



International Study Group On the Relations Between
HISTORY and PEDAGOGY of MATHEMATICS

NEWSLETTER

AN AFFILIATE OF THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION

No. 9

May 1985

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This Newsletter may be entirely or partially duplicated or reproduced. Please send Editor a copy of excerpted material.

Calendar

Meetings with HPM components are highlighted.

1985 June 2-4 Montréal
Canadian Society For History and Philosophy of Mathematics. Contact: Louis Charbonneau, Dept Math, U du Québec, CP 8888, Montréal, Qué H3C 3P8

1985 July 21-25 Havana
Primer Congreso Latinoamericano de Historia de las Ciencias y la Tecnología y la Segunda Asamblea General de la Sociedad Latinoamericana de Historia de las Ciencias y la Tecnología. Contact: Comité Organizador del Primer Congreso Latinoamericano de Historia de las Ciencias y la Tecnología, Palacio de las Convenciones, Apartado 16046, Zona 16, Habana, Cuba. (Télex 511609 PALCO CUBA)

1985 July 31-August 8 . . . Berkeley, Calif.
XVII International Congress on the History of Science (HPM tentative)

1985 November 1-3 Bloomington, Ind.
History of Science Society. Contact: HSS, 215 South 34th St/D6, Philadelphia, Pennsylvania 19104.

1985 November 13-16 New Delhi
Colloquium on the History of Oriental Astronomy. Contact: Dr A.K. Bag, Indian National Science Academy, Bahadur Shaf Zafar Marg, New Delhi 110002 India.

1986 January 9-11 New Orleans
Mathematical Association of America.
(HPM tentative; see inside)

1986 Mar 31-Apr 5 . . . Washington, D.C.
National Council of Teachers of Mathematics (details inside)

May 1985 / No. 9

2

HPM Newsletter

1985 August 3-11 Berkeley, Calif
International Congress of Mathematicians
(HPM tentative)

1988 July 27-Aug 3 Budapest
International Commission on Mathematics
Education (ICME 6). Contact: A.G. Hovson,
Secy. ICMI, Univ Southampton, Centre
For Mathematics Education, Southampton,
SO9 5NH, U.K. HPM will participate; watch
for details.

Editorial

The Cultural Dimension of Mathematics
(Ubiratan D'Ambrosio—) The distinguished historian J. Morgan Kousser wrote recently "history is better than ever," and he attributes this to the growing interaction of historians with other social scientists which becomes evident after the 1950's. We may paraphrase him and claim that history of mathematics is growing to a firm and mature position in mathematics education, and that it won't take much longer for us to see history of mathematics having a place in the mathematics curriculum. Once again paraphrasing J. Morgan Kousser, this is partly due to a broader vision of history of mathematics leading to an externalist approach.

It now becomes accepted and encouraged to look at mathematics in its socio-cultural, economic and anthropological contexts. Mathematicians now pay renewed attention to the references in the works of anthropologists such as H. Kroeber and L. White, historians such as A. Toynbee and sociologists as M. Hesse—hitherto practically unnoticed. The pioneering work of R.L. Wilder has become better known and a range of works on mathematics and art history, mathematics and music, and mathematics and the artesans receive considerable attention among mathematicians. Publications such as *Leonardo and Art Journal* (the official journal of the College Arts Association of America) welcome articles on historical aspects of mathematics. Books such as Hofstadter's *Gödel, Escher and Bach* and

Linda D. Henderson's *The Fourth Dimension and Non-euclidean Geometry in Modern Art* reach mathematicians in a marked way and are used as course references.

The relations of mathematics to cultural behaviour are clear. It is no novelty that much of what we may call astronomical mathematics of the ancient civilizations developed in close relation to socio-cultural structures allowing the ruling classes to control the means of agricultural production. It is no novelty that the revolution in the history of ideas which took place in the Christian reform movement, opened and paved the way for modern sciences through the exploration of more ideas in mathematics in the XVI, XVII and XVIII centuries. But not much has been emphasized about this in writings on the history of mathematics. Similarly, the fact that different civilizations and different cultural groups practice different modes of mathematics has only recently been recognized, mainly by cultural anthropologists, and consequently rarely reaching mathematicians and mathematics educators. In the few cases in which these facts are recalled they are used to illustrate mathematical curiosity in a different culture and never as an instructive component of the curriculum.

