!

Course on Ancient Greek Mathematics

This summer the course "Ancient Greek Mathematics" will be offered in Greece. Lectures will cover the development of mathematics from its legendary beginnings with Thales and Pythagoras through the work of Euclid, Archimedes, and Diophantus, viewed in the context of Greek culture and history. The lectures will be given in Athens and on the island of Samos, with a trip also included to Miletus and some other important early Greek cities in Turkey. Three undergraduate or graduate credits are available for completing this three-week course, July 6–27. Professionals who take the course for credit may find this a deductible business expense or that expenses will be defrayed by an employer.

The cost of the program is estimated to be $2000, including airfare from New York, lodging, transportation within Greece and to and from Turkey, and several tours. Most meals are not included, nor is tuition ($273 for 3 undergraduate credits or to audit, $432 for 3 graduate credits). For further information, write to the instructor: Professor Paul Wolfson, Department of Mathematics and Computer Science, West Chester University, West Chester, PA 19383.

8ème Colloque Inter-I.R.E.M.

Evelyne Barbin

La Commission inter-I.R.E.M. "Epistémologie et Histoire des Mathématiques" et l'Institut de Recherche sur l'Enseignement des Mathématiques de Lyon organisent. les 31 Mai et 1er Juin 1991, le 8ème Colloque National sur l'Histoire des Mathématiques. Les Colloques inter-I.R.E.M. sur l'Histoire des Mathématiques rassenblent tous les deux ans, dans une ville universitaire, plus d'une centaine de participants. Ces participants sont des professeurs de collège, de lycée et d'école normale, des universitaires, et des chercheurs en histoire des sciences. Ce sont des enseignants de mathématiques, mais aussi des enseignants de philosophie, d'histoire et de sciences physiques.

Le thème du Colloque inter-I.R.E.M. de Lyon sera celui de "La figure et l'espace." Il fait référence au quatrième centenaire de la naissance du géomètre lyonnais Desargues, dont les travaux constituent les débuts de la géométrisation des travaux sur la perspective des peintres de la Renaissance. Le Colloque sera l'occasion de resituer les travaux des géomètres pour représenter les objets de l'espace dans l'histoire de l'art et de l'architecture.

Le thème du Colloque correspond aussi à un certain nombre de préoccupations actuelles de l'enseignement de la géométrie. En effet, beaucoup de travaux récents des I.R.E.M. s'interrogent sur le rôle de la figure dans le raisonnement des élèves, sur l'importance à accorder aux configurations géométriques ou aux transformations géométriques, sur les relations entre algèbre et géométrie.

Les thèmes historiques et épistémologiques suivants seront abordés dans les ateliers et les exposés:

- le rôle de la figure géométrique

- la représentation de l'espace

- la géométrie: science de figures ou science de l'espace

- le mouvement et la géométrie

- la géométrie et le calcul

- la géométrisation comme langage et comme représentation.

Les personnes intéressées par ce Colloque doivent s'inscrire très rapidement en écrivant à:

I.R.E.M. de Lyon, 43 bd du 11 Novembre 1918, 69622 Villeurbanne Cedex, FRANCE.

Annual Meeting of HPM Americas Section

The Americas Section of HPM will meet in conjunction with the annual meeting of the National Council of Teachers of Mathematics in New Orleans from April 17 to 20, 1991. In contrast to previous years, however, this year's HPM meeting will be during the NCTM meeting, instead of before, in order to attract more teachers. Sessions are scheduled for Thursday, April 18 from 4:30 to 7:30 p.m. and Saturday, April 20 from 1:00 to 5:00 p.m. Both sessions will meet in Room 22 of the New Orleans Convention Center. Speakers at this session will include Israel Kleiner (York University), "Paradoxes in the History of Mathematics and Their Use in the Teaching of Mathematics;" Kattye Sowell (East Carolina University), "Hamilton's Icosian Calculus and his Icosian Game;" Frank Swetz (Pennsylvania State University), "The Sea Island Classic;" Victor J. Katz (University of the District of Columbia), "Themes from History in Precalculus Courses;" Florence Fasanelli (MAA SUMMA project), "Benjamin Banneker and his Mathematics;" V. Frederick Rickey (Bowling Green State University), "A Selection of Historical Ideas for Your Algebra Classroom;" and William Dunham (Hannover College), "The Bernoullis on the Harmonic Series." There is still time to arrange to present a paper. Contact Erica Voolich, 244 Summer St., Somerville, MA 02143 (617-666-0666) or V. Frederick Rickey (419-372-7452 (office), 419-352-4194 (home), or email at rickey@andy.bgsu.edu) as soon as possible. A complete program will be sent to individuals who contact either of the above. The programs will also be available in New Orleans. If you have never been to an HPM meeting, you are in for a pleasant surprise. The people are interesting, the discussions are lively, and we always continue at an excellent local dining spot. So come to New Orleans and share

your ideas.

