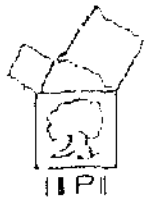

International Study Group on the Relations Between HISTORY and PEDAGOGY of MATHEMATICS NEWSLETTER

AN AFFILIATE OF THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION

No. 17 Winter 1989

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The *Newsletter* is the communication of the International Study Group on the Relations Between History and Pedagogy of Mathematics, an affiliate of the International Commission on Mathematical Instruction. Edited and produced in the Department of Mathematics, College of Physical Science, Engineering and Technology, University of the District of Columbia, Washington, D.C. 20008, USA. The *Newsletter* is available free of charge upon request. Distributors: US: Editorial Office; Canada: David Wheeler (Concordia University, Montreal, Qué H4B 1R6); Mexico: Alejandro Garciadiego (UNAM - contact at José M. Velasco 71, Del. Benito Juárez, 03900 Mexico, D.F.); South America: Ubiratan D'Ambrosio (Pró-Reitor de Desenvolvimento Universitário, Universidade Estadual de Campinas, CP 6063, 13081 Campinas SP Brazil); Australia: George Booker (Brisbane Coll Adv Educ, 130 Victoria Park Rd, Kelvin Grove, Queensland 4059); New Zealand: Andy Begg (Math Curr Off, Dept. Educ., Private Bag, Wellington); elsewhere: Edw. Jacobsen (Div Sci. Tech. & Envrmtl Educ., UNESCO, B.P 3.07 Paris). Send requests and address changes to the Editor.

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Calendar

Meetings with HPM components are highlighted

- 1989 April 11-15 ..... Orlando
  Annual Meeting of the National Council of Teachers of Mathematics and the Americas Section of HPM. (See inside for call for papers.)

- 1989 May 29-30. .....Quebec
  Annual meeting of the Canadian Society

for History and Philosophy of Mathematics. For information about the society, contact the Secretary, Prof. A. Malik, Department of Mathematics, Concordia University, 7141 Sherbrooke ST W., Montreal, Quebec H4B 1R6, CANADA. (See inside for further details.)

- 1989 August 1–9 ..... Hamburg & Munich
  18th International Congress on the History of Science. Contact: ICHS Congress 1989, CPO HANSER SERVICE, Postfach 1221, D-2000, Hamburg-Barbüttel, FRG

- 1989 August 7–10.....Boulder
  Summer meeting of the American Mathematical Society and the Mathematical Association of America. Contact AMS, P.O. Box 6248, Providence, RI 02940, USA

- 1990 January 17–20.....Louisville
  Annual Meeting of the American Mathematical Society and the Mathematical Association of America. Contact: AMS, P.O. Box 6248, Providence, RI, 02940, USA.

- 1990 Late June.....Sao Paulo
  An international meeting of HPM to be organized by Ubiratan D'Ambrosio. Further details later.

- 1990 August 21–29.....Kyoto
  The International Congress of Mathematicians. Contact ICM-90 Secretariat, Research Institute for Mathematical Sciences, Kyoto University, Kitashirakawa, Sakyo-ku, Kyoto 606, Japan.

From the Chair

Florence D. Fasanelli

We have only anecdotal evidence that teaching mathematics *not* separated from its history improves learning. Of course, those of us who are enthusiastic about history easily pass this enthusiasm on to our students. But somehow this interest and enthusiasm has promoted a broader effect. Specifically, many more texts, at all levels, contain both problems and narrative wherein the history of mathematics is designed pedagogically to clarify understanding. Furthermore, several publishers have undertaken contracts to (1) develop new history of mathematics texts from which mathematics can be learned as well as its history, (2) create posters which provoke students to ask how various concepts developed, and (3) reprint with and without alterations the classic earlier texts which were used to teach the history of mathematics. The year 1989 should show a large increase in material available for improving teaching and learning by use of historical materials. The international conferences held last summer made it clear that this is not unique to any one country. We have similar problems and seek both similar and diverse solutions.

The history of a topic often begins a plenary session or an invited talk at a conference for research mathematics because the subject would not be at all accessible without a setting, and the most pertinent introduction is often the history of the problem to be presented. Listening to the first fifteen minutes of an hour talk can be very revealing about how much there is to learn about the history of mathematics, particularly contemporary mathematics.

Studying the history of a field such as vector spaces or number theory enables us to have a better understanding of what happened as the theory developed. Occasionally, new material sheds light on old stories, such as the discovery of the *Method* of Archimedes, the lost Diophantine books, or the contract between Bernoulli and L'Hospital. More history is always interesting.

But that is not the role of HPM. The harder role adopted by this organization is to use the history to enrich the curriculum, to join the learning of mathematics to its historical and cultural background in order that students value the discipline as a cultural achievement which is at all times part of their lives.

Doors are open today. Many countries are developing national curricula. New texts will

be developed. Publishers and authors of old texts are seeking appealing, marketable revisions. IHPM can play an active role in arguing that history becomes one more tool to improve understanding.

In the past four years Ubi D'Ambrosio has led IHPM to a broader understanding of the history of mathematics. Speaking at the IHPM satellite meeting in Australia in 1984, he said there is more than one history of mathematics. There is classroom mathematics, there is research mathematics, and there is the mathematics that we use, each with its own distinct history. From each of these we can develop better ways to reach students and make mathematics meaningful to them. Thank you, Ubi.

IHPM, in some sense, is its newsletter. This excellent piece of work has been in the hands of Charles Jones for the last 13 issues. Over a five year period he has increased the scope of the content and developed a thoughtful and thorough Newsletter. He brought clarity, organization, and style to his task of editing. Thank you, Charles.

