International Study Group on the Relations Between HISTORY and PEDAGOGY of MATHEMATICS NEWSLETTER

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Calendar

Meetings with HPM components are highlighted.

- 1990 April 7–9 ..... Leicester
  British Society for the History of Mathematics conference/workshop: The Uses of History in Mathematics Education. Contact Dr. Steve Russ, HIMED 90, Department of Computer Science, University of Warwick, Coventry CV4 7AL U.K. (See inside for program information.)

- 1990 April 17–21 ..... Salt Lake City
  Annual Meeting of the National Council of Teachers of Mathematics and the Americas Section of HPM. (See inside for further information.)

- 1990 May 31–June 1 ..... Victoria
  Annual meeting of the Canadian Society for History and Philosophy of Mathematics. For information about the society, contact Professor A. Malik, Department of Mathematics, Concordia University, 7141 Sherbrooke St. W., Montreal, Quebec H4B 1R6, CANADA. (See inside for further details.)

- 1990 June 14–16 .... Bandar Seri Begawan
  Fifth South East Asian Conference on Mathematical Education (SEACME 5). The conference theme is the "Enchantment of Mathematics". Contact Dr. David Daniels, Secretary, SEACME 5, Universiti Brunei Darussalam, Gadong, BSB 3186, BRUNEI DARUSSALAM.

- 1990 June 26–28 ..... Campinas
  HPM Symposium on Using History in the Teaching of Mathematics. Contact Professor Itala L. D'Ottaviano, Director, CLE, UNICAMP, 13081 Campinas, SP, BRAZIL. (See inside for further details.)

- 1990 July 17–20 ..... Oxford
  Conference on the Scientific Revolution organized by the British Society for the History of Science. Contact Wing Commander Bennett, BSHS, 31 High St., Stanford in the Vale, Faringdon, Oxon, SN7 6LH U.K.

- 1990 August 8–11 ..... Columbus
  Summer meeting of the American Mathematical Society and the Mathematical Association of America. There will be events commemorating the 75th anniversary of the Association. Contact MAA, 1529 Eighteenth St., NW, Washington, DC 20036 U.S.A.

- 1990 August 21–29 ..... Kyoto
  The International Congress of Mathematicians. Contact ICM–90 Secretariat, Research Institute for Mathematical Sciences, Kyoto University, Kitashirakawa, Sakyo-ku, Kyoto 606, JAPAN. There will be an HPM session on August 29. Details will be in the next newsletter.

- 1990 August 31– September 1 ..... Tokyo
  Tokyo History of Mathematics Symposium 1990. The topics of this meeting include the History of Modern Mathematics, Mathematical Traditions in the East, and the Interaction between Mathematical Researches and the History of Mathematics. Contact Professor Chikara Sasaki, Department of History and Philosophy of Science, College of Arts and Sciences, University of Tokyo, Komaba, Meguro-Ku, Tokyo 153, JAPAN.

- 1990 September 18–20 ..... Cambridge
  Joint meeting of the British Society for the History of Mathematics with the Thomas Harriot Seminar. Contact Dr. R. W. Bray,

Department of Mathematics, University of Essex, Wivenhoe Park, Cochester CO4 3SQ, U.K.

- 1991 August 3–7 ..... Coral Gables
  Eighth Interamerican Conference on Mathematics Education. Contact Patrick Scott, Programas Latinoamericanos de Educación, Faculty of Education, University of New Mexico, Albuquerque, NM 87131 U.S.A.

- 1991 September ..... Cambridge
  A meeting on Renaissance Mathematics cosponsored by the British Society for the History of Mathematics. Contact R. W. Bray at address above.

- 1992 August ..... Quebec
  Seventh International Congress on Mathematics Education (ICME 7). This meeting will be preceded by an international HPM meeting in Toronto. Details will be announced later.

From the Chair

Florence Fasanelli

While browsing in the book exhibitions at recent AMS, MAA, and NCTM meetings in 1989 and 1990, I spoke with the editors about their new publications which contain notes on the history of mathematics. The notes appear in a variety of formats. Some notes are added to previously developed texts either as separate highlights, as footnotes, or as end of chapter notes. Some thoughtful texts have incorporated the notes into the "required reading" as a motivational device for both the student and teacher.

Virtually all the publishers say they are doing this more and more, either with outside consultants or with the author making the additions. In a few cases, unfortunately, historical myths are perpetuated in these notes. For the most part, though, the notes do provide glimpses of the latest historical scholarship in the area to be covered. It is a major step forward to have these historical notes, especially in the American context where the books tend to determine the curricula. Books with these notes are moving in the direction of the NCTM standards which suggest that the learning of mathematics should not be separated from its historical and cultural context.

Recent books whose latest editions contain historical notes include the following. (This list is by no means exhaustive. Readers are encouraged to submit names of other such books.)

