International Study Group on the Relations Between HISTORY and PEDAGOGY of MATHEMATICS NEWSLETTER

No. 18	September 1989

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The *Newsletter* is the communication of the International Study Group on the Relations Between History and Pedagogy of Mathematics, an affiliate of the International Commission on Mathematical Instruction. It is edited and produced in the Department of Mathematics, College of Physical Science, Engineering and Technology, University of the District of Columbia, Washington, D.C. 20008, USA. The *Newsletter* is available free of charge upon request. Distributors: US: Editorial Office; Canada: David Wheeler (Concordia University, Montreal, Qué H4B 1R6); Mexico: Alejandro Garciadiego (UNAM - contact at José M. Velasco 71, Del. Benito Juárez, 03900 Mexico, D.F.); South America: Ubiratan D'Ambrosio (Pró-Reiter de Desenvolvimento Universitário, Universidade Estadual de Campinas, CP 6063, 13081 Campinas SP Brazil); Australia: Robert L Hayes (Mathematics Department, Hawthorn Institute of Education, Private Bag 12, Hawthorn, Victoria 3122); New Zealand: Andy Begg (University of Waikato, Private Bag, Hamilton); elsewhere: Edw. Jacobsen (Div. Sci. Tech. & Envrntl Educ., UNESCO, Paris 75700, FRANCE). Send requests and address changes to the Editor.

Calendar

Meetings with HPM components are highlighted.

- 1989 October 26-29 ...... Baltimore Annual Conference of the American Mathematical Association of Two-Year Colleges. Contact Barbara Gale, Prince George's Community College, 301 Largo Road, Largo MD 20772 U.S.A.

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• 1989 November 5–10 ...... Tallahassee
First International Conference on the History and Philosophy of Science in Science Teaching. Contact Center for Professional Development, Florida State University, Tallahassee, FL 32306-2027 U.S.A. (See inside for further details.)

• 1989 December 21 ...... London
Annual General Meeting of the British Society for the History of Mathematics. The program will include a talk on some aspect of the mathematics of Greek music. Contact R. W. Bray, Department of Mathematics, University of Essex, Wivenhoe Park, Colchester, CO4 3SQ U.K.

• 1990 January 17–20 ...... Louisville
Annual Meeting of the American Mathematical Society and the Mathematical Association of America. There will be a special session in the History of Mathematics organized by Victor Katz and Florence Fasanelli. Contact AMS, P.O. Box 6248, Providence, RI 02940 U.S.A.

1990 April 7–9 ...... Leicester
British Society for the History of Mathematics conference/workshop: The Uses of History in Mathematics Education. Contact Neil Bibby, School of Education, University of Exeter, Exeter EX1 2LU U.K. (See inside for further details.)

• 1990 April 17–21 ...... Salt Lake City
Annual Meeting of the National Council of Teachers of Mathematics and the Americas Section of HPM. (See inside for call for papers.)

• 1990 May 31–June 1 ...... Victoria
Annual meeting of the Canadian Society for History and Philosophy of Mathematics. For information about the society, contact Professor A. Malik, Department of Mathematics, Concordia University, 7141 Sherbrooke St. W., Montreal, Quebec H4B 1R6, CANADA. (See inside for further details.)

• 1990 June 25–28 ...... Campinas
HPM Symposium on Using History in the Teaching of Mathematics. Contact Professor Itala L. D'Ottaviano, Director, CLE, UNICAMP, 13081 Campinas, SP, BRAZIL. (See inside for further details.)

• 1990 July 17–20 ...... Oxford
Conference on the Scientific Revolution organized by the British Society for the History of Science. Contact Wing Commander Bennett, BSHS, 31 High St., Stanford in the Vale, Faringdon, Oxon, SN7 6LH U.K.

• 1990 August 8–11 ...... Columbus
Summer meeting of the American Mathematical Society and the Mathematical Association of America. There will be events commemorating the 75th anniversary of the Association. Contact MAA, 1529 Eighteenth St., NW, Washington, DC 20036 U.S.A.

• 1990 August 21–29 ...... Kyoto
The International Congress of Mathematicians. Contact ICM-90 Secretariat, Research Institute for Mathematical Sciences, Kyoto University, Kitashirakawa, Sakyo-ku, Kyoto 606, JAPAN.