As IHPM changed leaders in Budapest at the end of the quadrennial session at ICME 6, a new board was announced and shortly thereafter an invitation was made to Victor J. Katz, Professor of Mathematics at the University of the District of Columbia, Washington, D.C., USA, to become editor of the Newsletter. Many of you are familiar with Victor's writings and expositions about the use of the history of mathematics in his teaching. We look forward to his views.

In order to share our own ideas as well as have the opportunity to develop them together, IHPM will begin to meet every two years. The next meeting will be in June, 1990, in Brazil. Ubi D'Ambrosio is the organizer of this meeting. A satellite meeting will also be held in conjunction with ICME 7 in Quebec, Canada in 1992. At both of these meetings members of IHPM will be urged to give papers on how they have used the history of mathematics in their teaching.

If we can assist students in learning to value mathematics by relating it to its historical and cultural evolution, then they will gain the power to apply mathematics to many situations. As research has shown, students master procedures but are unable to use them because their learning is contextually bound. Mathematics should not be separated from its scientific, cultural, and historical contexts.

From the New Editor

Victor J. Katz

As Charles Jones noted in the last issue of the Newsletter, he has decided to step down as editor. I have taken up the job and only hope that I can fill his shoes. We both apologize for the long delay in getting Newsletter 17 into production, but there were certain technical difficulties connected with the changeover which had to be overcome. I hope to be able to put out the Newsletter on a regular basis from now on, one issue each in the winter, spring, and fall. But that depends on you, the readers. I can only publish material which you send me. I therefore encourage you to send information you have about future meetings, in time for readers to make plans to attend or to present papers. Also send reports of meetings which have occurred. Include the list of speakers with perhaps a few lines about each talk, and, most importantly, the address of the speakers to whom readers can write for further information. I am also interested in short book reviews on books of interest as well as brief articles on topics of interest to the readership. Since more and more textbooks are appearing, in the U.S. at least, with historical notes, reports on these would also be welcome.

The financial support for the Newsletter, without which it could not be produced, has now shifted to the College of Physical Sciences, Engineering, and Technology of the University of the District of Columbia. I want to thank the Dean, Philip Brach, and the Associate Dean, Theodola Milligan, for making arrangements to provide funds for the printing and mailing of the Newsletter.

Kristiansand History of Mathematics Summer Workshop

Victor J. Katz

The history of mathematics workshop in Kristiansand, Norway from August 7–13, 1988, organized by Otto Bekken of Agder College, Kristiansand and Bengt Johansson of Göteborg University, Göteborg, Sweden was designed to explore how teaching of mathematics in schools and colleges can be improved through the use of historical material. Speakers from some eleven countries were invited to present ideas from the history of mathematics which could be used to motivate, illustrate, and enhance the understanding of important concepts and methods in the mathematics curriculum. The meeting was held in the delightful surroundings of the Gimlekollen Mediasenter.

The speakers interpreted the general theme of the meeting in different ways and ever by the end of the conference there was no firm agreement on exactly how best to use history in teaching, whether to use historical examples as motivational ideas, to trace the beginnings of various topics in the curriculum from their origins, or even to organize the presentation of courses from an entirely historical point of view. As a result, there were many lively discussions as the various papers were presented.

Two talks on trigonometry, one by Frank Swetz of Pennsylvania State University (U.S.A.) and one by Jan van Maanen of the University of Utrecht (Netherlands), illustrated some of these different approaches. The former gave a brief history of surveying techniques, including what he called 'prototrigonometry', and illustrated how these techniques can be used to motivate the study of trigonometry. The latter discussed the history of one particular trigonometric relationship, the double angle formula for tangent, traced its history from ancient times, and showed how he uses this history in his course for prospective high school teachers of mathematics. Both participants gave other talks as well, with Swetz showing how he used problems from ancient Chinese texts in his courses to develop not only mathematical techniques but also an understanding of ancient Chinese society, while van Maanen demonstrated how some of his research discoveries, including a medieval law treatise on the division of alluvial deposits, can be used with success in the high school classroom.

The theory of logarithms was the subject of two talks, one by John Fauvel of the Open University (England) who discussed how the various conceptual streams leading to Napier's discovery can be used to develop students' intuition, the other by Victor Katz of the University of the District of Columbia (U.S.A.) who presented a modern interpretation of Napier's own work as a possible way of introducing natural logarithms to a precalculus class. Other talks dealing with precollege materials were given by Otto Bekken, on the oldest Scandinavian arithmetic work using Hindu-Arabic numerals, by Bengt Johansson, on the development of reckoning algorithms in Swedish schools, and by Roland Stowasser of the Technical University of Berlin on some organizing ideas from history useful in teaching certain topics in high school algebra and geometry.