- *Differential Equations with Boundary Value Problems*, by Dennis Zill. Notes by Zill. (PWS-Kent)
- *Calculus*, by Howard Anton. Notes by Valarie Hunter. (Wiley)
- *College Algebra*, by Michael Sullivan. Notes, including comments and problems, by Sullivan. (Dellen)
- *After Calculus: Algebra*, by David Foulis and Mustafa Munem. Notes by William Schutz. (Dellen)
- *After Calculus: Analysis*, by David Foulis and Mustafa Munem. Notes, including annotated bibliographies, by Gerald Alexanderson and Al Bednarek. (Dellen)
- *Calculus and Analytic Geometry*, by Lynn Garner. Notes by William Schutz. (Dellen)
- *Calculus*, by Dennis Berkey. Lengthy notes by Duane Deal. (Saunders)
- *Algebra and Trigonometry, College Algebra, Trigonometry, Precalculus*, by Stanley Grossman. Notes by Grossman. (Saunders)
- *Fundamentals of College Algebra*, by Charles Miller, Margaret Lial, and David Schneider. Notes by Florence Fasanelli. (Scott, Foresman - Little, Brown)
- *Linear Algebra*, by John Fraleigh and Ray Beauregard. Notes by Victor Katz. (Addison-Wesley)

- *A First Course in Abstract Algebra*, by John Fraleigh. Notes by Victor Katz. (Addison-Wesley)
- *Calculus with Analytic Geometry*, by John Fraleigh. Notes by Victor Katz. (Addison-Wesley)
- *Elementary Number Theory*, by David Burton. Notes by Burton include a two-page time line of mathematicians important in number theory, lengthy biographies, background information about problems, and bibliography. (Brown)
- *Abstract Algebra*, by David Burton. Notes by Burton include biographies, pictures, bibliography. (Brown)

To Be Or Not To Be

V. Frederick Rickey

Having just returned from the annual joint meetings of the American Mathematical Society and the Mathematical Association of America, I was most impressed with the considerable amount of effort being devoted to pedagogical issues by the community of mathematicians.

- Calculus reform was the biggest single item. Everyone is talking about how our lumpish and listless calculus courses can be transformed into lean and lively ones.
- The NCTM *Standards* were discussed, because some in the university community realize that a vital component in the successful implementation of those standards is a major overhaul of the curriculum that prospective teachers study.
- The use of computers and writing in the classroom were widely discussed. All agree that it is a good idea, so the discussion is now down to specifics.
- The history sessions, which are described elsewhere in this newsletter, were extremely well attended and there is lots of interest. Most of the talks dealt with curent research in history rather than its pedagogical applications, but there was plenty of interest in it in the informal discussions.

It is my firm belief that in all of these areas of acticvity, there is a significant place for history. When suggestions are made for improving what is taught and how it is taught, we need to show that there is a place for history. We need to argue, not by philosophical principles, but by concrete examples, that history can play a significant role in the reform of mathematics education at all levels.

If your students have trouble completing the square, show them the diagram of Al-Khwarizmi. If they dislike notation, show them some propositions from Euclid, and challenge them to interpret them. If they can't stand abstraction, explain to them that it was only after trigonometry became abstract and started to deal with functions, that it became useful in real world applications.

We, the members of HPM, must get involved. HPM is the community of mathematicians that are interested in using history in the classroom to motivate, to inspire, and to teach our students. There is no choice. If we do not get involved, then others won't either, and the reform movement will not be as strong as it could be.

What can we do? First, we need to get involved in our own classrooms. What topics do we have the most trouble teaching? Can you think of a way that history would help? Some biography? A picture? An example from an old text? Try out your ideas, but don't stop there, share them with a colleague. Talk about how they could be improved.

Second, go to meetings. Get involved with your colleagues from other schools. Listen to talks, learn all that you can, ask questions.

Finally, and most importantly, share your ideas. Explain how you used a historical example in the classroom, and then persuade a colleague to try it. Convince a colleague that having students write about famous mathematicians will

improve both their writing skills and their interest in mathematics. Then, after you have honed your ideas, give a talk at a local meeting, a state meeting, or a national meeting. When this meets with success, write up your ideas for publication. Editors and publishers are clamoring for historical items. We should accommodate them!

The history of mathematics can be a positive force in the reform of mathematical education at all levels. We can contribute. We must contribute.

From the Editor

Victor J. Katz

For many years, this newsletter has been supported through the generosity of various educational institutions. The University of the District of Columbia is still willing to do so. However, as many of you have probably noticed, third class mail, the only class the university is willing to pay for – at 8.5 cents per piece – leaves much to be desired. As an example, the last newsletter took $2\frac{1}{2}$ weeks to reach my home in the suburbs from the post office in Washington, a distance of perhaps 10 miles. To improve the mail delivery, therefore, and insure that U. S. readers get notices of meetings in time to make plans, it has been decided to solicit contributions from those on the mailing list who want the newsletter by first class mail. (This request is made only of those in the United States; those in other countries in general already receive the newsletter sooner than those in the U.S.) If you send a check for $ 5 to the editorial office, made payable to H.P.M., your newsletters for the next three years (approximately) will be sent by first class. (The remainder of the charge is to pay for the increased clerical work.) I hope that this method will solve the mailing problem.

With this newsletter, we have instituted a change in the distribution system for countries in Europe, Asia, and Africa. Instead of mailing these from one central source, a representative has been found in each country or small group of countries who will handle the distribution. These representatives are listed on the front page. Please contact them if you have an address change or to request being added to the mailing list. As you will note, we still need representatives from Eastern Europe, including the Soviet Union, and from most countries in Africa and Asia. If you would like to volunteer, please contact me. There is very little work involved and you get to contact all sorts of interesting people who are interested in history and pedagogy of mathematics.

As I noted in the last newsletter, I need written contributions from all of you to make this venture a success. In particular, I would like reports on the various meetings listed in the calendar from those of you who are able to attend. Did any of you attend the First International Conference on the History and Philosophy of Science in Science Teaching which was held in Tallahassee in November? If so, and if there was anything of interest about the history of mathematics at that meeting, I would appreciate a brief report. Similarly, if someone would report on activities of interest at the meetings in Japan in August, that material would appear in a newsletter in the fall. And again, reports on how you use historical material in your classes are always of interest as would be a brief review of any of the books or articles mentioned in Have You Read.