• 1991 August 3–7 ...... Coral Gables
Eighth Interamerican Conference on Mathematics Education. Contact Patrick Scott, Programas Latinoamericanos de Educación, Faculty of Education, University of New Mexico, Albuquerque, NM 87131 U.S.A. (See inside for further details.)

• 1991 September ...... Cambridge
A meeting on Renaissance Mathematics co-sponsored by the British Society for the History of Mathematics. Contact R. W. Bray at address above.

• 1992 July ...... Quebec
Seventh International Congress on Mathematics Education (ICME 7). Details later.

From the Editor

Victor J. Katz

As many of you are aware, the publication of Newsletter 17 was severely affected by the application of Murphy's Laws. One of the problems was that an electronic monster devoured the entire U.S. mailing list. Though the list was eventually recovered from hard copy, there are probably many people who had been on the list who were inadvertently removed. If any of you know anyone who did not receive Newsletter 17, or, naturally, anyone who would like to be on the mailing list, please let me know.

As I noted in the previous newsletter, this publication can only work if I receive contributions to share. In particular, I would like volunteers to report on papers of interest in any of the meetings mentioned in the calendar. Please let me know which ones you are planning to attend. I would also appreciate reports on how you have used historical material in the classroom. Such reports need not be long, but should indicate the sources of your material and how you adapted it. Interested readers could then write for more detailed information. Finally, please send information for the Have You Read section to Professor Calinger. Include not only detailed bibliographical information, but also a few lines of description of the book or article. If you would like to review a book in more detail, please let me know and that can also be arranged.

Canadian Society for History and Philosophy of Mathematics Annual Meeting

Victor J. Katz

The Canadian Society for History and Philosophy of Mathematics had its annual meeting at Université Laval, in Quebec City, Quebec on May 29 and 30, 1989. Some nineteen talks were presented over the two day session, many of which of interest to those involved in HPM. The keynote address of the special session on 18th

century mathematics was given by Joan Richards (History Department, Brown University, Box N, Providence, RI 02912, USA) on the topic *Rigor and Revolution*. Richards discussed the changing approaches to rigor in mathematics throughout the eighteenth century, with particular attention to the work of Clairaut and Lacroix. She provided a fascinating comparison of how the two mathematicians approached the foundations of geometry in their didactic works on the subject in 1741 and 1797 respectively and then showed how Lacroix modified his approach in his works on analysis around the turn of the nineteenth century. According to Richards, the French Revolution helped to change the status of mathematics from a central to a more peripheral role in our culture; as part of this change the concern with rigor became more important. The work of Cauchy in comparison with that of Lacroix provided an excellent example of this change. A study of the ideas of these mathematicians would certainly provide help in any current discussion of how to present rigor in a mathematics classroom.

A related talk, *Rigor and Proof in Mathematics: An Historical Perspective*, was presented by Israel Kleiner (Department of Mathematics, York University, North York, Ont. M3J 1P3, Canada). Kleiner, noting that "mathematical rigor is like clothing – in its style it ought to suit the occasion; it diminishes comfort and restricts freedom of movement if it is either too loose or too tight", gave a historical sketch from Greek times to the age of the computer of how the idea of rigor has changed.

Glen Van Brummelen (Department of Mathematics, Simon Fraser University, Burnaby, BC V5A 1S6, Canada) in *Reasonable Expectations: 18th Century Solutions of the St. Petersburg Paradox* discussed the various attitudes toward anomalies in eighteenth century mathematics, with special emphasis on ways of resolving the St. Petersburg paradox in terms of the meaning of expectation in probability. The approaches of such mathematicians as Jakob Bernoulli, Nicolaus Bernoulli, Daniel Bernoulli, Buffon, Cramer,

Fontaine, and D'Alembert were detailed. A discussion of this paradox and its resolution is clearly warranted in any course in probability.

Professor J. Seldin (Department of Mathematics, Concordia University, Montreal, Que H3G 1M8, Canada) provided a somewhat different perspective by dealing with *Reasoning in Elementary Mathematics*. He showed by some clever examples how reasoning in mathematics differs from that in other fields and how this recognition helps one to deal with students learning beginning algebra. James Bidwell (Department of Mathematics, Central Michigan University, Mt. Pleasant, MI 48859, USA), on the other hand, looked at *Babylonian Geometrical Algebra* and concluded that many of the Babylonian methods could have been derived from simple geometrical manipulations. He presented a number of these manipulations, all based on rearranging rectangles and squares in a particular diagram. These geometrical methods may in fact be useful in teaching methods of solution of quadratic equations.