Fred Rickey of Bowling Green State University (U.S.A.) discussed many of the pedagogical points in Euler's three great calculus texts which can still be used in calculus classrooms. In another talk, he discussed various examples from other parts of the history of calculus which he has also used in illuminating certain calculus concepts for his students. Joel Lehman of Valparaiso University (U.S.A.) discussed the development of concepts of series as he showed how to motivate that often very technical chapter in the calculus text. In a related talk, Lars Mejlbo of Aarhus University (Denmark) traced the history of the infinitely large and small in mathematics and showed how various thinkers have interpreted these notions. Otto Bekken demonstrated how Abel's letters and notebooks provide many examples useful in teaching these same concepts of convergence, while Michel Helfgott of the University of San Marcos (Peru) outlined the calcu-

his course he teaches in Lima, based on both a historical approach to the material and a strong effort at integrating the mathematics with the natural sciences. Man-Keung Siu of the University of Hong Kong illustrated how the historical development of the notion of a function can be incorporated into mathematics teaching at various levels, and also illustrated the teaching of heuristic reasoning via a study of Euler's solutions of various problems of analyis, geometry, and number theory. A talk by Florence Fasanelli of the National Science Foundation (U.S.A.) surprised the group by bringing in recent research to show how Viviani's problem led to the earliest work on surface integrals and how a letter of Johann Bernoulli to L'Hospital discussing this work also included the function which L'Hospital used as the first example in his text of his famous rule. Two talks on aspects of celestial motion, one by Eric Aiton of the University of Manchester (England) on the work of Newton and Leibniz and one by Steinar Thorvaldsen of the Tromsø Teacher's Institute (Norway) on the work of Kepler, showed how the mathematics involved can illustrate various concepts in today's classes.

There were three talks dealing with aspects of linear algebra, one by Katz on a general historical approach to the teaching of the subject, one by Bekken on Caspar Wessel and vectors, and one by Karin Reich from Stuttgart (West Germany) on the development of vector calculus. The three speakers on abstract algebra disagreed somewhat, however, on how or even whether history could be used in teaching that subject. Donovan van Osdol of the University of New Hampshire (U.S.A.) showed how the definition of an abstract ring emerged from various sources during the early part of the twentieth century and suggested the use of this development in teaching the subject. Israel Kleiner of York University (Canada) discussed how he uses certain concrete historical problems in teaching abstract algebra. Thus, each topic in the syllabus is introduced by the discussion of a particular problem which, historically, was the motivation for the development of the topic. Tony Gardiner of the University of Birmingham (England) was not convinced that a historical treatment of group theory was possible; he discussed various approaches to the subject which have been tried, with varying rates of success, and concluded that the best approach might be to combine historical material with the students' own explorations of certain standard problems. But he worried that putting in historical material could easily require the student to learn not one course but two. And there is never enough time.

Finally, several speakers outlined courses they had taught which included historical aspects. Abe Shenitzer of York University (Canada) discussed the general question of what a graduate in mathematics should 'know' and how to incorporate such ideas into a various courses using history. Shmuel Avital of the Technion (Israel) discussed the use of various problems from the past in his teaching of future teachers. Lars Mejlbo noted that the history of mathematics is now required for teachers of mathematics in Danish gymnasia and discussed how this requirement was now being implemented. And John Fauvel showed us certain of the videos from the new Open University course in the history of mathematics he was instrumental in developing.

Plans are now underway for the proceedings of this meeting to be published in Sweden. The volume should be available in mid-1989. More details will be provided when they become known.

One of the high points of the week in Kristiansand was a pilgrimage to Froland, where Niels Henrik Abel spent the last summer of his life and where he died and was buried in 1829. At his gravesite Man-Kiung Siu, the participant who had traveled the furthest to reach Norway (16,919 km.), gave a moving speech, of which the following is a brief excerpt: "In this wonderful gathering we share the conviction that mathematics is not just a collection of theorems and formulae; there is a cultural and human element to it. So we come here today not just for a technical mathematical connection. For myself, and I believe also for others, I harbour a deep

admiration and respect for Abel, not just for the contributions he made in mathematics, but also for his qualities as a human being.... I first read of the life of Abel in the popular account "Genius and Poverty" by E.T. Bell when I was a first year undergraduate. At that time, young and ignorant as I was, I was already moved and inspired by this story of a young man who gave so much of himself despite the adversity he had to face, so much so that he was robbed of his life in his prime. Schopenhauer maintained that a strong motive that led people to art and science is flight from the harshness of everyday life. I think there is another, perhaps even stronger motive, namely, an inner call of intellectual curiosity and quest for learning. ... This feeling and admiration can only be strengthened especially after I learnt of the painful hardship most of my mathematician friends in China had gone through in those infamous years labelled as the Cultural Revolution.... History will surely reward scholars with such devotion, just like Abel is remembered today.... But wouldn't it be infinitely better if they can work and contribute in a more agreeable environment which they more than deserve? A regard for learning is what we need in the society of today."

British Conference on the Uses of History in Mathematics Teaching

John Fauvel

This year's annual residential conference of the British Society for the History of Mathematics, held in Leicester on September 1–3 1988, was devoted to exploring *the relations between the history and teaching of mathematics*. By the end of the conference one thing at least was clear; these relations are so complex and have so many different aspects that participants in the conference were by no means agreed. But it became clear, too, that there is much interest, goodwill, and energy ready to be channeled into exploring these areas further.

The four talks on the opening day highlighted four different conceptions of the relation between history and pedagogy. First Eric Aiton described the history of mathematics course which he taught in Manchester, a course taught as one component of a mathematics degree, for complementing and enriching students' understanding of mathematics.

Neil Bibby exemplified a different approach, in showing how the teaching of hyperbolic functions might be improved through taking seriously their historical development – an internalised rather than a bolt-on style of history input. John Fauvel's concern was different again, to ask how today's teaching practices could benefit from awareness of the great mathematics teachers of the past, which he illustrated through analysis and dramatic performance of the work of Robert Record.

The account of the mathematical education of Bertrand Russell given by Albert Lewis showed a fourth important strand in history-pedagogy relations, the history of pedagogy itself, seen here in the weird but thought-provoking practices of late nineteenth-century Cambridge.