HIMED 90

The British Society for the History of Mathematics has organized a History in Mathematics Education conference which will be held in Beaumont Hall at the University of Leicester in Leicester, England on April 7–9, 1990. The program begins on Saturday afternoon, April 7 with a talk by John Fauvel (Open University, England) on *Using history in mathematics education*, followed by several contributers answering the question, *How has the history of mathematics mattered to me in my mathematics teaching?* After tea, Ivan T. Jakobsen (Statsgymnasiet, Aarhus, Denmark), Maryvonne Hallez (Collège Paul- Bert, Paris, France), and Jan van Maanen

(Christelijk Gymnasium, Utrecht, Netherlands) will speak on *Teaching mathematics with a historical perspective*. David Singmaster (Polytechnic of the South Bank, London) will conclude the day's program with a presentation of *Recreational problems down the ages*.

On Sunday, April 8, George Ghevarghese Joseph (University of Manchester, England) will deal with *History in the multicultural classroom*, Marylynne Lolley (Sydenham School, London) with *Learning mathematics through herstory*, and Yannis Thomaidis (Lycée Ilioupolis Thessaloniki, Greece) with *Historical diversions in Greek geometry lessons*. After lunch, Lutz Führer (Albert Einstein Gymnasium, Hameln-Afferde, Germany) will speak on *Historical stories in the mathematics classroom* and Ron Hilfer (Tabor Community School, Lower Galilee, Israel) will deal with *Historical happenings in the mathematics classroom*. During the morning and afternoon, there will also be time for two sets of workshops which will explore in practice what can be done in the classroom. On the final morning, Monday, April 9, Leo Rogers (Roehampton Institute, England) will discuss *Resources: what is available?* and Neil Bibby (University of Exeter, England) will conclude with a presentation of *Mathematics, history and the National Curriculum: what can we learn from the European experience?*

Anyone who is not registered and who would like to attend should contact Dr. Steve Russ at the address given in the Calendar or telephone him at 0203 523681 as soon as possible. A full report on what promises to be a very stimulating meeting will appear in the next newsletter.

Annual Meeting of HPM

The annual meeting of the Americas section of HPM will be on April 17–18 in Salt Lake City, just prior to the annual meeting of the NCTM. On April 17 the meeting will take place from 9:00 am to 5:00 pm in Canyon Room # 3 of the Red Lion Hotel. On April 18, the group will continue the tradition of meeting at a museum by gathering at the Pioneer Memorial Museum, 300 N. Main, from 9:00 am to 12:00 noon. The program for the meeting is not yet finalized, but there will be talks by Ubiratan D'Ambrosio, Alex Garciadiego, and Steve Heath, among others. There will also be a session discussing the ways of teaching a course in the history of mathematics. This special session will be led by William Campbell of the University of Wisconsin, Platteville. All who are interested in participating should bring syllabi and class materials in multiple copies so they may be shared. Anyone wanting to give a talk on any aspect of the relationship of history and pedagogy of mathematics should communicate promptly with Erica Voolich, 244 Summer St., Somerville, MA 02143 or by phone at (617) 666-0666. You may also contact V. Frederick Rickey at (914) 938-2559 (office), (914) 938-2409 (messages) or (914) 446-2009 (home).

If you have never been to an HPM meeting, you are especially encouraged to participate. Usually the meetings are small (between twenty and thirty), there is a mix of teachers from all levels, the group is very friendly, the talks are informative, and the discussion is lively. Naturally we go out to dinner together to continue the informal discussion. In conclusion, the message is simple: Come to Salt Lake. Get involved in HPM. Share your ideas.

Canadian Society for History and Philosophy of Mathematics

The annual meeting of the CSHPM will be held May 31–June 1, 1990 at the University of Victoria in Victoria, British Columbia. There will be a special session on history and pedagogy of mathematics organized by Victor Katz. The keynote speaker will be Judy Grabiner (Pitzer College, Claremont, CA) whose topic is *Was Newton's Calculus a Dead End? The Question of the Continental Influence of Colin Maclaurin*. Other speakers at that session include Erica Voolich (Wheelock College, Boston, MA) on *Mathematics: A Multicultural and Historical Approach in the Elementary Classroom*, Is-

rael Kleiner (York University, Toronto, Ontario) on *Themes in the Evolution of Number Systems*, Sam Kutler (St. John's College, Annapolis, MD) on *Why Study Ancient Mathematics?*, Charles Jones (Ball State University, Muncie, IN) on *The Beginnings of the New Math Movement: The Ball State Program*, and Victor Katz (University of the District of Columbia, Washington, DC) on *Non-Western Mathematics in the College Classroom.*

The remainder of the program is being organized by Francine Abeles (Kean College of New Jersey, Union, NJ); abstracts should have been sent to her by February 28.

Membership in the Canadian Society is open to all and costs $ 15 Canadian or $ 11 U.S. To join, send your name and address with the appropriate remittance to Professor M. Malik, Department of Mathematics, Concordia University, Montreal, Quebec H3G 1M8, CANADA.