There are many sessions at the NCTM meeting itself which deal with the use of history in teaching. These include Frank Swetz, "Seeking Relevance? Try the History of Mathematics," Wednesday, 10:30–11:30 a.m.; Dolores Granito, "A History of Women in Mathematics from the Time of Pythagoras to the Present," Wednesday, 10:30–11:30 a.m.; Don Gallagher, Michael Sequeira, and James F. Kiernan, "Math Monuments: A Historical Tour through Great Britain, France, and Germany," Wednesday, 12:00–1:00 p.m.; William L. Campbell, "Ancient and Not-So-Ancient Algorithms – Using Math History and Old Texts," Wednesday, 3:00–4:00 p.m.; James M. Rubillo, "Just in Time for 1992: The Calculations of Columbus – a 500-Year Retrospective," Thursday, 9:00–10:00 a.m.; Michael A. Iannone, "Historical Aspects of Math Appropriate for the Elementary and Middle Grades," Friday, 12:00–1:00 p.m.; James E. Lightner, "Meet the Great Contributors to Geometry during a Brief Historical Tour," Friday, 1:30–2:30 p.m.; J. Philip Smith, "Mersenne Primes: A Historical Problem-solving Odyssey," Friday, 3:00–4:00 p.m.; Rick Armstrong, "Who Was Who in Trigonometry – the Greeks, Indians, Arabs, and Europeans," Friday, 3:00–4:00 p.m.; and Bruce F. White, "Archimedes: His Life and Mathematics– Use His Mathematics to Motivate," Friday, 3:00–3:30 p.m.

HIMED 91 Workshop in Leicester

The British Society for History of Mathematics and the Department of Mathematics, University of Leicester are cosponsoring a workshop on History in Mathematics Education as a followup to the highly successful HIMED 90. This one, on a somewhat smaller scale than the one last year, will take place on May 4, 1991 from 11:00 to 17:00 at the Department of Mathematics, University Road, Leicester, U.K. The workshop is aimed at anyone interested in using history to improve the learning of mathematics, including every level from primary school and basic numer-

acy work, to degree level courses on the history of mathematics. The aims of the workshop are to provide basic information about history of mathematics relevant to teachers; to let the HIMED 90 Action Groups present what they have been doing; to make it easy for those with experience of teaching mathematics, and those with knowledge of history to exchange information, experiences, ideas, problems, questions, materials, etc; and to work in groups or individually at preparing, trying out, commenting on, improving, various materials for teaching use at all levels of age and ability.

The schedule includes two talks, one in the morning on "Stamping through Mathematics (a philatelic tour of the history of mathematics)," by Robin J. Wilson, Open University, and one in the afternoon on "Teaching Mathematics with a little help from History," by Mike Dampier, University of Leicester. The rest of the day will be spent in group work on various activity themes, including Basic Skills/Adult Education, National Curriculum, both Primary and Secondary, Initial Teacher Training, Use of Computers, Higher Education Courses, and Recreational Topics.

Anyone interested in attending should contact Steve Russ, Department of Computer Science, University of Warwick, Coventry CV4 7AL, U.K. (phone 0203 523681 (work), 0203 672517 (home), fax 0203 525714, or e-mail sbr@uk.ac.warwick.cs) without delay.

Canadian Society for History and Philosophy of Mathematics

The 1991 meeting of the C.S.H.P.M. will be held May 27 to 29 at Queen's University in Kingston, Ontario. There will a special session on the theme of Women in Mathematics. The program chair is Professor Erwin Kreyszig, Department of Mathematics and Statistics, Carleton University, Ottawa, Ontario, K1S 5B6. CANADA. (Phone: (613) 788-2145) Abstracts for contributed papers should be sent directly to him.