Victor Katz (University of the District of Columbia, USA) presented *Why Mathematics*, an historical look at how applications did or did not influence the development of various parts of mathematics, noting that even when there was little applicability of the mathematics, textbook writers often included many artificial applications.

Other talks at the meeting included *Pure and Applied Mathematics in the Philosophies of D'Alembert and Kant* by Craig Fraser (IHPST, University of Toronto, Canada), *Lewis Carroll's Method of Trees: Its origins in the work of C.S. Pierce* by Fran Abeles (Kean College, USA), $\sqrt{2}$: *A Foundational Perspective* by J. Loase (SUNY Westchester Comm. Coll., USA), *The Story of Diophantine Geometry* by Abe Shenitzer (York University, Canada), *The British Influence on Euler's Early Mathematical Discoveries* by P. L. Griffiths (UK), *From Manoel de Azevedo Fortes to Joze Fernandes Pinto Alpoym: the Emergence of Mathematics in Eighteenth Century Colonial Brazil* by Ubiratan D'Ambrosio

(Univ. Estadual de Campinas, Brazil), *What the Manuscripts Tell. The Scientific Work of Abu Sahl al-Kuhi* by Len Berggren (Simon Fraser Univ., Canada), *Developments in the Solution of the Differential Equation* $x^n y^{(n)}(x) - x^m y(x) = 0$ by Sharon Kunoff (Long Island University, USA), *The Shape of the Great Pyramid: A Mathematical Hysteria* by Roger Herz-Fischler (Carleton Univ., Canada), *Mathematics in Man's Universe* by A. K. Kay (Fundamental Research Institute, Canada), *Mathematics is a Sphere, not a Klein Bottle* by Robert Thomas (Univ. of Manitoba, Canada), *Pappus's Notes to Euclid's Optics* by A. Jones (IHPST, University of Toronto, Canada), and *New Theorems involving the Golden Section* by Samuel Kutler (St. John's College, USA).

The proceedings of the meeting will be published in early 1990. Members of the Society will receive the proceedings as a privilege of membership. Details as to the availability and price of the proceedings to non-members will be provided later. Membership in the society costs $ 15 Canadian or $ 11 U.S. If a subscription to *Historia Mathematica* (the official journal of the society) is desired, the total cost is $ 45 Canadian or $ 34 U.S. To join, send you name and address, with the appropriate remittance to Professor M. Malik, Department of Mathematics, Concordia University, Montreal, Que. H3G 1M8, Canada.

HPM in Orlando

Erica Voolich

The annual Americas section meeting of HPM was held in Orlando, Florida on 11 April 1989, the day before the NCTM meetings began. All six talks had both a historical and a pedagogical focus to them and at the same time were quite different.

Charles V. Jones (Ball State University, Muncie, Indiana 47306) spoke about the history of mathematics and American curriculum reform. He focused on the recently released NCTM Standards and their stated goals and

emphases. These include, for example, having the students value mathematics, understand the cultural/historical/scientific evolution of it, be confident in their abilities, be problem solvers, and be able to communicate and reason mathematically. He looked at what was being de-emphasized, for example, paper/pencil computation and two column proofs. He pointed out many opportunities for writing and history within the new curriculum and encouraged the inclusion of math history in the training of teachers. His talk ended with the audience discussing the effects of the proposed application-driven curriculum recommended by NCTM. There were concerns expressed about teaching math in preparation for future jobs while seeming to leave out applying math to math problems. Historically, we know that many math ideas were discovered without a specific application in mind; years later a practical use was realized.

Joel Lehmann (Valparaiso University, Valparaiso, Indiana 46303) spoke about using history as a bridge to understanding infinite series. He explained how he took the pedagogical and historical approach to teach the idea of infinite series. He pointed out that there are different levels of understanding beginning with trying to solve a single problem, to generalizing the technique used to solve other similar problems, to abstracting the technique, to redefining, refining and adjusting methods and definitions when the original method no longer applies. He illustrated these various levels of understanding of infinite series historically with Alhazen, Bernoulli, Leibniz, Newton, and Nicomachus.