HPM Conference in Campinas

HPM, together with the Center of Logic, Epistemology and History of Science (CLE) at the State University of Campinas (UNICAMP), near Sao Paulo, Brazil, is organizing a Symposium on "Using History in the Teaching of Mathematics." The event will take place in Campinas on 26, 27, and 28 June, 1990. Campinas is the second largest city in the State of Sao Paulo (about 1,000,000 inhabitants), about 50 miles northwest of the city of Sao Paulo. Participants may want to stay for the 42nd Annual Meeting of the Brazilian Society for the Advancement of Science which will take place in Southern Brazil in the second week of July. This is the largest regular scientific meeting in Latin America, with about 5,000 participants. It should be noted that air fares to Brazil are reduced this part of the year. Although it is winter, the weather in Campinas is mild, making the season very attractive from the tourist viewpoint. Interested individuals should consult VARIG, the Brazilian airline, or PAN AM, or CANADIAN, or most European and Latin American airlines, for fares and information on tourist packages. The CLE will provide accommodations and meals for the three days of the conference. Those wishing to attend should send an abstract of the paper they intend to present to Professor Itala L. D'Ottaviano, Director, CLE, UNICAMP, 13081 Campinas, SP, BRAZIL or to Professor Ubiratan D'Ambrosio at the address on the first page.

Mathematics on the Mall

Mathematics on the Mall is a tour and a tour book written and organized by Florence Fasanelli, V. Frederick Rickey, and Richard Thorington to provide the reader with a mathematical introduction to the Mall in Washington, DC. The purpose of the tour and the book is to convince the reader "that mathematics permeates the world around us." As the authors write, "our purpose here is to convince you. We will point out many little mathematical things. Some are very significant, but most are trivial. We do hope to sensitize you, so that you too can see math, not just on the mall, but wherever you go." The book provides a self-guided tour with five stops on the Mall. At each stop, the mathematics in the various buildings visited is explored, from both a historical and a technical viewpoint. If you want to bring a group to Washington and take the tour under the guidance of Florence Fasanelli, contact The Reading Center, George Washington University, Washington, DC 20052 or call (202) 676-5189. Copies of the tour book may be obtained at cost from the same address if you wish to take the tour yourself or if you wish to see how to design a similar tour in your own home town.

Why do we use 'm' for Slope?

V. Frederick Rickey

The question teachers most frequently ask historians is "Why do we use 'm' for slope?" One might first guess that Descartes or Fermat, the coinventors of Analytic Geometry, introduced the notation. But Descartes did not consider

equations of lines at all, and Fermat treated lines in terms of proportions, not equations.

The point-slope equation of a line first appeared in a paper of Monge in 1784, so it is doubtful that 'm' was used earlier. Amazingly, the earliest use of the equation "$y = mx + b$" *that we have located* is in *A Treatise on Analytic Geometry*, 1866, by Howison. The earliest use of the word "slope" *that we have located* is in *Elements of Analytic Geometry*, by Church. [The *Oxford English Dictionary* cites an 1889 Physics text as using the word, but we have not seen a copy.]

A plea for help: Please check your library for nineteenth century works dealing with lines. Try to locate a use of 'm' before 1866, or a use of "slope" before 1897. If you find an example, please photocopy the title page and the relevant pages and send them to V. Frederick Rickey, Department of Mathematical Sciences, United States Military Academy, West Point, NY 10996 USA.

Reform In American Education

Charles V. Jones, Ball State University, Muncie, IN 47306, USA

American education – by which we mean education in the United States but not in Canada, Mexico or any other country in the Americas – is undergoing a self-appraisal and evaluation. Several recent studies and reports conducted by public and private commissions have been very critical of the quality of education that American students are receiving, and mathematics education in particular has been singled out for a lot of attention. Beginning with *A Nation At Risk*[1] in 1983, and continuing on through the most recent report from the National Research Foundation, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*[2], criticism has been leveled at both the content and pedagogy of mathematics. This reform movement, if it runs its course, will have a profound impact on the teaching of mathematics in the United States and, if successful, will raise the mathematical performance of school children and direct more people into mathematics as a profession.

So far this movement has been long on criticism and short on action. However, the National Council of Teachers of Mathematics – the premier American organization for primary and secondary mathematics education (kindergarten through grade 12) – has moved boldly to establish new standards of education. In the just-released document, *Curriculum and Evaluation Standards For School Mathematics*[3], the NCTM has called for both a shift in educational philosophy and changes in curriculum that address the concerns voiced in several of the reform reports. Many non-American HPM readers would be surprised to see some of the apparently necessary changes – some of the recommended changes have been in place for quite some time in other countries – and many more readers would find the *Standards* a fascinating insight into the state of American education.

Why this deterioration of American education has occurred is not widely agreed upon. However, I think an understanding of how it came about must take into account the American political system. First, and foremost, public education in America – which means education paid for by taxes and free to all residents – is an extremely decentralized social institution in which citizens in every town and city exercise control through democratically chosen, local boards of education. Over the past decade or so, in my opinion, American politics in general has shown a great deal more xenophobia and political jingoism, and this has given rise to a diminished interest in other cultures and a general shallowness in political thought. Elected members of boards of education reflect this general public attitude and, therefore, tend to be complacent and fail to respond to a changing world. For good or ill, public education in America mirrors the popular state of mind.

Second, education is not the responsibility of the federal government seated in Washington, D.C.; it is the responsibility of each of the fifty individual states. Still, there are national agen-

das in education. The federal government, with its troika-like executive, legislative and judicial branches, has established its own bureaucracies of education, the National Science Foundation being perhaps the most easily recognized outside the United States. By granting funds with conditions attached, the federal government influences the direction of education. For example, the above report, *A Nation At Risk*, is the product of one of these federal agencies and it is exercising considerable influence on the direction of education.