We now see a clear indication of change of attitude in the history of mathematics regarding external components and a definitive acceptance among mathematics educators of the fact that mathematics education rests on a socio-cultural basis. These are the components of the optimism revealed in the first sentence of this editorial. Let us bring history of mathematics to its fundamental role in the curriculum and thereby reveal the central role of mathematics in the complex cultural intertwining which is the framework upon which our educational systems are built. □

From the Editor

Please note the new address for the Editor:
Department of Mathematical Sciences, Ball
State University, Muncie, Indiana 47306, U.S.A.

This has been a year of progress and expansion for HPM. We see increased interest in HPM evident in the expanding mailing list for the *Newsletter*, which now has about 630

names. Some recipients duplicate and send the *Newsletter* to others, so the total mailing list is in excess of this number and the readership is larger still. In addition, the restructuring of the organization at the Adelaide meeting in 1984 reflects the need for an expanded organization with a wider audience. The North American Edition of the *HPM Newsletter* was adopted as the international edition, and provisions were made for regional supplements which will provide detailed coverage of regional events. A regional supplement is already being distributed in the Americas and other are planned for Australia and Europe. It is hoped that these regional supplements will provide even better communications by tailoring the reporting to regional needs.

This also completes the first year for the *Newsletter* editorial offices at Ball State University, and in this retrospective note I wish to call attention to the support that we have received. The Ball State Department of Mathematical Sciences completely underwrites the cost of producing the *Newsletter* and the Americas "Supplement". This represents a considerable financial and resource commitment, including the use of a computer, printer and a xerographic duplicating machine, a supply of paper and envelopes, toll phone calls, postage (we mail 375 to 400 newsletters in the U.S. and to those not on our mailing list who request copies, and send materials to distributors in other countries), and providing some student help. This has made it possible to expand the *Newsletter* and thereby establish a framework within which the Study Group, HPM, can function in a truly international manner. As Professor D'Ambrosio has pointed out in the Editorial in this issue, these sociological and institutional factors are recognized to be of fundamental importance. I therefore think it is fitting that we note this vital support for HPM and its *Newsletter* as we come to the end of this first year of operation at Ball State University.

As you will see in the reports in this *Newsletter*, much progress has been made in the Americas section toward producing materials for the classroom teacher. There should be tangible results to report in the near future with the appearance of a model classroom module and with the *Proceedings* of the

1985 Americas Section meeting in San Antonio. Here it should be noted that HPM is not a monolithic structure which requires threading one's way through bureaucratic procedures in order to begin a project. All that is needed is someone willing to take the initiative. So, if you have an idea for a project to develop historical material, or if you are already in the midst of one, the *Newsletter* is available to help you publicize and organize.

Once again I wish to note the assistance of Wanda Back in producing this issue. She has taken an active interest in HPM and will continue to contribute as she begins a new career. □

L'Université D'été Sur L'Histoire des Mathématiques du Mans

(Evelyne Barbin-La Rest) — La Commission nationale inter-I.R.E.M. "Histoire et Epistémologie des mathématiques" a organisé en Juillet 1984 une Université d'été consacrée à l'histoire des mathématiques, la première en France. Cette Université d'été, subventionnée par le Ministère de l'Éducation Nationale, a reçu l'appui financier de la Société Mathématique de France, de la Société Française d'Histoire des Sciences et des Techniques et de l'Association des Professeurs de Mathématiques. Elle a accueilli pendant une semaine, sur le campus de l'Université du Maine, une centaine de participants français et étrangers (Europe et continent nord-américain).

L'Université d'été a été organisée dans le but de répondre à une demande de formation des enseignants qui correspond à une rénovation en cours de l'enseignement des mathématiques, à la mise en place d'activités interdisciplinaires et au désir d'insérer l'histoire des sciences dans l'enseignement. Cette Université d'été, à vocation interdisciplinaire, était ouverte à des enseignants de collèges, de lycées et d'Universités des disciplines mathématique, philosophique, historique et physique.

L'université d'été avait pour objectif de replacer la production mathématique dans son contexte historique, philosophique et social, d'étudier la place des mathématiques et des sciences dans la culture et de mettre en évidence les rapports entre mathématiques et société. Les objectifs pédagogiques étaient de

lutter contre un enseignement dogmatique et formel des mathématiques, de définir à partir de l'histoire des mathématiques, des pratiques renouvelées d'enseignement en montrant que le savoir mathématique se construit à partir de problèmes et de situations et en précisant le rôle de l'erreur, de la conjecture et de la rigueur.