David Kullman (Miami University, Miami, Ohio 45056) looked at the historical use of prosthaphaeresis, a labor saving calculational device used before the invention of logarithms. Prosthaphaeresis refers to the use of trigonometric identities such as $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$ to change multiplication and division into addition and subtraction. He gave a chronology showing the development and use beginning with Johannes Werner in 1510. These identities became obsolete in 1614 with Napier's development of logarithms. He pointed out that they are used today with calculus students to help find antiderivatives but that this is being rendered obsolete yet again by contemporary software and calculators.

Joan Countryman (Germantown Friends School, Philadelphia, Pennsylvania 19144) demonstrated the drama of mathematics. After speaking briefly about journal keeping and dramatic readings in math class, she distributed selections from I. Lakatos' *Proofs and Refutations* (Cambridge U. Press, 1976), assigned parts, and conducted a dramatic reading. She pointed out how this could introduce passion and interest into proofs. She mentioned that *School Science and Math* used to publish plays and gave a list of resources.

Duane E. Deal (Ball State University, Muncie, Indiana 47306) humorously reported that $x = 4$, but only in Indiana. He distributed a copy of the infamous House Bill no. 246, "A bill for an act introducing a new mathematical truth and offered as a contribution to education to be used only by the State of Indiana free of cost ..." This was authored by Dr. Edwin Goodwin, a physician who enjoyed math and who proclaimed he had squared the circle and trisected the angle. The local papers in 1897 reported on this discovery that was being donated to the state free of charge, everyone else having to pay. He told about the strange choice of committee referals (swamplands and temperance) before Professor C. A. Waldo came to check on the budget for Purdue University and told the senators of their folly.

V. Frederick Rickey (Bowling Green State University, Bowling Green, Ohio 43403) spoke about conic sections. He gave a historical development starting with Menaechmus and progressing through Hippocrates, Apollonius, Diocles, Kepler, Galileo, Tartaglia, van Schooten (the younger), Gregory, Newton, Cassegrain, Cowley, and Whitt. Conics arose out of geometric construction problems, but can be defined either as sections of a cone or in terms of loci. In his de-

velopment, he used a variety of examples such as cube doubling, projectiles, telescopes, paths of planets, suspension bridges, water fountains, sewer pipes, and the quarter on the elbow trick. He conducted a paper folding activity, each time beginning with a circle and a point. By placing the point in different positions either a circle, an ellipse, a parabola, or a hyperbola was generated.

First International Conference on the History and Philosophy of Science in Science Teaching

The conference will be held from Sunday, November 5 to Friday, November 10, 1989 at Florida State University, Tallahassee, Florida. The conference is supported by the National Science Foundation (U.S.A.) and by a wide range of professional associations, teaching groups, and research associations. The conference aims to demonstrate the contribution that can be made by the History and Philosophy of Science to the enhancement of school science teaching. We also hope to further understand the place science education has in the overall development of the individual. This will be a working conference involving science educators, historians of science, philosophers of science, school science teachers, scientists, and educational administrators. Participation by all attending will be encouraged. A unique feature of the conference will be the Review Groups. These will provide the opportunity for developed, cross-disciplinary reflection on the conference presentations. Concurrent sessions will be held on a wide range of topics, including discussions of papers published in special issues of the following Journals: *Educational Philosophy and Theory, Synthese, Interchange,* and *Studies in Philosophy and Education.* (From the conference announcement.)

AMS Meeting in Louisville

There will be a special session in the history of mathematics at the annual meeting of the American Mathematical Society to be held in Louisville, Kentucky on January 17-20, 1990. Among the speakers who have agreed to participate are Marcia Ascher (Ithaca College, Ithaca, N.Y. 14850) on mathematical ideas of traditional peoples, Craig Fraser (IHPST, University of Toronto, Toronto, Ontario, Canada M5S 1K7) on Lagrange's contributions to the calculus of variations and differential equations, Roger Cooke (University of Vermont, Burlington, VT 05405) on N. N. Luzin and set theory, Charles Ford (Saint Louis University, St. Louis, MO 63103) on mathematics and social conflict in 20th century Moscow, Sergei Demidov (Institute for the History of Natural Science and Technology, Academy of Sciences of the USSR, 103012 Moscow K-12, USSR) on the early history of the Moscow school of mathematics, Francine Abeles (Kean College of New Jersey, Union, NJ 07083) on the mathematics of Lewis Carroll, and Ubiratan D'Ambrosio (Universidade Estadual de Campinas, 13081 Campinas SP, BRAZIL) on aspects of mathematics in colonial Latin America. Other speakers will be announced later. The full program will appear in the December issue of the AMS *Notices*.