Through agencies of our federal government, we Americans have identified the symptoms, if not the causes, of our declining educational system. It hardly needs to be said that America is among the most technologically advanced and science-dependent countries. So the issues surrounding mathematics education are seen in the context of nothing less than national security – a term which now generally refers to economic security rather than military (although no American politician could long survive in office if he or she ignored national defense). What then should be the response?

One sees something of the American character in the response to this crisis in education. The NCTM *Standards* are explicitly founded on democratic ideals and economic needs. The responsible citizen must be educated to make decisions in a democratic society – a theme in American history from Thomas Jefferson through John Dewey and on into the present – and a democratic society promotes economic well-being. So the American education system must prepare its citizens with adequate mathematical training to deal more and more with economic and scientific decisions. The explicitly stated implication here is that mathematics is for all students, not just a few; there must be universal access to mathematics. If there isn't, then economic and scientific development will be for just a few privileged individuals, which is not the American image of democracy.

This goal of the *Standards* marks a basic shift in pedagogy away from a system that permits both the unprepared student and his or her teacher to avoid dealing with new concepts. The *Standards* advocates exposing all students to the same basic topics, while tailoring the presentation to the students' capacity. The *Standards* recommends changing both what is taught and how it is taught. The American student will cultivate an appreciation of the importance of mathematics in culture, and will learn mathematics by first being introduced to problem situations and then secondly developing computational skills.

Here is where history can come into the picture. Historical anecdotes have always been in the teacher's arsenal, but to show the cultural significance of mathematics requires going somewhat deeper into the story. In addition, the history of mathematics can provide problem situations for concepts before all the machinery of notation and procedures is introduced – just as the *Standards* recommends. Further, the *Standards* advocates a lot more use of reading and writing in the mathematics classroom. And what better place to go for materials than to the history of mathematics. All of this adds up to a very challenging opportunity for using history in the teaching of mathematics.

There is no ready resource for this kind of history. However, the writers of the *Standards* provided a couple of good examples of what might be done. In the discussion of problem solving, the problem of dividing the stakes in an interrupted game of chance is analyzed and solved. This is a historical problem but the treatment is not a slave to the history. In fact, it is a very easily understood lesson that conveys genuine mathematical content while illustrating how a practical problem can generate new mathematics. In this particular example, no new notation is introduced. The discussion begins with a reference to the history of the problem and to Pascal and Fermat.

History is only one of several approaches exemplified in the *Standards*. However, to use history in a fashion suggested in the *Standards* requires developing historical modules, writing a

different kind of history of mathematics book, and utilizing history in textbooks at a more profound level than biographical or historical vignettes. These are all activities that the HPM in the United States and Canada, in particular, has promoted – although not with very remarkable results. With the publication and adoption of the *Standards* I would hope that we would see – at long last in America – more writing of historical materials suitable for use directly in the mathematics classroom in the spirit of the *Standards*. It is clearly the time for the Americas Section of HPM to take a leading role in the American education reform movement.

1. National Commission on Excellence in Education. *A Nation At Risk: The Imperative for Educational Reform.* Washington, DC, U.S. Government Printing Office, 1983.

2. National Research Council. *Everybody Counts: A Report to the Nation on the Future of Mathematics Education.* Washington, DC, National Academy Press, 1989.

3. National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA, The National Council of Teachers of Mathematics, Inc., 1989.

From a talk given at the 1989 HPM meeting in Orlando.

The Drama of Muthematics

Joan Countryman, Germantown Friends School, 31 W. Coulter St., Philadelphia, PA 19144, USA

Writing in the *New York Times Book Review* about the Watson and Crick books on the discovery of the structure of DNA, George Johnson, an editor of *The Week in Review* of *The New York Times*, states that there are "two ways to write about science. A writer can focus on the science, as though it were received wisdom, like a Coke bottle that fell from the sky, or on the scientists, laying bare the emotions and drives that are part of the haphazard process of discovery." The problem for teachers is that if we want our students to become thinkers and knowers, to construct their own knowledge, then that dichotomy, science on the one hand, scientists on the other, though common, is not particularly useful. Students need to see that people *do* mathematics (and biology and physics and chemistry) and that they do not pluck theorems out of the sky, receiving wisdom like anointed sages.

As Florence Fasanelli points out in the HPM Winter 1989 newsletter, the role of this organization is "to use history to enrich the curriculum, to join the learning of mathematics to its historical and cultural background in order that students value the discipline as a cultural achievement which is at all times part of their lives." The challenge is that we must look at the history *and* look for ways of conveying that history to a wide range of students at all levels of schooling.

Two years ago I read a piece in *Mathematics Magazine* entitled "Presenting a Mathematics Play." David Conibear and John Poland from Carleton University in Ottawa, Canada there suggest that Imre Lakatos' *Proofs and Refutations* makes good dramatic reading – his ideas, his wit, and his interest in the history of mathematics combining to make portions of the text worth staging. In *Proofs and Refutations* Lakatos confronts the classical picture of mathematical development as a steady accumulation of established truths [i.e., Coke bottles], showing that mathematics grew instead through a richer, more dramatic process of successive improvements of creative hypotheses, by attempts to prove them, and by criticism of those attempts.

As it happened, my 12th grade calculus class that year was filled with budding thespians; so, I thought, why not close out the year with a reading of the first four sections of *Proofs and Refutations*? This paper is a brief description of what happened, how the students responded, and why I think that drama should be added to the list of activities for the mathematics classroom.