The differences between these four contributions helped to make everyone aware of the range of different ways in which history and mathematics teaching can relate. The next day's opening talk, by Steve Russ, was timely in raising the moral and political concerns which must lie underneath and beyond the subject of the conference, and in pointing to ways in which the BSHM itself could act as an agent of change. This led to a fruitful debate between the proponents of different tactical approaches: the grass-roots approach in which change towards more history-permeated teaching practices comes about through reaching the hearts of practising teachers, versus the top-down approach in which change is effected through the existing power hierarchy. It would be sufficient, on the latter analysis, to secure the attention and support of just one person, our Prime Minister (who is known to enthuse over mathematicians born near Grantham, at least).

In a characteristically provocative address, the

BSHM's President, Ivor Grattan-Guiness, sharpened the discussion by pointing out the characteristics of history that preclude its coming too close to the heart of the political educational mathematics establishment: it is intrinsically too subversive and asks basic questions which no entrenched power wants to hear. Indeed, as Tony Gardiner later pointed out, history is moral-intellectual dynamite in the hands of 18-year olds, and we should not be surprised at high-level resistance to its incorporation in mathematical pedagogy. Grattan-Guiness argued that the role of history of mathematics is not to be bolted onto nor inserted into mathematics courses, but lies in the design of the curriculum in the first place, illustrating this argument from the history of set theory and of its role in teaching.

Mike Dampier then described the thinking behind the new history of mathematics course at Leicester, which unabashedly aims to change students' perceptions of mathematics, and is in part trying to encourage mathematics students to consider teaching as a career. This nicely illustrates the ethical-political-social considerations which are inseparable from pedagogical activity in this area.

In a spirited appeal which illustrated further the claim that history should have more influence over the syllabus, Walter Ledermann urged that modern linear algebra courses should not lose contact with the older subject of matrix algebra. Ledermann, who is through his teacher Issai Schur a mathematical grandchild of Frobenius, pointed out the immense motivational difficulties which are created for students by a too abstract approach to linear algebra and by taking as teaching paradigms for a first course texts (such as van der Waerden's *Modern Algebra*) which were never intended as such. Teachers would do much better for their students by going back to Cayley's 1858 treatment of matrix multiplication, for instance.

Much went on at this conference which I have not space to report: Tony Gardiner's rich explorations of the role of history in motivating the teaching of group theory; Colin Fletcher's elegant clarification of what led Fermat to his 'little theorem'; Peter Griffith's revelations about the British input to Euler's early mathematics; Ian Bradley's entertaining dialogue about historically informed rigour; Otto Bekken's report on his productive Kristiansand conference [see elsewhere in this Newsletter]; the visit led by Mike Price and Mike Dampier to the Mathematical Association's library, now kept in the library of Leicester University; David Singmaster's beautiful mediaeval manuscript illustrations of mathematical problems.

The high level of interest and concern which the theme of the conference generated was evident from the high standard of the presentations, both intellectually and in provoking serious consideration of fundamental questions. The excitement and variety of the discussions which the papers stimulated, during the formal sessions and over meals and in the bar and late into the night, testify to the way in which the conference's theme is of increasing importance. A disappointing feature of the conference, though, was the low attendance by school-teachers - only two participants out of the thirty-five conferees would admit to currently teaching in schools. It is hoped that this experience will be put to good use in devising another conference on this theme in a year or two's time, building upon the success of this one, when the programme and the publicity will be devised to ensure greater participation by school mathematics teachers. (Or perhaps, if one takes the other view of the best way forward, to arrange for the attendance of Mrs Thatcher.)

HPM in Florence

The satellite meeting of HPM before ICME 6 took place in the congenial setting of the Palazzo Medici-Riccardi in Florence on July 20, 21, and 22, 1988. The first day's speakers included Catherine Perrineau (France): "Towards an Historical Perspective in the Teaching of Mathematics", John Fossa (Brazil): "The Use of the History of Mathematics in Teaching Mathematics", Ubiratan D'Ambrósio (Brazil): "The Work

of the International Study Group on the History and Pedagogy of Mathematics", David Wheeler (Canada): "Pedagogy and the History of Mathematics", James Tattersall (USA): "Geometry in Jeffersonian Architecture", and Michael Serfati: "About so-called "Elementary" demonstrations". The day concluded with a reception in the apartment of Florence Fasanelli, the program chair.

The second day saw a tour of the Palazzo di Storia della Scienza led by the Associate Director, Professor Miniati. It also included three talks, by Jacques Borowczyk (France): "Budan's synagmatic sequences and Horner's method", Ubiratan D'Ambrosio: "Mathematics in Brazil and Portugal in Colonial Times", and Bob Hayes (Australia): "Uses of the History of Mathematics in Australian schools".

The final day included several museum visits as well as talks by Benedito Castrucci (Brazil): "The use of history of mathematics in elementary school", Israel Kleiner (Canada): "The concept of proof in mathematics as it was viewed in various periods", Maryvonne Hallez (France): "The work of IREM", and V. Frederick Hickey (USA): "Using History in Teaching Calculus". Most of the participants in the meeting then proceeded to Budapest to attend ICME 6.

HPM in Budapest

Ubiratan D'Ambrosio

The scientific program of HPM was organized in four one-hour sessions, distributed in two symposia on *non-Euclidean geometries and their adoption in the school systems* and *the evolution of algorithms for use in schools*, a panel on *history of mathematics in the teaching of mathematics* and a session of short communications.