British Society for the History of Mathematics conference/workshop — The Uses of History in Mathematics Education

In recent years it has become clear that several other European countries have taken a lead over Britain in developing historical perspectives within the mathematics curriculum. The British Society for the History of Mathematics is therefore planning a conference/workshop on this topic, for April 7-9, 1990 at Beaumont Hall, University of Leicester. Its main purpose is to bring to the attention of mathematics teachers some of the important work being done in this area, and the fresh ideas it has stimulated in mathematics teaching.

The format is a mixture of talks from teachers in Britain and abroad who have developed historical perspectives in their mathematics teach-

ing, and workshop sessions to consolidate and develop the ideas further. The result will be a sharing of experiences of classroom activities and initiatives, which will enrich and support the mathematics teacher in the classroom in various situations: for teaching in middle school or at sixth form level; in cross-curricular initiatives involving mathematical activity; to encourage girls into mathematics; to teach in a multicultural setting; as a weapon in remedial and numeracy teaching.

In particular, the wide range of different uses of history within each of these situations will be addressed, and its benefits for improving students' understanding of and enthusiasm for mathematics, and their ability to participate in mathematical activity, as well as their appreciation of mathematics' role in society. The conference is not though, about 'teaching the history of mathematics in schools' – a quite different proposal from that of *using* history *in* mathematics teaching. Some of the ways of using history which will be explored are: telling historical stories to motivate students; using critical examples from the past to illustrate techniques or methods; devising the pedagogical approach in sympathy with historical development; setting projects about past mathematical activity in the locality; directing dramatic activity which reflects mathematical interaction; encouraging students to empathise with the problems of past mathematicians.

Speakers and workshop leaders include: Maryvonne Hallez (College Paul-Bert, Paris), Ivan Tafteberg Jakobsen (Statsgymnasium Aarhus), George Ghevargese Joseph (Manchester), David Kaye (Willesdon College), Marjolein Kool (Middle School, Utrecht), Marylynne Lolley (Sydenham School), Jan van Maanen (Christelijk Gymnasium, Utrecht), Leo Rogers (Digby Stuart College), David Singmaster (South Bank Polytechnic).

The full program and other information will be available in the autumn. For more details, contact Neil Bibby, School of Education, University of Exeter, Exeter EX1 2LU, U.K. (From the preliminary announcement.)

Annual NCTM and HPM meeting

The annual NCTM meeting is scheduled for April 18–21, 1990 in Salt Lake City. As usual, the Americas section of the HPM will meet just prior to that meeting, on April 17–18. Papers on any aspect of the relationship of history and pedagogy of mathematics are welcomed. Please send abstracts to Erica Voolich, 244 Summer St., Somerville, MA 02143 U.S.A. to reach her no later than February 15, 1990.

Canadian Society for History and Philosophy of Mathematics

The annual meeting will be held May 31–June 1, 1990 at the University of Victoria, in Victoria, British Columbia. There will be a special session on history and pedagogy of mathematics organized by Victor Katz. Anyone who wishes to be considered for presenting a 20–30 minute paper in this session should send an abstract to him at the address on the front page by February 28. The remainder of the program is being organized by Professor Francine Abeles, Mathematics Department, Kean College of New Jersey, Morris Ave., Union, NJ 07083 U.S.A. Abstracts of presentations dealing with any aspect of the history of mathematics should be sent to her, also by February 28.

HPM Conference in Campinas

HPM, together with the Center for Logics, Epistemology and History of Science (CLE) of the State University of Campinas (UNICAMP), near Sao Paulo, Brazil, will organize a Symposium on "Using History in the Teaching of Mathematics". The event will take place in Campinas on 26, 27, and 28 June, 1990. Campinas is the second largest city in the State of Sao Paulo (about 1,000,000 inhabitants), about 50 miles northwest of the city of Sao Paulo. Participants may want to stay for the 42nd Annual Meeting of the Brazilian Society for the Advancement of Science which will take place in Southern Brazil in the second week of July. This is the largest regular

scientific meeting in Latin America, with about 5,000 participants. It should be noted that air fares to Brazil are reduced this part of the year. Although it is winter, the weather in Campinas is mild, making the season very attractive from the touristic viewpoint. Interested individuals should consult VARIG, the Brazilian airline, or PAN AM for fares and touristic packages information. For further information, write to Professor Itala L. D'Ottaviano, Director, CLE, UNICAMP, 13081 Campinas, SP, Brazil, or to Ubiratan D'Ambrosio (address on the first page). Further information will appear in the next HPM newsletter.