As you know, *Proofs and Refutations* ad-

dresses the questions: what is the nature of mathematics? how is it created? what is its [real] history? how should it be taught? Lakatos described the work as "an approach to some problems of the methodology of mathematics." Objecting to the tendency of formalists to disconnect history, philosophy, and practice, he saw formalism as leading to two views of mathematics, neither one acceptable. On the one hand, there are right answers to be discovered algorithmically; on the other hand is the discovery of theorems, guided by insight, luck and guessing. But, according to Lakatos, real mathematics is neither mechanical nor irrational. It grows by the improvement of guesses, by speculation and criticism, by the logic of proofs and refutations.

The dialogue is meant to contain a reconstructed history of mathematics. It takes place in an imaginary clasroom, where the class gets interested in a problem: Is there a relation between the number of vertices, edges and faces of solids, particularly regular solids, similar to the relation between the number of vertices and edges of polygons? After trial and error they notices that for all regular polyhedra $V - E + F = 2$. The dialogue begins with the teacher offering a proof.

Our work, however, began much earlier, with some simple library research. Each student was asked to track down a person, or concept, that appeared somewhere in the text of the dialogue: Euler, Descartes, Euclid's Proposition 17 in Book VII, Poincaré, Hardy, induction, analysis. They were to try to become "expert" enough on the subject to explain it to the rest of us when it appeared as we read the text.

The text, about forty pages, was distributed, and students were asked to read it with the caveat that they were not to expect that they would make much sense of it on the first reading. Initially, in class reading was very informal – parts were interchanged, and there were numerous interruptions, questions, and comments. The "experts" attempted to explain who Leibniz was, or Euler, or what a lemma was. Eventually, the class, our class, began to sort themselves into actors, stage managers, model makers, lighting directors and, with some help from the drama teacher, put on a rough reading, with costumes and props, for members of the other 12th grade math classes and the math department. I'm not sure the audience made much sense of it, but by that time we understood a fair amount.

I know what my students were making of the experience because they kept journals throughout the period and they shared with me in their entries their collective passage through five or six stages:

- I can't wait until the play.
- I don't understand a word of it.
- Well, now I see how that network thing works.
- It's beginning to make sense.
- This is really interesting stuff.
- I'm glad we did the play.

We used overhead projector transparencies and toothpick and gumdrop models and had some heated arguments of our own about the nature of mathematics. A second reading with "nonmathematical" adults (philosophy graduate students at Teachers College in New York) was much briefer but had similar results. [The gumdrops helped.]

Conibear and Poland suggest other mathematical dialogues suitable for dramatic treatment. Don Allen of St. George's School of Montreal and editor of *The Mathematical Log*, the publication of Mu Alpha Theta, mentioned to me in conversation recently that *School Science and Mathematics* used to publish mathematical plays appropriate for classroom use. An hour in the stacks of the Teachers College library turned up a few of these and some other resources, listed below.

1. Conibear, D and Poland, J. 1987 "Presenting a Mathematics Play" *Mathematics Magazine* 60:4.

2. Galileo 1914 *Dialogues Concering Two New Sciences* (Dover). (See also Stillman Drake's translation in dialogue form; also his *Galileo Against the Philosophers*.)
3. Lakatos, I. 1976 *Proofs and Refutations* (Cambridge University Press)
4. Macdonald, Alex 1984 *A 1-Act Play about Euclid* (Polygonal)
5. Plato 1937 *Dialogues* (2 volumes, translated by Jowett) (Random House)
6. Portage 1982 *Geometrical Investigations* (Addison-Wesley)
7. *School Science and Mathematics*
   - Flatland – A Mathematics Play, 14: 583–7
   - A Mathematical Victory, A Play in Two Acts, 17: 475–82
   - Mock Trial of B Versus A, 18: 611–21
   - Euclid Dramatized, 21: 381–2
   - How Much? A Mathematics Playlet, 54: 628–34
   - Variable Paradox: A Dialogue in One Act, 59: 461–4
   - Dramatizing Mathematics, 60: 99–104

(From a talk given at the 1989 HPM meeting in Orlando)

Vakantiecursus 1989

A two day summer course, entitled "Mathematics in the Golden Age" (17th century in the Netherlands), was offered last summer through the Stichting Mathematisch Centrum to science teachers in the Netherlands at two sites, the Technical University of Eindhoven and the Center for Mathematics and Computer Science in Amsterdam. The program attracted 80 participants at Eindhoven on August 17 and 18 and 110 participants in Amsterdam on September 1 and 2. The programs at the two sites were the same. The opening lecture, a Survey of Seventeenth Century Mathematics, was presented by A. W. Grootendorst of the Technical University of Delft. Jan van Maanen, of the University of Utrecht, spoke on The Seventeenth Century in Ten Mathematical Problems from Ten Decades. He then led the group in a practical exercise in reading seventeenth century texts. H. J. M. Bos, also from Utrecht, spoke about Descartes and the Beginnings of Analytic Geometry. He was followed by C. de Pater, of the Institute for the History of Science in Utrecht, who dealt with the Relation between the Sciences and Mathematics in the Seventeenth Century. The final lecture, by J. P. Hogendijk of the University of Utrecht, was on the work of Desargues. The program concluded with a presentation of two of the videos prepared by the mathematics faculty of the Open University in England for their new course in the History of Mathematics.