L'organisation de l'Université d'été comprenait des conférences, des exposés, des ateliers, des compte-rendus d'expériences pédagogiques, une table ronde et une exposition.

Quatre conférences avaient pour but de donner les éléments essentiels et les idées générales permettant d'entreprendre les travaux en atelier. Deux conférences portaient sur les conditions philosophiques et sociales d'émergence d'une pensée scientifique et de travaux mathématiques: dans la Grèce antique (F. De Gandt, C.N.R.S.) et dans les sociétés arabo-musulmanes (A. Djebbar, Université d'Orsay). Deux conférences abordaient deux grands thèmes des mathématiques envisagés dans une perspective historique: algèbre (C. Houzel, Université de Paris Nord) et analyse (J. Dhombres, Université de Nantes). Deux conférences avaient pour objet l'enseignement des mathématiques: l'une traitait de l'utilisation de l'histoire des mathématiques dans cet enseignement (R. Stovasser, Université technique de Berlin) et l'autre de l'histoire de la réforme des "mathématiques modernes" (B. Charlot, Ecole Normale du Mans). Quatre exposés ont traité de points précis de l'histoire des mathématiques: les fractales, le nombre en Chine, l'infini au Moyen-Âge et l'oeuvre de Martin Ohm.

Dix ateliers ont fonctionné pendant l'Université d'été. Le travail en atelier avait pour but de donner une formation en histoire des mathématiques mais aussi une formation méthodologique à la lecture des textes mathématiques: étude du contexte scientifique, philosophique ou social, recherche d'une problématique épistémologique ou didactique. Les ateliers étaient animés par les formateurs I.R.E.M. ou des chercheurs en histoire des sciences. Les participants y ont travaillé à partir de documents originaux et de textes historiques et épistémologiques élaborés dans les I.R.E.M. Les thèmes des ateliers furent les

suivants: algèbre, géométrie, représentation des mathématiques dans la pensée philosophique, analyse, algorithmes, mathématiques et société, arithmétique, astronomie et le statut des sciences au 17ème siècle.

Plusieurs compte-rendus d'expérience d'insertion de l'histoire des mathématiques en lycée et en collège ont permis d'étudier les motifs, les conditions et les résultats de ces expériences et d'engager une réflexion sur les rapports entre histoire et enseignement des mathématiques. Cette réflexion fut reprise lors d'une table ronde, placée au terme de l'Université d'été, ou furent confrontés les différents enjeux et conceptions de l'insertion de l'histoire des mathématiques dans l'enseignement et dans la formation des enseignants.

Pendant l'Université d'été une exposition permanente présentait des documents sur l'histoire des mathématiques élaborés dans les I.R.E.M. et des travaux élaborés lors de deux "Projets d'Action Educative", l'un sur Mersenne en collège et l'autre sur le Perugin en lycée.

L'Université d'été est apparue comme une expérience positive par la quantité d'information apportée et la qualité des échanges. Elle a créé une dynamique qui se traduit par l'élaboration de projets personnels, par la mise en place de nouvelles formations en histoire des mathématiques et par la constitution de groupes de recherche. La Commission Inter-I.R.E.M. se propose, en conséquence, d'organiser une nouvelle Université d'été en Juillet 1986.

Il est prévu une édition des Actes de l'Université d'été. Les personnes intéressées doivent écrire à l'adresse suivante: Université du Maine, I.R.E.M., Route de Laval, 72000 LE MANS, France. □

Post-doctoral Fellow at Ball State

Abraham Arcavi of The Weizmann Institute in Rehovot, Israel, has been awarded a post-doctoral Fellowship by Ball State University in Muncie, Indiana, for the academic year 1985-1986. His appointment will be in the Department of Mathematical Sciences.

Dr. Arcavi's doctoral research has included developing course materials for training teachers of mathematics which use the history of mathematics. This work was supervised by Professor Maxim Bruckheimer and Dr. Ruth Ben-Zvi of the Science Teaching Department of The Weizmann Institute. (A report was in *Newsletter* No. 6.) At Ball State University, Dr. Arcavi will collaborate with Professor Charles V. Jones. □

Canadian Historians of Mathematics Meet in Montréal

The Canadian Society for History and Philosophy of Mathematics/Société canadienne d'histoire et de philosophie des mathématiques will hold its twelfth annual meeting at the Université de Montréal on June 2-4, 1985. The society meets in conjunction with the Conference of Learned Societies and all sessions will be held in Pavillon Lionel Groulx, room LG B101. A number of speakers are scheduled and two talks on Monday afternoon will be on pedagogy: one session is devoted to developing a kit of pictorial material for use in a history of mathematics course.