The planning of the scientific program resulted from the recognition that teaching of geometry continues to be a major challenge to mathematics educators and that by bringing into the classroom the challenge that non-Euclidean geometries represented throughout history we may introduce an element favorable to a revival of interest in geometry in the school system. The same with respect to algorithms, which have for years dominated school mathematics in the elementary levels and now may face a lessening of importance in view of the rapid introduction of calculators and computers. Major events in the history of mankind are closely related to major changes in algorithms and the very strong cultural roots of algorithms justify an historical overview of their presence in the school system. The theme for the panel was suggested by the very essence of this International Study Group. To reserve one session for short contributed papers, without bounds on their themes except to be relevant and of interest to both History and Pedagogy of Mathematics, was quite natural in organizing the program.

Let us now report in specifics of each of these activities.

The Symposium on *non-Euclidean geometries and their adoption in the school systems* had three speakers: Nikos Kastanis (Greece), Massouma Kazim (Qatar), and Tibor Wessely (Romania). The session was presided over by Benedito Castrucci (Brazil). Nikos Kastanis spoke on *the concept of space before and after the non-Euclidean geometries: an approach for didactic reasons*. Drawing from an analysis of the architectural forms of the Parthenon in Athens, he stated that this was the first historical hint of the weakness of identifying or reducing the contemplation of space to the Euclidean model. The second hint is drawn from Albert Einstein's view that the conceptual system of Euclid does not include space, which is first introduced only by Descartes. Kastanis proposed the following periodization for the study of space:

1. from myths to logos;
2. from scholasticism to analytic thought;
3. the epistemological revolution (19th century);
4. the shift in scientific outlook (early 20th century);

5. structuralism.

Massouma Kazim's presentation on *an educational unit in non-Euclidean geometry for secondary schools* proposes a program based on the following steps:

1. stating the objectives of the unit;
2. developing contents as follows:
   (a) how non-Euclidean geometries have emerged from the geometry of Euclid through history;
   (b) present Euclidean logic system and non-Euclidean systems as well;
   (c) give some modern applications of non-Euclidean geometries;
3. suggesting methods of teaching the unit, emphasizing the nature of proof

Tibor Wessely spoke on *the Bolyais*, first introducing the main ideas of János Bolyai's *Appendix* and then telling about the life and careers of Farkas and János Bolyai. The lecture was illustrated by a collection of slides showing the places where the Bolyais lived and worked, as well as monuments and museum pieces honoring them.

The Symposium on *the evolution of algorithms for use in schools* had only one formal presentation, by Lawrence Shirley (Nigeria), and was presided over by Victor Katz (USA). In his talk on *historical and ethnomathematical algorithms for classroom* use Lawrence Shirley suggested that mathematical techniques taken from historical and diverse cultural ambiances can be used in the classroom as alternative algorithms. This results in cognitive gains. Also, children feel a sense of closeness with their ancestors by bringing ethnomathematical practices into the classroom and learning the methods and approaches they have developed in their cultural context to deal with problems facing them. An extensive intervention by George Ghevarghese Joseph (UK) further exemplified the drawing from ethnomathematical sources by reporting on algorithms used by street prodigies on mental arithmetic in India. The algebraic explanation of the algorithms they use can be an important element in teaching algebra.

The panel on *history of mathematics in the teaching of mathematics* had four participants. Evelyne Barbin (France) spoke on *a case for the teaching of mathematics in a historical perspective*. Her presentation described, with several illustrative examples, the experience of the IREM in France in teaching the history of mathematics based on the reading of old texts. Helena M. Pycior (USA) spoke on *mathematics teaching with history*, which reported on a project offering a course on History of Mathematics for teachers of grades 7 through 12 and on the preparation and testing, for this same course, of sample projects on history applied to mathematics instruction. Arpad Szabó (Hungary) spoke on *relations between history and pedagogy of mathematics*, claiming that historical viewpoints in the teaching of mathematics help also to deepen the understanding of mathematics itself, and giving examples supporting this claim. Hans Wüssing (DDR) spoke on *the teaching of history of mathematics*, emphasizing the importance of bringing the social context into the historical presentation and of discussing the social momentum in which advances in mathematics take place. The panel was presided over by Ubiratan D'Ambrosio (Brazil).

The fourth time slot was devoted to short contributed papers and was also presided over by U. D'Ambrosio. Ten minute presentations were given by László Filep (Hungary) on *using the history of mathematics in teacher training*, Ryosuke Nagaoka (Japan) on *mathematical education beyond the training of mathematical literacy*, Zofia Golab-Meyer (Poland) on *some difficulties in understanding mechanics notions in history and in the school*, Rudolf Bkouche (France) on *ce que l'histoire des mathematiques peut apoporter a l'enseignement: l'exemple de la geometrie*, Robert Hayes (Australia) on *history as a way back to mathematics*, and Circe M. Silva da Silva (Brazil) on *Forschungsprojekt über Geschichte der Mathematik und Mathematikunterricht in Brasilien*.

The presentations drew in general much interest and discussion.

(as included in the Proceedings of the Sixth International Congress on Mathematical Education)

Call for Papers

The annual meeting of the Americas section of HPM will take place in Orlando on April 11-12, just prior to the annual meeting of the National Council of Teachers of Mathematics. Papers on any aspect of the relationship of history and pedagogy of mathematics are welcomed. Please send abstracts to Erica Voolich, 244 Summer St., Somerville, MA 02143 to reach her no later than March 15.