The program was quite successful in interesting the participants in the history of mathematics. The lectures have been published by the Amsterdam Center for Mathematics and Computer Science, Kruislaan 413, Amsterdam (as C. W. I. syllabus n. 25). More information is available from Jan van Maanen, Department of Mathematics, University of Utrecht, P.O. Box 80.010, 3508 TA Utrecht, THE NETHERLANDS.

AMS Meeting in Louisville

Victor J. Katz

There was a Special Session on the History of Mathematics at the annual meeting of the American Mathematical Society held in Louisville, Kentucky on January 17–20, 1990. The session was organized by Victor J. Katz and Florence Fasanelli. Ten talks were presented, several of which will have special interest to those involved in HPM. Professor Ed Barbeau (Department of Mathematics, University of Toronto, Toronto, Ontario M5S 1A1, CANADA) spoke on the topic *Lagrange Multipliers: A Study in the History of Ideas*. Barbeau noted that Lagrange himself introduced the idea of multipliers through a phys-

ical idea. Namely, he considered a system of particles subject to various forces and, in addition, subject to various physical constraints, and wanted to determine the equilibrium position of the system. With his replacement of each equation of constraint by a force acting normal to the constraint, he was able to determine this equilibrium by minimizing a certain set of functions, a technique which became our modern method of Lagrange multipliers. As Barbeau pointed out, the technique has been applied in various situations over the two hundred years since it was first developed. Thus in the late 19th and early 20th centuries, the technique was used as part of the calculus of variations, from about 1930 it was appropriated by economists, given an economic interpretation, and used in connection with linear programming, and in the period beginning around 1950, it became important in optimization questions involving the tools of convex functions and functional analysis.

In another vein entirely, Professor Marcia Ascher (Department of Mathematics, Ithaca College, Ithaca, NY 14850, USA) gave a talk entitled *Ninety-four years ago and now*, in which she pointed out the pernicious influence over the last century of the 1896 work of L. Conant, *The Number Concept: Its Origin and Development*. The message of that book, with its very structured view of the kinds of mathematical ideas which could be understood by traditional peoples, has permeated even modern histories of mathematics, even though more recent field research has demonstrated that traditional peoples in fact often have very deep and rich mathematical ideas embedded in their own cultures. Ascher demonstrated, for example, that the clan structure of the Warlpiri of Australia includes a fairly explicit understanding of the dihedral group of order 8. Other traditional cultures similarly demonstrate the understanding of various aspects of what are today called group theory and the theory of graphs.

Professor Michael Closs (Department of Mathematics, University of Ottawa, Ottawa, Ontario K1N 6N5, CANADA), in a talk entitled *Mathematics as a Profession in Ancient Mayan Society*, surprised and delighted the audience with his slide show taken from paintings on Mayan ceramic vessels in which he demonstrated that females as well as males served as mathematical specialists in that society. Although there is still much to be done in interpreting the various glyphs appearing in Mayan inscriptions, it is clear that the mathematicians were a distinctive subgroup of the class of scribes and painters and were educated in special schools in which they mastered their craft.

The Special Session included a series of talks on Russian mathematics. Irving Anellis (Iowa State University, Ames, IA 50010) spoke on *The Roots of Mathematics and Mathematics Education in Russia in the Age of Peter the Great*. Charles Ford (Saint Louis University, Saint Louis, MO 63103) discussed *Dmitrii Egorov and the Roots of Modern Soviet Mathematics*. Roger Cooke (University of Vermont, Burlington, VT 05405) dealt with *N. N. Luzin and the Persistent Problems of Set Theory*. And finally, Sergei S. Demidov (Department of the History of Mathematics, Institute for the History of Natural Science and Technology, Academy of Sciences of the USSR, Staropanskii per. 1/5, Moscow K-12, 103012, USSR) gave a fascinating report on *The Moscow School in the 1930's*. He showed how the mathematical school headed by Luzin escaped the fate of biology and managed to survive the various political pressures endemic to Russian society in the decade before the Second World War.

Other speakers at the Special Session included Craig Fraser (IHPST, University of Toronto, Toronto, Ontario M5S 1K7, CANADA) on the *Calculus of Variations and Differential Equations in Lagrange's Mathematics, 1765–1785*, Ubiratan D'Ambrosio on *Algebraic Structure and the Concept of Time*, and Stanley H. Lipson (Kean College of NJ, Union, NJ 07083, USA) on *Lewis Carroll's Encryption Algorithms*.

It is hoped that there will again be a Special Session on the History of Mathematics at the next annual meeting of the AMS in San Francisco

in January, 1991. Suggestions are welcomed for topics for that session.

Have You Read

Readers are asked to send titles to be listed in this column. References from across the history or pedagogy of mathematics are welcome, as well as other works with sections that have potential for encouraging and motivating students to learn mathematics or that enrich courses. Please supply complete bibliographic information to the section editor: Dr. Ronald Calinger, Department of History, Catholic University of America, Washington, DC 20064, USA.