The president of the Society, Professor Edward Barbeau of the University of Toronto, has announced plans to develop a series of monographs published jointly by the society and Wilfred Laurier University Press. He also announced plans to collect and distribute classroom historical notes or surveys of general interest. Further information about the Society may be obtained from Prof. Louis Charbonneau, Département de mathématiques et informatique, U.G.A.M. (Université du Québec à Montréal), C.P. 8888, Succ. A, Montréal, Québec H3C 3P8 (phone 514/282-3217). [Taken from the *CSHPM-SCHPM Newsletter*.] □

Americas Section Meets and Plans Publications

The Americas Section of HPM met in San Antonio, Texas, April 15 - 17, 1985, immediately prior to the annual meeting of the National Council of Teachers of Mathematics. Florence Fasanelli arranged and chaired the sessions, which included speakers from Canada, United States, Brazil, and Germany on a variety of topics: Linda Dalrymple Henderson, "Art and

mathematics: the fourth dimension"; Victor Katz, "The development of a module on power series"; V. Frederick Rickey, "Modules and module writing: material on what a module should contain"; Israel Kleiner, "A module on the evolution of the function concept"; Roland Stovasser, "Organizing ideas from history for the classroom"; Barbara Gale, "A Junior College Math Course for Liberal Arts Majors", and Ubratan D'Ambrosio, "ICME 5". Charles V. Jones led a discussion on planning a module which resulted in those present agreeing to put together a model module on the quadratic equation.

It was decided to undertake the publication and distribution of two works: the proceedings of the conference and the module on the quadratic equation. The module will consist of historical and background material including a biographical sketch of al-Khwarizmi, descriptions of al-Khwarizmi's solutions to the quadratic, illustrations, exercises, and a bibliography. There will also be a computer program with some graphics (in BASIC) which will guide a student through one of al-Khwarizmi's solutions. A discussion and critique of a draft of the module is planned for the New Orleans meeting of the Mathematical Association of America in January, 1986 (see elsewhere in this issue). The finished module should then be available for distribution at the Americas Section meeting in Washington, D.C. next April.

The second publication, the proceedings of the 1985 meeting, will contain the papers presented at San Antonio. It will be printed and bound in an inexpensive manner and distributed by HPM. Proceedings of future meetings are possible, if enough interest exists, and they perhaps will be produced and distributed through an established publisher.

Charles Jones will act as editor of the proceedings and will also coordinate the writing and compiling of the module on the quadratic equation. Announcements of the availability of both the module and the proceedings will appear in the *Newsletter*. □ If you are interested in receiving either or both, or would want a copy for your library, write to the editor, Charles V. Jones, Department of Mathematical Science, Ball State University,

Muncie, Indiana 47306, U.S.A. This will help gauge the amount of support and interest there is for each of these projects. You will be notified when each is available. □

Future Meetings of HPM

Several meetings are already in the plans for HPM, some of a tentative nature. This information is provided to help you make your conference and travel plans in order to take advantage of HPM activities.

The next major international meeting of HPM will be at the ICME 6 conference in Budapest, Hungary, 27 July to 3 August, 1988. No program plans are available at this time. Some less ambitious international meetings of HPM will take place at international conferences planned for this summer and next. The International Congress on the History of Science will meet at Berkeley, California, 31 July to 8 August, 1985. Co-chairman Ubratan D'Ambrosio will arrange for a meeting room and announce the location. As reported in the last *Newsletter* the meeting is planned for the afternoon of August 8th, from 3 to 7, immediately following the symposium on the transmission of mathematical sciences. Co-chairman D'Ambrosio is also planning a meeting of HPM for the International Congress of Mathematicians the following August 1986, again in Berkeley, California. (This is a confusing coincidence of month and city, so please note carefully.) The International Commission on Mathematical Instruction, the parent organization of HPM, will meet as part of the International Congress, and it is hoped that a meeting of HPM may be scheduled at about the same time. Details on these meetings will be reported as available.