Mathematics Teacher (India) Reorganized

According to a new plan, the *Mathematics Teacher (India)*, published by the Association of Mathematics Teachers of India, will be composed of various sections, each to be put under a sectional editor. Dr. R. C. Gupta has been appointed to look after the section on History of Mathematics. He takes this opportunity to invite articles and other items on history of mathematics especially useful for educational purposes. His simultaneous editorship of the journal *Ganita Bharati* makes it possible also to accept material covering research and other activities in the field of the history of mathematics. Contributions to either journal may be sent to Dr. R. C. Gupta, Professor of Mathematics, Birla Institute of Technology, P.O. Mesra, Ranchi - 835 215, INDIA.

TME-Conference Proceedings Available

The International Study Group "Theory of Mathematics Education (TME)", founded in 1984 at ICME 5 in Adelaide, Australia, has thus far held three international conferences. The proceedings of TME-1, a post-congress conference held in Adelaide in 1984, are available from: IDM, University of Bielefeld, P.B. 8640, D-48 Bielefeld, F.R. Germany (Prof. H. G. Steiner). The proceedings of the 1985 Bielefeld conference TME-2: "Foundations and Methodology of the Discipline Mathematics Education (Didactics of Mathematics)" (ed. by H. G. Steiner and A. Vermandel) and of the 1988 Antwerp conference TME-3: "Investigating and Bridging the Teaching Learning Gap" (ed. by A. Vermandel and H. G. Steiner) are available from: Universitaire Instelling Antwerpen, Universiteitsplein 1, B 2610 Wilrijk, Belgium (Prof. A. Vermandel). The prices are resp. US $ 5, 10, 5 plus postage.

Symmetry of Structure

An interdisciplinary symposium on the Symmetry of Structure will be held in Budapest, Hungary from August 13-19, 1989. The fields of interest of the symposium include geometric-morphologic-architectonic aspects of symmetry (dissymmetry, asymmetry) in different disciplines, arts, and skills, emphasizing concrete science-art, nature-technology, or man-machine relationships, and intercultural (e.g. East-West) as well as historic approaches to symmetry. Extended abstracts of proposed contributions (3-4 pages with short bibliography and black-and-white drawings) should be submitted by March 15, 1989 to Dénes Nagy and György Darvas, Co-chairmen, Institute for Research Organization of the Hungarian Academy of Sciences, P.O. Box 4, Budapest, H-1361, Hungary. The final, camera-ready versions of the accepted abstracts are then expected by April 30, 1989.

Canadian Society for History and Philosophy of Mathematics

The Canadian Society welcomes members from all over the world. Papers presented at the annual meetings, held in conjunction with the Canadian Learned Societies meeting in late May or early June, include both original research in the history of mathematics as well as papers dealing with the application of history to teaching at various levels. The 1989 meeting will be

held on May 29 and 30 at Université Laval, in Québec. In commemoration of the bicentennial of the French Revolution, a special session dealing with eighteenth century mathematics will be organized by Professor Craig Fraser, Institute for the History of Science, University of Toronto, Toronto, Ontario, M5S 1K7, CANADA. The remainder of the program is being organized by Professor R. H. Eddy, Department of Mathematics and Statistics, Memorial University of Newfoundland,, St John's, Newfoundland A1C 5S7, CANADA, to whom abstracts should be sent before the end of February.

Culture Break!

Roger Herz-Fischler, Dept. of Mathematics and Statistics, Carleton University, Ottawa, Ontario K1S 5B6, Canada

Two of the difficulties facing anyone trying to introduce history in a mathematics course or curriculum are:

1. colleague resistance;
2. student resistance.

I personally have abandoned all hope as regards the first item and have adopted a devious approach to the second. In most of my courses I simply stop the lesson half way through each period and, under the guise of giving the students a breather and increasing their "culture", I spend anywhere from one to five minutes talking, very informally, about some sort of historical topic. Subjects that I have discussed include

1. How wallpaper helped Sofia Kovaleskaya become a mathematician!
2. Peruvian quipus.
3. Counting systems in Papua New Guinea.
4. Babylonian cuneiform, base 60 and the calculation of the diagonal of a square. (I do this topic over several periods.)

On each test I give a "cultural question" worth one mark.

The effect of these culture breaks can be measured by the remarks of students whom I have met several years after they have taken my course. Invariably they do not remember much of the mathematics, but do remember the culture breaks. In a recent course survey the students commented almost as much on the Babylonians as on the course and my teaching!

A Mathematical Tour of England

A tour of England which will trace the paths of mathematicians such as Newton, Babbage, Wren, and Somerville as well as learn about the British educational system and mathematics curricula will be conducted from July 10-24, 1989 by Susan Indorf, Skagit Valley College, 1201 E. Pioneer Way, Oak Harbor, WA 98277. The comprehensive cost from Seattle is $ 1600; arrangements can be made to begin the trip from other places as well. Direct all inquiries to Prof. Indorf at the address above or by phone at (206) 675-6656.

Colonial Mathematics in Brazil

Ubiratan D'Ambrosio

"In the summer of 1950, "American Mathematics" had experienced a metamorphosis. The "colonial period" had ended by the time of the Oslo Congress." (Springer's ICM Album – 1986)

Although this paper refers specifically to Brazil, the general problems posed by it are characteristic of the period between the so called discoveries and the establishment of colonies in the region now called the Americas, up to the independence of these colonies. The model of conquest and colonization conducted by Portugal and Spain and to a certain extent followed by England, Holland and France in this region have some similarities which pose historiographical and methodological problems for the History of Mathematics in that period. Even if the mathematical contribution of these regions are close to negligible, it is worth looking into what was

going on there. The reasons are clear. It contributes to a better understanding of the social and institutional history of mathematics by raising questions such as those following. How did Mathematics fit into such major national enterprises as the colonial movement? What was the place of Mathematics in the cultural and economic projects underlying the colonial adventure? This seems to have been somewhat neglected in treating the History of Mathematics in the XVIth, XVIIth and XVIIIth centuries, the height of the colonial venture in the Americas.