International Study Group on Ethnomathematics (ISGEm)

The International Study Group on Ethnomathematics was founded in 1985, under the guidance and inspiration of the internationally famous Brazilian mathematician Ubiratan D'Ambrosio. Since that time it has sponsored programs and business meetings at the annual conferences of the National Council of Teachers of Mathematics and at the Sixth International Congress of Mathematical Education in Budapest in 1988. In 1990 it became an affiliate of the National Council of Teachers of Mathematics.

What is ethnomathematics? The term was coined by D'Ambrosio, and implies the influence of sociocultural factors on the teaching and learning of mathematics. The prefix "ethno" encompasses "identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on," and includes "their jargon, codes, symbols, myths, and even specific ways of reasoning and inferring" (quoted from the first ISGEm Newsletter). Alan Bishop suggests that six environmental activities lead to the development of mathematical practices and concepts: counting, locating, measuring, designing, playing, and explaining. There is now ample evidence that people in all societies devise their own ways of doing mathematics, independently of their technological level or what they may have learned in school. Yet these practices are rarely recognized in formal school mathematics.

The semiannual Newsletter includes concept papers on current work in the field of ethnomathematics, information on research projects, book reviews, annotated bibliographic entries, and ideas for promoting the study of ethnomathematics. The editor is Patrick Scott, College of Education, University of New Mexico, Albuquerque, NM 87131, U.S.A. To join the ISGEm and to receive the Newsletter, contact Gloria Gilmer, President, Math-Tech, Inc., 9155 N. 70th St., Milwaukee, WI 53223, U.S.A. (Tel: (414) 355-5191). Dues are five dollars per year,

to cover the cost of the Newsletter.

HPM and ICME-7

HPM has been allocated four 90 minute sessions at ICME-7 in Quebec. In addition, HPM will hold its quadrennial international meeting in Toronto during the week preceding ICME, specifically on August 11, 12, and 13 at Victoria College of the University of Toronto. Discussions are under way to have the Canadian Society for History and Philosophy of Mathematics sponsor at meeting in conjunction with HPM on August 9 and 10. The quadrennial business meeting of HPM will also take place during that week in Toronto. At present, fourteen members of the French Inter-IREM Commission on History and Epistemology of Mathematics have indicated their intention of attending the Toronto meeting and presenting papers. All readers are of course invited to attend and contribute. Recommendations for speakers for the sessions in Quebec are also solicited. Please send your recommendations to either Florence Fasanelli or Victor J. Katz. The chosen speakers can then be officially invited.

Third University of Wisconsin LaCrosse Mathematics History Conference

Irving Anellis, Modern Logic Publishing

The conference, held on October 5-6, 1990, was organized by J. D. Wine (University of Wisconsin-LaCrosse), Irving H. Anellis (Modern Logic Publishing and Des Moines Area Community College), Douglas Cameron (University of Akron), and Charles Jones (Ball State University).

In an hour-length talk, "Mathematics teaching with history from the perspective of some pre-college teachers," Helena Pycior (University of Wisconsin-Milwaukee) discussed a National Science Foundation program which she and Robert L. Hall directed in 1985-87. The program, designed to revitalize mathematics teach-

ing with the use of history, involved secondary-school mathematics teachers in the preparation of teaching modules for the classroom use of history of mathematics as a teaching tool. Reactions of the participants was also discussed. One of the leading reactions was that the preparation time proved to be a heavy burden on the teachers, but that it was well worth the effort inasmuch as it revitalized the interest of the teachers in their subject.

Gerald Meike (Wright State University) spoke on "A lesson of (mathematical) history," in which he discussed the results of an upper division undergraduate history of mathematics course. Most of the members of the course were secondary mathematics education students. Meike described a phenomenon in which students had little difficulty understanding the mathematical concepts involved in discussions of mathematics through the nineteenth century, but used the same words with meanings different from those used by the instructor in considering mathematical concepts developed in the twentieth century. This phenomenon, which was noted to occur for several independent offerings of the course, persisted despite a change in the methodology by which the material was taught.

Jerry Lenz (St. John's University), David Kullman (Miami University of Ohio), Rich Maresh (Viterbo College), and Dick Jarvinen (Winona State University) participated in a panel discussion on "The use of history of mathematics in mathematics education," in which it was agreed that the use of history in the teaching of mathematics was an effective and useful way to show students that mathematics is a human endeavor which has cultural relevance and remains a vital pursuit which continues to grow.