The Americas Section of HPM, through the efforts of Florence Fasanelli and Fred Rickey, has a very unusual meeting planned for 31 March to 2 April 1986 in Washington, D.C. Tours and special talks have been arranged with several institutions in the U.S. capital: the Smithsonian Institution, Naval Observatory and Library, the Rare Books Room of the Library of Congress, the National Gallery of Art, and the Taylor Model Basin are among the attractions. This meeting immediately precedes the annual meeting of the National Council of

Teachers of Mathematics (April 2-5). In a break with the pattern of the past four years, the Americas Section is also in the process of making plans for a session at the New Orleans meeting of the American Mathematical Association (January 1986, with the American Mathematical Society). A short program will be planned including a discussion of the draft of the classroom module on the quadratic equation which the group is developing.

Mark your calendars for these meetings. If your plans should include travel near these locations, make plans to attend. Anyone interested in pedagogy and history is welcome to attend HPM meetings. □

Conference On History of Science In Central America

The First Central American Conference on the History of Science and Technology is planned for 10-14 June 1985 at the University of Costa Rica in San José. The central theme of the conference is "The History of Science and Technology in Teaching" and a variety of sub-themes will be explored. The chairman of the organizing committee is Prof. Angel Ruiz-Zúñiga (Escuela de Matemática, Universidad de Costa Rica, San José; tel: 25-55-55 ext 742 - Of. 420). □

History of Modern Mathematics Conference

A conference on the History and Philosophy of Modern Mathematics was held at the University of Minnesota, May 17-19, 1985. It was sponsored by the Alfred P. Sloan Foundation and the University of Minnesota and organized by William Aspray and Philip Kitcher. Talks were on logic, nineteenth century mathematics, Felix Klein, non-standard analysis, among others, and one specifically on teaching: "Writing and teaching history of mathematics: a case study", by Michael Crove of Notre Dame. [From the conference program.] □

Education Group Will Meet in Québec

The ninth annual meeting of the Canadian Mathematics Education Study Group will be held at the Université Laval, Québec, Québec, June 7-11. For more information and to register, contact Charles Verhille (506/453-3500) or

David Wheeler (514/482-1320, ext 727 or 362), during office hours. One of the Topic Groups is related to history: Topics Group B: La genèse trébule de la géométrie analytique et de la géométrie sythétique depuis l'antiquité en passant par Descartes, von Staudt et autres. The Topic Group leader is Fernand Lemay (Laval). [Taken from the CSNPM-SCHPM Newsletter.] □

Bibliography Available

All citations in the "Have You Read?" section of this Newsletter have been accumulated and are available in a single alphabetical listing. 135 citations have appeared, some with annotations, in the following issues: October 1983; February 1984; October 1984; February 1985; and May 1985. If you would like to receive a copy of this bibliography, send one dollar (\$1) to cover costs of copying and postage to: Charles V. Jones, HPM Editor, Department of Mathematical Sciences, Ball State University, Muncie, Indiana 47306. □

New Journal in Costa Rica

Desarrollo is a journal published jointly by the Asociación Costarricense de Historia y Filosofía de la Ciencia (ACOHIFIC) and the Grupo de Estudio en Ciencia, Tecnología, Planificación y Política (CITEPPOL). Contact at Apdo. 388, San Pedr 2050, Costa Rica. □

Notes and Queries

A new feature of the Newsletter is being instituted at the suggestion of the Americas Section of HPM: "Notes and Queries". This section is reserved for questions that readers would like to have answers for; these are the "queries". The answers (which at times may be for questions that have not been asked, yet) will be the "notes". The notes and queries should be basically historical or pedagogical.

If you have questions or answers about sources, dates, names, titles, facts, or other such matters, frame them in clear and concise language and send them to the Editor. If you are answering a question, make a clear reference to the question. All readers may send both questions and answers. Each will be published with the name of the sender. □

Have You Read?

Readers are encouraged to submit contributions to "Have You Read?". References need not deal exclusively or explicitly with history in the mathematics classroom, but should have the potential for motivating or enriching. *N.B.* Supply complete bibliographic information: names of author(s); complete titles of books or of both the article and journal; for journals include both the volume and date; for books, edition, copyright date, publisher and place of publication. Accuracy in spelling and wording is critical. Please provide concise annotations whenever possible. —Ed.