VIII CIAEM – IACME

The organizing committee announces that the 8th Interamerican Conference on Mathematics Education will be held at the University of Miami, Coral Gables, Florida, USA, from August 3-7, 1991. We invite you to participate in this Conference, which like past Interamerican Conferences on Mathematics Education, should help to improve the teaching of mathematics. There will be three invited major addresses during the 8th IACME delivered by internationally recognized invited specialists. There will also be four panels: Integration of the Sociocultural Context in the Teaching of Mathematics; Innovative Uses of Calculators and Computers in Mathematics Teaching; Effective Teaching of Mathematics; Curricular Changes for the 21st Century. Other modalities will be 30 minute Oral Communications, Discussion Groups, and Poster Sessions. To receive the Second Announcement of the 8th IACME, contact Patrick Scott, Programas Latinoamericanos de Educación, Faculty of Education, University of New Mexico, Albuquerque, NM 87131, USA. (From the First Announcement.)

ICME 7

The International Programme Committee for ICME-7 will hold its first meeting in September, 1989. Since the committee is anxious to have

input as it makes its decisions concerning the structure of the scientific program, suggestions are welcomed through the National Representatives. In particular, the committee wishes to know which features of the program of ICME–6 should be retained, which modified, and what innovations should be considered for ICME–7. The chairman of the International Program Committee is David Wheeler, Concordia University, 7141 Sherbrooke St. West, Montreal, Quebec H4B 1R6, CANADA. The U.S. representative on the Committee is J. Kilpatrick, 105 Aderhold Hall, University of Georgia, Athens, GA 30602, U.S.A.

La Crosse Math History Conference II

The proceedings for the La Crosse Math History Conference II is now ready. It is 64 pages long and contains presentations by Irving Anellis, Charles V. Jones, Richard Maresh, Carroll Rusch, and Mark Saegrove. The price is $5.00 U.S. If you wish to order one, contact J. D. Wine, 4033 Cowley Hall, University of Wisconsin – La Crosse, La Crosse, WI 54601.

Mathematical Aspects of Traditional African Games

Claudia Zaslavsky suggests that it could be important in the reconstruction of the history of mathematics in Africa to investigate mathematical aspects of traditional African games. As a starting point, she indicates the following literature:

- Russ, Laurence *Mancala games*, Reference Publications, 218 St. Clair Drive, Box 344, Algonac, MI 48001, U.S.A. (1984). Rules and brief history of many versions of the game known also as Ayo, Bao, Wari, and Mwese.

- Klepzig, Fritz *Kinderspiele der Bantu*, Verlag Anton Hain, Meisenheim am Glan, Federal Rep. of Germany (1972). Includes games of chance, string figures, board games, and riddles.

• Beart, Charles *Jeux et jouets de l'Ouest Africain* IFAN, Dakar (1955). Children's games, including finger counting rhymes, magic squares, string figures, cards, dominoes, games of chance, mancala-type games, riddles, and arithmetic problems.

• Centner, Th. *L'Enfant Africain et ses jeux* CEPSI, 17, Lubumbashi, Zaire (1963). Games of francophone Central Africa: sand drawings, games of chance, counting chants, kisolo (mancala) game, string figures, memory games, etc.

• Pankhust, Richard "Gabata and related board games of Ethiopia and the Horn of Africa" in *Ehiopia Observer* 14, 3 (1971), 154-206. History and rules of play of 103 versions of the mancala-type game.

• Driedger, Walter "The game of Bao or Mankala in East Africa" in *Mila* 3, 1 (1972), 7-17. Institute of African Studies, University of Nairobi.

The preceding note is taken from the newsletter of the African Mathematical Union's Commission on the History of Mathematics. Interested readers may receive the newsletter free of charge by writing to Paulus Gerdes, C. P. 915, Maputo, Mozambique (English version), Ahmed Djebbar, Department de Mathématiques, Université Paris-Sud, 91405 Orsay Cedex, France (French version) or Mahdi Abdeljaoud, I.S.E.F.C., 43 rue de la Liberté, 2019 Le Bardo, Tunis, Tunisia (Arabic version).