Hour long talks were given by Irving H. Anellis on "The roots of mathematics and mathematics education in Russia in the age of Peter the Great," Thomas Drucker (Dickinson College) on "The roots of model theory," and Charles Duffy (Massachusetts Maritime Academy) on "N. I. Lobachevsky." Additional talks were given by Nathan Houser, "The difference a no-

tation can make," John A. Synowiec, "Harmonic analysis, partial differential equations and symbolic methods: interacting still," William S. Mutch, "The historical development of Jordan content and Lebesgue measure," Michael Millar, "Curves from ancient times," Matilde Macagno, "Leonardo da Vinci and transformation geometry," Valentine A. Bazhanov, "A history of mathematics at Kazan State University," Charles Ford, "Crisis in the Moscow school of mathematics," Francis Florey, "Galois and the theory of groups," Helen Skala, "A concise history of cryptology," and Walter S. Sizer, "Mathematics before the age of writing." The scheduled talks by Robert Brabanec, "A tale of two mathematicians - Fourier and Cantor," and Ali Zakeri, "A mathematician who enjoyed life, Omar Khayyam," were cancelled. The publication of a volume of proceedings is being planned.

History Talks in San Francisco

V. Frederick Rickey

There were an amazing number of talks dealing with history at the combined American Mathematical Society and Mathematical Association of America annual meetings held in San Francisco California on January 16–19, 1991. An AMS Special Session on History of Mathematics was organized by Florrence Fasanelli, Victor Katz, and David Rowe. Abstracts of the thirteen talks appeared in the *Abstracts of Papers Presented to the American Mathematical Society*, 12(1), January 1991. The talk in this session dealing most closely with the ideals of HPM was one by Barnabas Hughes, California State University/Northridge, entitled "Genesis of the equation," which dealt with the origin of the word "equation."

David Zitarelli organized an MAA Session on Using History in the Teaching of Mathmatics. The seventeen talks presented there are abstracted in the meeting program distributed in San Francisco, but these will be hard to obtain, so we indicate their contents here.

"Great Problems of Mathematics: A course

Based on Original Sources" was presented by Reinhard C. Laubenbacher and David J. Pengelley of New Mexico State University. This was a freshman honors class (only half of the students had some calculus) which dealt with selected texts of Euclid, Archimedes, Cardano, Fermat, Euler, and Cantor. The students were to read these original sources and did exercises based on them.

"History from Three Viewpoints" was the contribution of Gary McGrath of Pittsburg State University. This course was based on Burton's *History*, Struik's *SourceBook*, and a variety of papers dealing with the history and philosophy of mathematics. One goal of the course was to get students to see that debate exists between mathematicians and also between histoians.

"The History of Mathematics and Philosophy" was the title of a course presented by James W. Petticrew of the University of Texas—Pan American. This course included information on the epistemology of mathematics including the various views of axiomatic systems, the changes that Non-Euclidean geometries introduced, and the impact of Gödel on our understanding of mathematics. In order to keep the student's papers on track Petticrew had them come in individually and consult with him on their contents.

"A Writing Intensive History of Mathematics Course" by John F. Berglund of Virginia Commenwealth University. A variety of writing projects was assigned in this class: one addressed to an "intelligent but not overly interested High School Student," another based on an original source where the historical context was to be stressed, and even a review of a research article. In all cases rewriting and criticism of each other's work was encouraged.

"History of Mathematics: A time-Hopping Problem-Solving Approach with Maple Flavoring" by David Graves of Elmira College. A survey course for teachers using Eves's book and also using the computer program Maple to help with the computations and to encourage the students to explore mathematics on their own.

"A graduate Course in the History of Math-

ematics as a Cornerstone of a Teacher Training Program" was discussed by David Knee of Hofstra University. This was the first course in an NSF Institute for teachers whose goals included getting back to mathematics, seeing the big picture, problem solving, networking, and meeting mathematicians. He distributed a booklet describing the course in detail. This course proved to be a perfect introduction to a graduate program for teachers.