Albis G., Victor S., Editor 1983 *A.C.F. Gauss en el bicentenario de su nacimiento. Epistemología, Historia y Didáctica de la Matemática* V. Bogotá: Universidad Nacional de Colombia (iii+94 pp)
"To C.F. Gauss on the two hundredth anniversary of his birth".

Bekken, Otto B 1983 *Una Historia Breve del Algebra* Lima: Sociedad Matematica Peruana. Based on a series of lectures presented in Peru. See Newsletter No. 6, item on "Lectures on history of algebra presented in Peru".

1984 "On the problem of the reals—the problem of the limits" *Fagseksjon for matematikk* Skrifter: 1984:3.

In English; order from Agder Distrikthøgskole, Postboks 607, N-4601 Kristiansand, Norge (Norway). An appendix to the author's *Una Historia Breve del Algebra*.

1984 "Themes from the history of algebra" *Fagseksjon for matematikk* Skrifter: 1984:4.

In English; order as above. The lecture presented to the HPM meeting held in San Francisco in April 1984.

1984 "Readings from the Hindu arithmetic and algebra" *Fagseksjon for matematikk* Skrifter: 1984:5.

In English; order as above. Distributed at the HPM meeting in San Francisco in April 1984.

1984 "Matematikk I Utvikling" Bok I *Fagseksjon for matematikk* Skrifter: 1984:6. In Norwegian; order as above. A history based in part on works listed above. Treats Egyptian, Babylonian, Greek and tally mathematics with problems. Good bibliography.

Cajori, Florian 1890 *The Teaching and History of Mathematics in the United States* Government Printing Office, 400 pp (Reprinted 1974: Scholarly Res.)

Cooke, Roger 1984 *The Mathematics of Sonya Kovalevskaya* NY/Berlin: Springer-Verlag.

Coughlin, Mary, & Carolyn Kervin 1985 "Mathematical induction and Pascal's problem of the points" *Mathematics Teacher* 78:5 (May) 376-80.

Illustrates Pascal's use of mathematical induction in solving the problem of how to divide the stakes in an interrupted game of chance.

Crowley, Mary L. 1985 "The 'Difference' in Babbage's Difference Engine" *Mathematics Teacher* 78:5 (May) 366-72, 354.

D'Ambrosio Ubiratan 1985 "Ethnomathematics and its place in the history and pedagogy of mathematics" *For the Learning of Mathematics* 5:1, 44-48.

Develops the concept of ethnomathematics and its relation to the teaching of mathematics.

Dauben, Joseph W. 1985 *The History of Mathematics from Antiquity to the Present: A Selective Bibliography* (Bibliographies of the History of Science and Technology, 6; Garland Reference Library of the Humanities, 313) NY/London: Garland Publishing.

Dhombres, Jean 1978 *Nombre, mesure et continu. Épistémologie et histoire* Paris: CEDIC/Fernand Nathan.

A history of real numbers, with a chapter on China (in French).

DiDomenico, Angelo S. 1983 "Eureka! Pythagorean triples from the multiplication table" *Mathematics Teachers* 76:1 (Jan) 48-51.

1985 "Pythagorean triples from the addition table" *Mathematics Teacher* 78:5 (May) 346-48.

Engelsman, Steven B 1984 *Families of Curves and the Origins of Partial Differentiation* Amsterdam-New York: North-Holland Publishing Co.

Fleckenstein, J.O. 1977 *Johann und Jakob Bernoulli* 2nd Ed. (Kurze Mathematiker Biographien 6) Basel: Birkhäuser
A short biography, in German, with two portraits, four figures, and two facsimiles.

Glaeser, Georges 1983 "A propos de la pédagogie de Clairaut vers une nouvelle orientation dans l'histoire de l'éducation" *Recherches en didactique des mathématiques* 4:3, 332-344.

Clarification of Clairaut's teaching objectives within a general criticism of the history of education.

Gray, Jack 1985 A review of John Pottage's *Geometrical Investigations: Illustrating the Art of Discovery in the Mathematical Field*. (Addison-Wesley, 1983) in *For the Learning of Mathematics* 5:1, 12-14.

Hoggatt, V.E., Jr., & Marjorie Bicknell 1974 "Triangular numbers" *Fibonacci Quarterly* (Oct) 221-30.

Hovson, Geoffrey 1984 "On writing a history of mathematics education" *Recherches en Didactique des Mathématiques* 5:2, 239-52.