Have You Read?

Readers are asked to submit contributions. References need not deal exclusively or explicitly with history in the mathematics classroom, but should have the potential for motivating or enriching. Please supply complete bibliographic information: names of author(s); complete titles of books, articles and journals; for journals include both the volume and date; for books, edition, copyright date, publisher and place of publication. Accuracy in spelling and wording is critical. Please provide concise annotations whenever possible. The editor of this section is Professor Ronald Calinger, Department of History, Catholic University of America, Washington, D.C. 20064; please send all submissions to him.

• Arcavi, Abraham & Bruckheimer, Maxim "The Didactical De Morgan: a Selection of Augustus De Morgan's Thoughts on Teaching and Learning Mathematics", *For the Learning of Mathematics* 9:1, (1989) 34-39. An excerpt from this article is presented below.

• Ascher, Marcia "Graphs in Cultures (II): A Study in Ethnomathematics", *Archive for History of Exact Sciences* 39:1 (1988), 75-95.

• Bailey, D.F. "A Historical Survey of Solution by Functional Iteration", *Mathematics Magazine* 62:3, (1989) 155-166.

• Burton, Martha B. "The Effect of Prior Calculus Experience on Introductory College Calculus", *American Mathematical Monthly* 96:4, (1989), 350-354.

• Calinger, Ronald "Mathematical Innovator and Reformer" on George Boole and Modern Computers in *World & I* 3:8 (1988), 194-199.

• Calinger, Ronald "Analyses and Treatises' on Leonhard Euler in *World & I* 4:5, 312-317.

• Cipra, Barry A. "The Legacy of John von Neumann", *SIAM News* 21:5 (1988), pp. 28 and 22-23.

• Fauvel, John "Platonic Rhetoric in Distance Learning: How Robert Record Taught the Home Learner", *For the Learning of Mathematics* 9:1 (1989), 2-6.

• Glieck, James *Chaos: Making a New Science* (New York, Viking, 1987).

- Grattan-Guinness, Ivor, ed., *Cahiers d'Histoire et de Philosophie des Sciences* 21 (1988) with articles in English on uses of history of mathematics in mathematics education by A. Arcavi, B. Hughes, and H. Pycior. This volume may be ordered at a cost of $ 20.00 from Professor C. J. Scriba, International Commission on History of Mathematics, Bundesstrasse 55, IGN, D-2000 Hamburg 13, Federal Republic of Germany.

- Kleiner, Israel "The Evolution of Group Theory: A Brief Survey", *Mathematics Magazine* 59 (1986), 195-215.

- Kleiner, Israel "A Sketch of the Evolution of (Noncommutative) Ring Theory", *L'Enseignment Mathématique* 33 (1987), 227-267.

- Kleiner, Israel "Thinking the Unthinkable: The Story of Complex Numbers (With a Moral)", *The Mathematics Teacher* 81 (1988), 583-592.

- Kline, Morris *Mathematics and the Search for Knowledge* (Oxford: Oxford University Press, 1985).

- Michaelowicz, Karen Dee "Middle School Concepts and Related Historical Topics" *Virginia Mathematics Teacher* (1989), 27-29 and "Multiplication in Egypt, 1650 B.C." *Ibid.* 21.

- Otto, Michael "The Ideas of Hermann Grassmann in the Context of the Mathematical and Philosophical Tradition since Leibniz" *Historia Mathematica* 16:1 (1989) 1-36.

- Paulos, John Allen *Innumeracy: Mathematical Illiteracy and Its Consequences* (New York: Hill and Wang, 1988).

- Truesdell, C. *An Idiot's Fugitive Essays on Science: Methods, Criticism, Training, Circumstances* (New York: Springer Verlag, 1984).

- Wright, Margaret "A Brief History of Linear Programming - from Fourier to the Ellipsoid Algorithm" *SIAM News* 22:2 (1989), 10-11, 16.