"History of Mathematics: A Graduate Survey Course at Penn State" was discussed by John Dawson of Pennsylvania State University, York. An ambitious course for graduate mathematics students that included stress on bibliographic skills by having the students create a bibliography for a not too well known mathematician, for a contemporary mathematician, and for an individual topic. They read all of Lakatos's *Proofs and Refutations* and Struik's *Concise History*. A final project was to write a new chapter for Struik's book on a twentieth century topic. One student became so interested in the decimal history of time that she took an incomplete in the course so that she could work in the archives in Paris this summer.

"Hilbert's Problems, and a Play Based on Them," discussed an innovative course presented by Marjorie Senechal of Smith College. This honors junior-senior course was loaded with ideas that we should emulate: Agnesi's *Analytical Institutions* was the text for a portion of the course dealing with calculus, an historical banquet where students came dressed as their mathematician and had to speak in his style (there was an interesting discussion between Kepler and Cauchy on style and rigor), an exhibit of (fake) mathematical artifacts including Euler's eyepatch and Kepler's snowflake, and finally a play the students wrote on "All My Problems (as Göttingen Turns)" which included a conversation between a student and a docent, quotes from May Winston's letters about her student days in Germany (they are in the Smith Library), and a Pub scene where Hilbert explains axiomatics to the man on the street.

"Using Hilbert's problems to Teach History," by Sharon Kunoff, Long Island University, C.W. Post Campus. They used Hilbert's tenth problem on Diophantine equations to motivate learning about mathematics and its history, from the contributions of the Greeks to the work of Julia Robinson.

"The Problem of Points as a Case Study in Problem Solving," by David Kullman, Miami University, Oxford OH. The history of a problem important in the origins of probability was used to discuss problem solving by looking at both correct and incorrect solutions of the problem.

"An Alternative Approach to the History of Mathematics" by Claudia A. Henrion of Middlebury College. This course studied mathematical cultures and geometry to explore questions such as 'What is Mathematics?' and 'What is history?' For a final project the students had to create a timeline of mathematics. One student created a quilt with different cultures represented in different colors.

"Mathematics and the Bells" by Angela B. Shiflet of Wofford College. Discussed campinology as a way of motivating ideas in group theory.

"A History Course Designed Around Ideas," by Joseph F. Conrad, Pennsylvania State University, Altoona. This liberal arts course was based on concepts, not chronology. The threads included number, equations, infinity, geometry, function, and proof methods. Stress was given to the ways in which mathematicians convinced each other that their results were correct, and how these concepts changed over time.

"A Women's History Month Project: Famous Women in Mathematics" by Regina Baron Brunner of Cedar Crest College. To celebrate Women's History Month students in several undergraduate mathematics classes presented papers and talks on a several women mathematicians.

"Undergraduate History of Mathematics: Success from Team Teaching" by Stuart Anderson of East Texas State University, and "The History of Mathematics as Taught in a Seminar" by Herbert E. Kasube of Bradley University. An

interesting idea on keeping a small department active: jointly conduct a seminar on history with faculty members speaking about topics in their specialties. The next year this was presented as a team taught course for undergraduates.

"The Rise and Fall of Euclidean Geometry" by Mark Huibregtse of Skidmore College and Texas A & M University. A liberal arts course dealing with Non-Euclidean Geometry and its history.

History—A Way Back to Mathematics

Robert L. Hayes, Hawthorn Institute of Education, Australia

For the past several years the author has been teaching Mathematics and basic numeracy courses to various adult groups. Many students in the classes initially entered with a deep seated disliking of Mathematics – generally due to poor teaching, lack of success, and unpleasant memories during their primary and secondary school years.

The students were required to undertake Mathematics and basic numeracy studies because they were training as trade teachers in technical schools or TAFE colleges and may be required to teach some Mathematics content to apprentices and technician students in various trades such as Fitting and Machining, Electrical/Electronics, Automotive, Sheetmetal, Plumbing, Carpentry and Joinery, and Horticulture.

With each new intake it has been necessary to change the attitudes of self-conscious and embarrassed adults toward Mathematics and to develop an active interest in the subject. Simply reteaching and revising basic school Mathematics was not a suitable approach and a total waste of time. Most had experienced no success with the traditional presentation of content and were so defensive and inhibited that meaningful learning using such an approach was practically zero.

Thus the problem! Reluctant learners and a lecturer searching for an effective teaching and communication strategy.

In the early stages time in class was devoted to discussing Mathematics in a general way. Why did they not like Maths? What were their problems? What kind of Maths did they use in their trades? What Maths would they be required to teach? How had they been taught Maths in school?