The quote in the beginning synthesizes in its use of the quotation marks two basic problems. The first comes from the calling of a practice "American Mathematics", which suggests a challenge to the universality of Mathematics, at least as far as problems, priorities, and even methods are concerned. And the second is the meaning of "colonial period" when dealing with intellectual production, suggesting that the ties with the colonial power were not severed even after the bloody independence wars. The colonial adventure was indeed a long lasting, in some cases permanent, process of cultural imposition of certain modes of thought, such as art, religion, political and judiciary institutions, schooling, science and technology and, mainly and unquestionably, Mathematics. This might have been thought as the guarantee of continuing dominance even after the end of political dependence. I doubt this was a consicous act. This poses an additional problem for the philosophy of science, that is the control of practitioners over the dynamics of scientific development. Maybe Arthur Clarke's "2001" metaphor refers to Science itself.

Other historiographical challenges may be raised, such as what is the meaning of Mathematics in the colonial environment. Why do Mathematics there? Is there much meaning talking about Mathematics in such an environment? In Europe itself Mathematics was then barely identified as a discipline and its practitioners were more like amateurs, playing certain intellectual games, with professional careers of diverse nature. These careers were unlikely to be found in the colonies. Maybe some occasional visitors or servicemen or missionaries would have used their free time to play the games they liked. Maybe these early practitioners were geniuses in the wilderness? Maybe many of them still are nowadays! Clearly, a production system was in the build-up process in the colonies, requiring ingenuity and techniques with some characteristics of Mathematics, more properly of Applied Mathematics in the medieval meaning of the term, such as Merchant and Practical Mathematics for Construction, Mining and Agriculture, for Cartography, Land Surveying and Demarcation, and surely as well for divinatory purposes. All this was practiced by people drawing from their multivaried cultural background. We surely must take into account the cultural diversity of the colonial populations, with obvious implications and effects in their motivation, their creativeness and their entire cognitive processes. The *ethnomathematics* thus created and currently practiced was building up as a form of knowledge adequately related to the specifics of the colonial environment. They were and still are going on, but they have been, and still are, consistently unaccounted for.

The above remarks are the essence of a research program on "The Social History of Sciences in Colonial Brazil". See [1] for the so-called traditional approach and [2] for the ethnomathematical approach.

In order to look into Mathematics in colonial Brazil we have to take into account some important facts in the history of Portugal. Let us remind the reader that Portugal was scientifically quite developed in the middle ages and the early Renaissance, drawing from Arabic science and benefiting from its European connections and a rather stable political situation. The University, founded in 1290 in Coimbra and frequently moving to Lisbon and back to Coimbra, was an important cultural center. Under King John II (1481–1496) Portugal experienced important scientific advances related to its trading successes in the Atlantic markets. One of the key achievements of Portugal was the construc-

tion of the "caravela" (Portuguese Man of War) and all of the navigational techniques related to it. This involves a rather sopisticated knowledge of Algebra and Geometry and the name of Pedro Nunes (1492-1577) towers as the most important mathematician of the early XVIth century. The opening of the southern Atlantic routes by Bartolomeu Dias (1488) and Vasco da Gama (1498) are major achievements which culminated with the "discovery" of Brazil in 1500. The establishment of a colonial enterprise purely exploratory followed. For details see [3].

Meanwhile, moved by an unbelievable miscalculation of its force, Portugal was economically and politically dismantled in the disaster of Alcacer-Quibir in 1578, and its survival was only possible by drawing into the colonies. The limited economic and human resources of the Kingdom were directed to the protection of the colonies and to the royal hegemony. Jesuits were given a major role in keeping this hegemony through a well organized and very conservative educational system. Basic teaching included classical mathematics through Portuguese textbooks, for example *Engenheiro Portugues* (1728) and *Logica Racional, Geometrica e Analitica* (1744) by Manoel de Azevedo Fortes (1660-1749), and a Portuguese translation of the *Elements* of Euclid by Father Manuel de Campos, in 1735. Nothing of the impressive scientific developments going on in Europe was incorporated, although some capable mathematicians can be recognized among the Jesuit teachers. We mention Athanasios Kircher, whose studies in Sacred Geometry are quite relevant. The astronomer Cristovao Schneirer identified solar spots independently of Galileo. Better known is Valentin Estancel (1621-1705), whose observations and calculations of the 1688 comet, while living in Brazil, were given credit by Isaac Newton in the *Principia*. Also cartography was an important aspect of the Jesuitic presence. Obviously, the process of demarcation of the colonies was related to the political playing of the King of Portugal. The disputes with Spain, centered in the Tordesillas Treaty, called for manipulating longitude measurements. Important demarcations in Brazil were carried on from 1573 through 1578 by Luis Teixeira, by the Swiss priest Aloisio Pfeil (1638-1701), and by the so-called "mathematical priests", the Portuguese Diogo Soares and the Italian Domenico Capassi. The conservative mood of the Jesuits prevailed and culminated with an edict in 1746 prohibiting the teaching of Descartes, Gassendi, and Newton in Jesuitic schools.