De Morgan and Teaching Mathematics

Abraham Arcavi and Maxim Bruckheimer have collected a number of quotations relating to teaching and learning from the works of the English algebraist Augustus De Morgan (1806-1871) in an article "The Didactical De Morgan: a Selection of Augustus De Morgan's Thoughts on Teaching and Learning Mathematics" which appeared in *For the Learning of Mathematics* 9:1 (1989) 34-39. The following excerpt, taken from the work of his wife Sophia Elizabeth De Morgan, *Memoir of Augustus De Morgan* (London: Longmans, Green and Co., 1882), pp. 98-101, illustrates his devotion to teaching. It is based on a description from one of De Morgan's students:

"As Professor of Pure Mathematics at University College, London, De Morgan regularly delivered four courses of lectures, each of three hours a week, and lasting throughout the academical year. He thus lectured two hours every day to his College classes, besides giving a course addressed to schoolmasters in the evening during a portion of the year. His courses embraced a systematic view of the whole field of Pure Mathematics, from the first book of Euclid and Elementary Arithmetic up to the Calculus of Variations. From two to three years were ordinarily spent by Mathematical students in attendance on his lectures. De Morgan was far from thinking the duties of his chair adequately performed by lecturing only. At the close of every lecture in each course he gave out a number of problems and examples illustrative of the subject which was then engaging the attention of the class. His students were expected to bring these to him worked out. He then looked them over, and returned them revised before the next lecture. Each example, if rightly done, was carefully marked with a tick, or if a mere inaccuracy occurred in the working it was crossed out, and the proper correction in-

serted. If, however, a mistake of *principle* was committed, the words 'show me' appeared on the exercise. The student so summoned was expected to present himself on the platform at the close of the lecture, when De Morgan would carefully go over the point with him privately, and endeavour to clear up whatever difficulty he experienced. The amount of labour thus involved was very considerable, as the number of students in attendance frequently exceeded one hundred. ... The fundamental conceptions of each main department of Mathematics were dwelt upon and illustrated in such detail as to show that, in the judgement of the lecturer, a thorough comprehension and mental assimilation of great principles far outweighed in importance any mere analytical dexterity in the application of half-understood principles to particular cases. Thus, for instance, in Trigonometry, the wide generality of that subject, as the science of undulating or periodic magnitude, was brought out and insisted on from the very first. In like manner the Differential Calculus was approached through a rich conglomerate of elementary illustration, by which the notion of a differential coefficient was made thoroughly intelligible before any formal definition of its meaning had been given. The amount of time spent on any one subject was regulated exclusively by the importance which De Morgan held it to possess in a systematic view of Mathematical science. The claims which University or College examinations might be supposed to have on the studies of his pupils were never allowed to influence his programme in the slightest degree. He laboured to form sound scientific Mathematicians, and, if he succeeded in this, cared little whether his pupils could reproduce more or less of their knowledge on paper in a given time. On one occasion, when I had expressed regret that a most distinguished student of his had been beaten, in the Cambridge Mathematical Tripos, by several men believed to be his inferiors, De Morgan quietly remarked that he ' never thought -- likely to do himself justice in THE GREAT WRITING RACE.' All *cram* he held in the most sovereign contempt.

I remember, during the last week of his course which preceded an annual College examination, his abruptly addressing his class as follows: 'I notice that many of you have left off working my examples this week. I know perfectly well what you are doing; YOU ARE CRAMMING FOR THE EXAMINATION. But I will set you such a paper as shall make ALL YOUR CRAM of no use.' "

The following quotation is from Augustus De Morgan, *On the Study and Difficulties of Mathematics* (Chicago: Open Court, 1898) p. 184:

"We believe firmly, that to the mass of young students, general demonstrations afford no conviction whatever; and that the same may be said of almost every species of mathematical reasoning, when it is entirely new. We have before observed, that is is necessary to learn to reason; and in no case is the assertion more completely verified than in the study of algebra. It was probably the experience of the inutility of general demonstrations to the very young student that caused the abandonment of reasoning which prevailed so much in English works on elementary mathematics. Rules which the student could follow in practice supplied the place of arguments which he could not, and no pains appear to have been taken to adopt a middle course, by suiting the nature of proof to the student's capacity."

For the remainder of the quotations, consult the original article. The journal *For the Learning of Mathematics* is not as well known as it ought to be. A personal or library subscription is highly recommended. The rate is $ 30 for institutions and $ 21 for individuals. (Canadian dollars are accepted for subscriptions in Canada; elsewhere, these rates are in U.S. dollars.) Subscriptions should be sent to FLM Publishing Association, 4336 Marcil Avenue, Montreal, Quebec H4A 2Z8, CANADA.