Mauldin, Lyle E. 1985 "An exercise with polygonal numbers" *Mathematics Teacher* 78:5 (May) 340-44

Mauldin, Daniel R. 1981 *The Scottish Book. Mathematics from the Scottish Café* Basel: Birkhäuser

A re-edition of the problems included in the "book" of the Scottish Café, Lvov, Poland, from July 17, 1935 (problem 1, by Banach) to May 31, 1941 (problem 193, by Steinhaus).

Meschkovski, Herbert 1984 *Problemggeschichte der Mathematik I (Second Edition)* Mannheim: Bibliographisches Institut.

"Problem history of mathematics, I".

Olson, Melfried, Gerald K. Goff, & Murray Blose
1983 "Triangular numbers: the building
blocks of figurate numbers" *Mathematics
Teacher* 76:8 (Nov) 624-25.

Page, Warren, Editor 1985 *American
Perspectives on the Fifth International
Congress on Mathematical Education (ICME
5)* Washington, D.C.: Mathematical Association
of America

Schubring, Gert 1984 "Chronique historique"
*Recherches en Didactique des
Mathématiques* 5:2, 235-37.

Schultz, Donald B. 1985 "Using sweeps to find
area" *Mathematics Teacher* 78:5 (May)
349-51.

An idea for teaching area to 7th & 8th
graders using techniques based on a
theorem of Pappus. The same issue of
MT, pp. 355 ff, contains a Logo program
(for an Apple computer) illustrating one
of the techniques Schultz discusses.

Smith, Joe K. 1972 "The n th polygonal number"
Mathematics Teacher 65:3 (Mar) 221-25.

Stark, Alice J. 1978 "Figurate numbers: an
introduction for the elementary classroom"
*North Dakota Teachers of Mathematics
Journal* 1, 25-27.

Stowasser, R.J.K. 1985 "History of science a
critical and constructive tool for the
mathematics curriculum" Berlin: Technische
Universität Berlin (Didaktik der Mathematik
Fachbereich 3) no.4.

Using the history of mathematics to
organize the curriculum. Includes "An
idea from Jakob Bernoulli for the
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Theses and Extended Research This
section of "Have You Read?" is a file of
research on history and teaching. Send
both old and new sources. N.B. Supply
complete bibliographic information. For theses,
supply name of author; complete title;
university; date; *Dissertation Abstracts
International* abstract identifier and page
number—thesis order number or similar data
for theses not in DAL

Colette, Jean-Paul 1974 "L'histoire des
mathématiques dans l'enseignement des
mathématiques: histoire et attitude des
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Montreal.

Theses from Canadian universities are
available from the National Archives of
Canada, Ottawa, Canada.

The Treviso Arithmetic

(Duane E. Deal—) Although there are a few
mathematical references in printed books as
early as 1472 and perhaps even as early as
1469, the first printed work devoted entirely
to mathematics is the *Treviso Arithmetic*. This
little book has no title page—it simply begins:
"Here beginneth a Practica, very helpful to
all who have to do with that commercial art
commonly known as the abacus." The anonymous
author goes on to explain that he has been
asked by "certain youths in which I have much
interest" to write down some of the principles
of arithmetic. From this we deduce that he
probably was a teacher. Treviso at that time
was a small commercial town 30 kilometers north
of Venice. It was an ancient Roman town named
Tarsivium, and includes an 11th century
cathedral and a university founded in 1318. At
least three printers were set up in Treviso
by the end of the fifteenth century though
it is not positively known which of them printed
this work. We are certain of the date because
at the end of the 62 pages appears the quaint

admonition: "What availeth virtue to him who
does not labor? Nothing. At Treviso, on the
10th day of December, 1478."

The book was not particularly popular, perhaps
because its commercial value was diminished by
a lack of covering rates of exchange among
the many monetary systems in use at the time.
Hence it is extremely rare today.

It covers the use of what we call the Hindu-
Arabic numeration system, then fairly new. The
numerals look much as they do today, except
that there was no 1 in the font, and a small
i is used instead.

The beginning of the first page is reproduced.
The first paragraph is translated above, and
the last word, "abacus", may be seen in the
last word of the first sentence, "labbacho".
The language is Venetian, a dialect of what
came eventually to be modern Italian. Because
fancy illuminated letters had become
fashionable just before the invention of
printing, early printed works often left space
for this artistic embellishment. At some
unknown time in the last five hundred years,
someone has very crudely "illuminated" in the
space provided. Note that below this is a space
for a second illumination which is still available.