Most seemed to believe that Maths either dropped from heaven as a "God given gift" or ascended from hell as an instrument of torture. They had little knowledge of the origins of Mathematics or factors influencing its development. They were unaware of the relationship between branches of the subject. Many considered Maths to be mostly Arithmetic. Algebra was a form of unintelligible code purposely invented to tease students, and Geometry was something about angles, triangles, and parallel lines. One student recalled a theorem about the "square on the hippopotamus" – who was that Greek bloke? The discussion continued in this vein for some time.

After some reflection on these matters, their problems, misconceptions and anxieties, an historical approach was chosen as a means to lead the students "back to Mathematics." This was seen to provide a context and a legitimate way to revise "basic Maths" in, for this type of student, a new and novel manner. One of the topics was development of number (tallying, counting, numeral systems etc..., the UNESCO film "One, Two, Three" providing a motivating introduction). A second was calculating strategies and devices through the ages (the endeavour to find a way round the drudgery of "arithmetic," – the abacus, various algorithms and calculating methods, logarithms, slide rules, calculating machines, computers, calculators). The Mathematics of gambling and probability was the topic for several dynamic activity based sessions. Development of confidence was a prime consideration.

Cultures and personalities involved in Mathematical development at various points in history were investigated. The book by V. S. Groza, *A Survey of Mathematics*, was found to contain useful material and ideas at a suitable level. The approach was supplemented by aids such as MAB blocks, Algebraic Experience Material,

and Cuisenaire blocks. Films and videotapes such as "Donald in Mathemagic Land," "Music of the Spheres" (from Bronowski's "Ascent of Man" series), parts of Carl Sagan's "Cosmos" series, "The Mystery of Stonehenge," and "Flatland" were viewed, discussed, and enjoyed.

Using the approach, topics, and themes outlined above, a marked change in attitudes of the majority of students was observed. From a view of Mathematics as just a tool subject, they began to appreciate Mathematics as a subject to study and enjoy in its own right. Initially they said, "Why would anyone want to do that?" Now they themselves were doing it!

Where possible, various current themes were used to liven up the studies. As an example, you may recall that in April, 1988, Belayneh Dinsamo of Ethiopia ran a world best time for the marathon – 2 hours, 6 minutes, 50 seconds. Several in the class at that time, myself included, were keen fun runners and some even tackle marathons. An article appeared in the press speculating that we may soon see a Two Hour Marathon. This stimulated lively debate. The class members decided to analyse data contained in the article showing world best times for the period 1953–88, the era of the modern marathon. What trends? They are getting faster! Could we find a formula to predict when the task might be achieved? Using simple curve fitting techniques, applicable to upper secondary level Mathematics classes, three predictive formulae were found. A straight line, $T = 406.4 - 2.924t$, predicting the year 2005; an exponential fit, $T = 2.120 \times 10^{12} \times 10^{-0.0843t}$, predicting 2110; and a logarithmic fit, $T = 4.503 \times 10^{55} \times t^{-26.68}$, predicting 2130. At that stage all "models" seemed, in the view of the class, to be "equally likely." They did decide that the event was still many years in the future and probably beyond their lifetime. (In the above, T = years after 1950; t = marathon time in minutes.)

The discussion then drifted onto the nature of time and its measurement. Why are we chasing a two hour marathon? What if different time units were used? Decimal time? Daniel Boorstin's book, *The Discoverers*, was mentioned by one of the class members, generating more discussion on the nature of time and its measurement.

The point that is made here concerns the fact that the described study was enjoyed by mature age students who were practically innumerate and anxious toward Mathematics only a few months beforehand. In the above case they were able to use and apparently understand fairly advanced Mathematical concepts – exponentials, logarithms, curves of best fit and so on. The calculator was an invaluable aid in lessening levels of anxiety and made the above study possible. Teaching adults or any students efficiently to use a calculator results in an enhanced level of mathematical fluency and understanding. This must be acknowledged also concerning the mathematical progress of these classes.

In general, over an extended period there has been a very positive response by students to the outlined approach. The students said that in retrospect they would have preferred an injection of "history" into their school courses. Many teachers claim that they do not have sufficient time available ... not on the syllabus, tests, exams, pressure, etc., etc. I believe that it is a grave mistake and error of strategy to attempt to teach Mathematics without reference to its cultural, social, philosophical, and historical background.