Other mathematical related activities in Brazil were related to defense and two contributions of some importance for military training are due to Jose Fernandes Pinto Alpoym, *Exame de Artilheiro*, published in Lisbon in 1744, and *Exame de Bombiero*, published in Madrid in 1748. These are the first two books of Mathematics written in Brazil by a Brazilian. They were published in Lisbon and Madrid since the Portuguese kingdom, contrary to the Spanish, had prohibited printing presses in the colony. Both books use a method of questions and answers. The first one more elementary, has three parts: Arithmetic, Geometry, and Artillery. The second, somewhat more ambitions, has ten parts: Geometry, "New Trigonometry", Longimetry, Altimetry, On Mortars, On Masonry, On Obuses, On Bombs, On Batteries of Mortars, with two Appendices on piling of spheres, and On Pyrotheia, or the study of artificial fires for war purposes. Alpoym is also known for his architectural works, being responsible for several important governmental buildings, and for the planning and the urban and architectural design of the city of Mariana, in the State of Minas Gerais. The two books of Alpoym were used in the metropolitan military academies. It must be pointed out that the main centers for Mathematics in the Iberian peninsula in the XVIIIth century were military related institutions. Particularly in Spain, we mention the founding of the "Academia Militar de Matematicas" in Barcelona in 1739, and of the "Sociedad Matematica Militar" in Madrid in 1756.

The books of Alpoym are rather elementary, not using any Calculus. As we said above, the later one, *Exame de Bombeiro*, is somewhat more

ambitious, and claims to introduce some novelties. What the author calls "new trigonometry" is justified as a novelty because of the "use [of] only one theorem and a proposition of Euclid and logarithmic sines and natural numbers". The most interesting parts are the Appendices describing "the easiest method that is possible to invent" in order to know the number of balls and bombs in a pile, and to know the dimensions of any pile, triangular or quadrangular, which holds a given number of balls or bombs. The author says that this method was invented by him after his former book had been sent to press. These and other remarks suggest a concern with Mathematical research by its author. We could find no indication that this was shared through any form of institutional or even social arrangement.

Major changes occurred in Portugal in 1750, when King Joseph appointed Sebastiao Jose de Carvalho e Mello, count of Oeiras and marquis of Pombal, as prime minister. He took over the effective rule of Portugal with strong powers. His main action was the expulsion of the Jesuits from Portugal in 1759 and the promotion of deep modernization reforms in the educational system. Particularly affected was the University of Coimbra, with the creation of a chair in Mathematics in 1773. Among the major appointments was that of young José Anastácio da Cunha, who became known for his works in the convergence of series. For details see [4]

Pombal lost power in 1777. Although he did appoint his nephew Francisco Xavier de Mendonca Furtado to take charge of demarcation of Brazilian frontiers, which was done by hiring modern foreign cartographers, most of the Pombal modernization measures did not positively affect Brazil. The educational system suffered from not getting the needed replacements for the Jesuits, and incipient independence movements, somewhat related and inspired by the American revolution, were suffocated by the strong government of Pombal. It is true that a number of Brazilians going to study in Coimbra may have benefited from the modern curricula and courses taught there, but there is no evidence of any important Mathematical advancement in Brazil as a consequence of this. For more on this see [5]. A collection of data on Brazilian Mathematics students in Europe, particularly Coimbra, in the colonial period, both before and after Pombal, is long due.

This brief account of the colonial period in Brazil covers up to 1808, when the Royal Family and the entire Portuguese court moved to Brazil as a consequence of Napoleon's threat to invade Portugal. King John VI established his government in Rio de Janeiro and was naturally led to create cultural facilities and an infrastructure in the new capital of the Kingdom. The result was the beginning of higher education in Brazil, the establishment of a quite important Library and of several institutes. The Royal Family returned to Portugal in 1821, leaving the way paved for Brazil proclaiming its bloodless independence from Portugal in 1822. The country opted to become a constitutional Empire, with the Portuguese heir being crowned as Peter I. He abdicated in 1831 leaving in his place his Brazilian born son, who became Emperor Peter II and who reigned until the Republic was proclaimed in 1889. The republican movement was strongly influenced by positivism, with interesting consequences for the development of Mathematics. But this does not belong to what we have agreed to consider the colonial period.

We should mention that in the XVIIth century the Dutch had established a flourishing colonial enterprise in Pernambuco and other parts of Northeast Brazil, which lasted from 1630 through 1654. Some scientific accounts and technological developments remained after the Portuguese succeeded in retaking their possessions. An account of this is given by Dirk J. Struik in [6]. Attempts by the French and English to establish colonies in Brazil were also short lived. Deserving mention also is the so-called "Republic of the Guaranis", a unique colonial project conducted in the XVIIth century by the Jesuits in Southern Brazil and parts of today's Argentina and Paraguay. This project tried new models of property and production, with a high degree of

collectivism. The experiment is sometimes referred to as a model of a Christian communist republic. It was completely destroyed and in the few remains we could not identify as yet elements of importance for the History of Mathematics

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6. Dirk J. Struik: *Revista da Sociedade Brasileira de Historia da Ciencia*, v. 1, no. 2, 1987.

Have You Read?

Readers are asked to submit contributions. References need not deal exclusively or explicitly with history in the mathematics classroom, but should have the potential for motivating or enriching. Please supply complete bibliographic information: names of author(s); complete titles of books, articles and journals; for journals include both the volume and date; for books, edition, copyright date, publisher and place of publication. Accuracy in spelling and wording is critical. Please provide concise annotations whenever possible. (For various reasons, the listing here is very brief. Starting with the next issue, the editor of this section will be Professor Ronald Calinger, Department of History, Catholic University of America, Washington, D.C. 20064; please send all submissions to him.)

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