Also reproduced is a page illustrating
multiplication. The product of 314 by 934 is
done in four ways, the top two being variations
of the method commonly in use today. The
other two methods are variations on the
"gelosia" method, so-called because of its
fancied resemblance to the ironwork at a
window. (English still preserves the word
"jealousie" window today.) One can easily pick
out the method used by finding the 934 and
the 314, by noting that the divided squares
contain the product of numbers in a given row
and column, and by observing that one adds
diagonally to obtain the answer which is
displayed around the edge of the rectangle.
This addition is suggested by the word
"Somma" at the bottom of the page, preceding
the answer of 293 276.

Illustrated below is the rather absurd product
of a 14 digit number by one of six
digits—fortunately two of the latter are
zeroes!

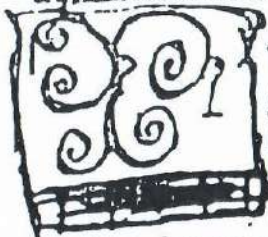
18949465889280	
360072	
7090771770160	
i 3264626124960	
i 1369679533568000	
56848397667840	
6023172081684828160	

From the Treviso Arithmetic

Although the *Treviso Arithmetic* was the
first printed book on mathematics, books on
the subject were produced incredibly often in
the remaining years of the fifteenth century
in Italy. By 1480 there were 38 such works,
there were 63 more by 1490, and 100 were
printed in the last decade, plus an additional
13 whose date is not known—an astonishing total
of 214 mathematical incunabula in Italy alone.

The following illustrations are taken from
David Eugene Smith's revision of Augustus
DeMorgan's *Rara Arithmetica*, published in 1908.
The quotations are due to DeMorgan and are
in the same work. □

Incomincia una practica molto bona et utile
a ciascaduno chi vuole exare l'arte dela mercha
d'agua chiamata vulgarmente l'arte de labbacho.



Regato piu e piu volte da alcuni
zouani a mi molto dilectissimi: li
quali pretendeuano a douer volere
fare la merchadantia: che per loro
amore me piace se affadigarme o
no puocho: de dargli in scritto qualche fundamēto
cerca l'arte de arismetria: chiamata vulgarmente
labbacho. Unde io costretto per amor di loro: et
etadio ad utilitati tutti chi pretendano a quella: se
gondo la piccola intelligenza del ingegno mio: ho
deliberato se non in tutto: in parte tanie satisfare a
loro. acio che loro virtuosi desideru vtile frutto re-
ceuerer posscano. In nome di dio adoncha: voglio
per principio mio el vnto de algorismo così dicēdo.

1. Tre quelle cose: che da la prima origine
hanno habuto producamēto: per ragione de
numero sono sta formate. Et così come so-
no: bano da fir cognoscuere. Per o ne la cognitione
de tutte le cose: questa practica e necessaria. Et per
inter nel pposito mio: primo sapi lectore: che qui-
to fa al proposito nostro: Numero e una moltitu-
dine congregata ouero insieme da molte vni-
tade. et al meno da dō vntade. come e. 2. el quale
e lo primo e minore numero: che se troua. La v-
ntade e quella cosa: da la quale ogni cosa si diua
vna. Segōdario sapi: che se troua numeri de tre
maniere. El primo se chiama numero simpliciter. l'al-
tro numero articulo. Et terzo se chiama numero

Voglio però che tu intendi che sono altri modi de
multiplicare per scachiero: li quali lassaro al studi-
o tuo: mettēdo li esempi soi solamente in forma.
come potrai vedere qui sotto
Oz toglì de fare lo preditto scachiero. 3oe. 3 i 4.
sia. 9 3 4. e nota de farlo per li quattro modi come
qui da sotto.

$$\begin{array}{r} 934 \\ 3236 \overline{) 4} \\ 934 \overline{) i} \\ 2802 \overline{) 3} \\ \hline 293276 \end{array}$$

$$\begin{array}{r} 934 \\ 3736 \overline{) 4} \\ 1934 \overline{) i} \\ 2802 \overline{) 3} \\ \hline 293276 \end{array}$$

Sca.

	9	3	4	
2	2	0	1	3
9	0	0	0	4
5	3	1	1	4
	2	2	6	

Somma.

	9	3	4	
	6	2	6	6
	9	3	4	2
	0	0	0	2
	7	9	2	2
	2	0	1	3
	2	9	3	