In school, students are most frequently given a sterile, polished, and clinical form of Mathematics, with no impression of the feeling, pain, and anguish connected with its development. Even the best have had to struggle and have made mistakes. The Mathematics of the textbook is not the Mathematics of history. The two perspectives need to be complementary. Mathematics should be taught as an exciting, dynamic part of human culture and all students should be allowed to share in its richness. An historical approach is a very good way of achieving this. Current approaches are very successful in teaching students to "hate" mathematics. I believe that using history as a matrix in which to embed content is a commendable way of avoiding this. I suggest that more should try it and enjoy the outcomes.

Furthermore it is suggested that history is an excellent medium through which to bring the "lost and disenchanted" and those needing remedial help back to Mathematics.

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From a talk originally presented at the H.P.M. session at ICME-6 in Budapest, August, 1988.

History in the Mathematics Classroom

The new book, *History in the Mathematics Classroom*, edited by John Fauvel (Open University), is a collection of essays translated from the French which deal explicitly with how one actually uses history in teaching mathematics at various levels, in particular, with how one uses historical materials in class. The essays were all written by members of an IREM – Institut de Recherche sur l'Enseignement des Mathématiques. these institutes were set up in the early 1970s for research into mathematics teaching in France. The French version of the book was originally put together by Evelyne Barbin, coordinator of the Inter-IREM Commission on the History of Mathematics and Epistemology.

From the Foreword, by Evelyne Barbin: "Today the new senior secondary school syllabus in France has encourged teachers to stress the cultural content of mathematics, and to introduce a historical perspective which, it is argued, helps pupils to grasp the meaning and scope of the topics they study, and to understand better how

science progresses. But why introduce a historical perspective, and how? This collection of articles brings some answers to both questions.... The different experiences reported here indicate the variety of resources a mathematics teacher can find in the history of the discipline, at all levels of teaching. It is important to recognise that the intention is not to teach the history of mathematics—nor even to mention historical dates for their own sake—but to *integrate the history of mathematics in one's teaching*.... It is true that the introduction of a historical perspective in teaching calls for some specific additional knowledge, but the work undertaken by the teacher also creates a personal enrichment to be shared with pupils and which can be the subject of enthralling research."

From the Editor's Introduction by John Fauvel: "An initial assumption about the use of history of mathematics may be that the teacher tells anecdotes about past mathematicians, to create a lively human atmosphere in the mathematics classroom which facilitates further teaching. There is undoubtedly a place for this approach. But the teachers who wrote the present chapters have been exploring deeper use of selected historical texts, working through them with pupils across a range of ages and levels."

The first article, "Teaching sixth form [age 17] mathematics with a historical perspective," by Jean-Pierre Friedelmeyer (Lycée Couffignal, Strasbourg) summarizes an entire year's worth of historical perspective introduced in dealing with the solution of equations, complex numbers, logarithms, integral calculus, the construction of regular polygons, and the conics. Henry Plane (Lycée Jacques-Amyot, Auxerre), in "To derive or not to derive," discusses his reading of a historic text with his sixth form class, *Rules for finding tangents to curved lines*, by Christiaan Huygens (1667). In another article, "Translating into algebra," he describes how he showed his students the methods various mathematicians of the past, including Pacioli, Lacroix, and Newton, used to translate problems into algebraic symbolic language. Simi-

larly, Marie-Françoise Jozeau (Lycée Gérard de Nerval, Luzarches) dealt with various texts of Fermat, Huygens, Clairaut, and Newton in her fifth form class in studying "A historical approach to maximum and minimum problems." Martine Bühler (Lycée Flora-Tristan, Noisy-le-Grand) read Archimedes' *Measurement of a Circle* with classes at both the sixth and fourth form level.

Jacky Sip (Collége Robert-Desnos, Masny) described her discussions of various historical conceptions of negative numbers with her class of 13-14 year olds in " 'But everybody accepts this explanation': operations on signed numbers." Xavier Lefort (IUT, Saint-Nazaire) showed how to use history in teaching adults, in particular to change their vision of mathematics in his article, "History of mathematics in adult continuing education." Maryvonne Hallez (Collège Paul-Bert, Paris) explored 17th century mathematics with students in both the third and fourth forms in her school located on the *rue Huygens*: "Teaching Huygens in the rue Huygens: introducing the history of 17th-century mathematics in a junior secondary school." Finally, Yvette Horain (Lycée Watteau, Valenciennes) used the resources of the nearby Benedictine Abbey, which from the seventh century had played an active role in the history of art, science, and the humanities in France, in developing "Polygonal areas: a historical project" with classes at the fourth, fifth, and sixth form levels.