- Abott, David, ed. *The Biographical Dictionary of Scientists* (New York: Peter Bedrick Books, 1985). Available through Carolina Biological. Contains short, accurate, informative biographies of about 200 mathematicians written at a reasonable level. Many are suitable for high school students.
- Bossi, Michele and Tucci, Pasquale, eds., *Bicentennial Commemoration of R. G. Boscovich, Milano, September 15-18, 1987, Proceedings* (Milan: Edizioni Unicopli, 1988).
- Burton, Martha B., "The Effect of Prior Calculus Experience on 'Introductory' College Calculus," *The American Mathematical Monthly*, 96:4 (1989), 350-354.
- Daston, Lorraine, *Classical Probability in the Enlightenment* (Princeton: Princeton University Press, 1988).
- Dieudonné, Jean, *A History of Algebraic and Differential Topology 1900-1960* (Boston: Birkhauser, 1989).
- Dudka, Jacques, "On the Gregorian Reform of the Julian Calendar," *The Mathematical Intelligencer*, 10:1 (1988), 56-61
- Ekeland, Ivar, *Mathematics and the Unexpected* (Chicago: University of Chicago Press, 1988).
- Fauvel, John and Gray, Jeremy, eds., *The History of Mathematics: A Reader* (Dobbs Ferry, N.Y.: Sheridan, 1987).
- Fraser, Craig, Review of *A Source Book in Mathematics, 1200-1800*, *The Mathematical Intelligencer*, 11:4 (1989), 68-70. Fraser's review of this recently reprinted classic is valuable for the additional references it suggests, particularly those regarding the calculus. An added bonus is a copy of the first page of the first paper on the differential calculus. Enlarge it with your photocopier and make an overhead to show to your classes.
- Gindikin, Semyon G., *Tales of Physicists and Mathematicians* (Boston: Birkhauser, 1987).
- Grattan-Guinness, Ivor, *Convolutions in French Mathematics 1800-1840* (Boston: Birkhauser, 1990).
- Harris, John, "Australian Aboriginal and Islander Mathematics," *Australian Aboriginal Studies* No. 2: 1987, 29-37. The author, a linguist with experience as a teacher in Aboriginal schools, presents an account of the usage of aboriginal numbers, and an instructive account of the origin and propagation of fallacious statements to be found in standard literature. (Neil Porter, Canberra College of Advanced Education).
- Hilton, Peter, "A Brief Subjective History of Homology and Homotopy Theory in this Century," *Mathematics Magazine* 61: 5 (1988), 282-291.
- Knorr, Wilbur, *Textual Studies in Ancient and Medieval Geometry* (Boston: Birkhauser, 1989).
- Kubik, Gerhard, "African graphic systems," *Muntu, revue scientifique et culturelle du Centre International des Civilisations Bantu (CICIBA)*, Libreville (Gabon), 4-5 (1986),

71–135. "In pre-colonial times, a varied range of graphic systems existed in Sub-Saharan Africa. The author presents the results of his own investigations made in Tanzania, Malawi, Gabon, Cameroun, Angola, and Zambia between 1962 and 1984." The author analyses also tusona-luchezi-ideographs. "The forefathers of the Eastern Angolan peoples discovered higher mathematics and a non-Euclidean geometry on an empirical basis applying their insights to the invention of these [tusona] unique configurations." (p. 108).

- Masani, Pesi R., *Norbert Wiener 1894–1964* (Boston: Birkhauser, 1990).
- Monastyrski, Michael, *Riemann, Topology, and Physics* (Boston: Birkhauser, 1987).
- Parshall, Karen H., "America's First School of Mathematical Research: John Sylvester and the Johns Hopkins University," *Archive for History of Exact Sciences* 38:3 (1988), 153–196.
- Richards, Joan, *Mathematical Visions* (San Diego: Academic Press, 1988).
- Rowe, David E., "Gauss, Dirichlet and the Law of Biquadratic Reciprocity," *The Mathematical Intelligencer*, 10:2 (1988), 13–25.
- Stern, Nancy, "Age and Achievement in Mathematics: A Case Study in the Sociology of Science," *Association for Women in Mathematics Newsletter* 18:2 (1988), 12–20.
- Stipanić, Ernest, "International Symposium of Boscović in Rome," *Dijaletika* 23: 3–4 (1988), 115–121.
- Thompson, Christopher L., "Nonstandard Continuity and Uniform Convergence," *The American Mathematical Monthly* 96:5 (1988) 443–445.
- Tignol, Jean-Pierre, *Galois Theory of Algebraic Equations* (Somerset, NJ: Wiley, 1988).
- Zaslavsky, Claudia, *Africa Counts* (Brooklyn, NY: Lawrence Hill Books, 1988) Order the paperback version from Independent Pulishers Group, 814 N. Franklin St., Chicago, IL 60610 – (800) 888–4741.
- Zaslavsky, Claudia, *Mathematics Comes Alive: Activities from Many Cultures* (Portland, ME: J. Weston Welch, 1987) Order from Educator's Choice, P.O. Box 96, Jericho, NY 11753 – (800) 631–0867.
- Zaslavsky, Claudia, "People Who Live in Round Houses," *Arithmetic Teacher* 37:1 (September, 1989), 18–21.

Electronic Mail

V. Frederick Rickey

Do you wish you could find a good general reference on Bolzano and his work? Something about the personality conflicts between the Bernoulli brothers? Something to give a student to read about Bourbaki? Whether the logician Paul Bernays (born 1888) is still alive? These are the kinds of questions that teachers interested in history are always asking. If you don't find an answer in your standard sources, then you have to look for someone to ask. There is now an easy quick way to ask lots of people at the same time: electronic mail.

Electronic mail is quickly becoming the handiest way of communication among individuals who have access to it. We who are interested in history should also be using it. All of us would benefit from the questions that others ask and the answers they receive.

Do you have access to electronic mail? If so, send a message to me: af4993%usma2@westpoint-emh1.army.mil Hopefully there will be enough interest so that we can start our own newsgroup for history of mathematics.