The beauty of the essays in *History in the Mathematics Classroom* lies in the fact that specifics of teaching are dealt with, rather than generalities. Each author discusses not only the historical material he or she used, but also how it was adapted to meet the needs of the class and how the class reacted to the material. John Fauvel in fact provides with the book a folder of the original source materials mentioned in the book. in a form suitable for making transparencies or multiple copies for distribution. (All of the original materials have been translated into English.) Anyone who is considering using historical materials in a class should obtain a copy

of this book to see how this has been successfully done in France. The book was published by The Mathematical Association (Great Britain) and is available from QED Books, 1 Straylands Grove, York, YO3 0EB, U.K. for seven and one-half pounds sterling. QED Books accepts charge orders from the United States. The dollar price, with shipping, should be about $17.

Creativity, Thought and Mathematical Proof

The Spring, 1990 issue of *Interchange*, a quarterly review of issues and trends in education published by The Ontario Institute for Studies in Education, was a special issue devoted to "Creativity, Thought and Mathematical Proof." Two of the articles had a historical flavor. First, Israel Kleiner (York University, Toronto) and Nitsa Movshovitz-Hadar (Israel Institute of Technology, Haifa) collaborated on "Aspects of the Pluralistic Nature of Mathematics." They showed by use of various historical examples how the notion of proof in mathematics has changed through the centuries, considering even the recent ideas of proof by computer and probabilistic proof. Second, J. Lennart Berggren (Simon Fraser University, Vancouver) analyzed the mathematical practice of certain Islamic scholars in "Proof, Pedagogy, and the Practice of Mathematics in Medieval Islam." In particular, he deals with the discussion and use of the binomial theorem in the works of al-Karaji and al-Samaw'al and the derivation of the volume of a segment of a paraboloid by Thabit b. Qurra and Abu Sahl al-Kuhi.

This special issue can be obtained for $12.50 (Canadian) from Publications Sales, O.I.S.E., 252 Bloor Street West, Toronto, Ontario M5S 1V6, CANADA. A subscription to *Interchange* can be entered for $35 (Canadian).

REPÈRES I.R.E.M.

Publiée   par   les   Instituts universitaires de Recherches sur l'Enseignement des Mathématiques, la revue *Repères I.R.E.M.*

est un bulletin trimestriel s'adressant à tous les professeurs, enseignants des Collèges, des Lycées, des Lycées Professionnels et des Universités. Son but est de tenir informé aux questions actuelles en didactique des mathématiques, qu'elles aient trait aux grands débats du moment ou plus simplement aux applications concrètes, pour les classes, de la réflexion menée en commun entre practiciens et chercheurs. Elle est donc destinée à devenir un outil indispensable aussi bien aux professeurs de mathématiques qu'aux formateurs spécialisés; ainsi qu'à tous ceux concernés par la pédagogie ou les Sciences de l'Education.

Cette revue comportera réguliérement des articles consacrés à l'histoire des mathématiques et à l'introduction d'une perspective historique dan l'enseignement des mathématiques.

Au sommaire du no. 1: J. Houdebine, "Démontrer ou ne pas démontrer, voilà la question;" A. Bodin, "Les nouveaux programmes de 6ème et de 5ème;" R. Noirfalise, "Aider les élèves à initialiser une procédure de résolution d'un problème;" J. C. Chabert, "Les géométries non euclidiennes;" R. Bkouche, "Enseigner la géométrie, pourquoi?" (110 pages)

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Les propositions d'articles sont à faire parvenir à Evelyne Barbin, IREM Université du Maine, Route de Laval, 72017 Le Mans cedex, FRANCE.

Have You Read?

Ronald Calinger, ed.

This column welcomes references from across the history or pedagogy of mathematics, as well as other works with sections that have potential for encouraging and motivating students to learn mathematics better or that enrich courses. Please send citations with complete bibliographic information to the section editor c/o Department of History, Catholic University of America, Washington, D.C. 20064, U.S.A.

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