International Study Group On the Relations Between
HISTORY and PEDAGOGY of MATHEMATICS

NEWSLETTER

AN AFFILIATE OF THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION

No. 8 February 1985

The *Newsletter* is the communication of the International Study Group On the Relations Between History and Pedagogy of Mathematics, an affiliate of the International Commission on Mathematical Instruction. Editorial and production offices are in the Dept of Mathematical Sciences, Ball State Univ, Muncie, Ind. 47306 U.S.A. The *Newsletter* is available free of charge upon request. Distributed in the U.S. by Duane E. Deal (Ball State Univ, Muncie, Indiana 47306), in Canada by David Wheeler (Concordia Univ, Montréal, Qué H4B 1R6), in Mexico by Alejandro Garciadiego (UNAM-contact at: José M. Velasco 71, Del. Benito Juárez 03900, Mexico, D.F.), in South America by Ubiratan D'Ambrosio (address above), and in Australia by George Booker (Brisbane Coll Adv Educ, 130 Victoria Park Rd, Kelvin Grove, Queensland 4059). Distribution elsewhere by UNESCO. Send requests to receive the *Newsletter* and address changes to the Editor.

Calendar

Meetings with HPM components are highlighted.

1985 April 15-20 . . . San Antonio, Texas National Council of Teachers of Mathematics (details inside)

1985 June 2-4. . . . . Montréal Canadian Society For History and Philosophy of Mathematics. Contact: Louis Charbonneau, Dept Math, U du Québec, CP 8888, Montréal, Qué H3C 3P8

1985 July 21-25. . . . . Havana Primer Congreso Latinoamericano de Historia de las Ciencias y la Tecnología y la Segunda Asamblea General de la Sociedad Latinoamericana de Historia de las Ciencias y la Tecnología. Contact: Comité Organizador del Primer Congreso Latinoamericano de Historia de las Ciencias y la Tecnolgía, Palacio de las Convenciones, Apartado 16046, Zona 16, Habana, Cuba. (Télex 511609 PALCO CUBA)

1985 July 31-August 8 . . Berkeley, Calif. XVII International Congress on the History of Science (tentative)

1985 November 1-3 . . . . . Bloomington, Ind. History of Science Society. Contact: HSS, 215 South 34th St/D6, Philadelphia, Pennsylvania 19104.

1986 Mar 31-Apr 5. . . . Washington, D.C. National Council of Teachers of Mathematics (details inside)

1986 August 3-11 . . . . . Berkeley, Calif International Congress of Mathematicians (tentative)

1988 July 27-Aug 3 . . . . . Budapest International Commission on Mathematics Education (ICME 6). Contact: A.G. Howson, Secy. ICMI, Univ Southhampton, Centre For Mathematics Education, Southampton, SO9 5NH, U.K. HPM will participate; watch For details.

From the Editor

Please note the new address for the Editor: Department of Mathematical Sciences, Ball State University, Muncie, Indiana 47306, U.S.A.

This issue of the *Newsletter* contains reports on the meetings of HPM held at the ICME 5 conference in Adelaide last August. George Booker has provided an invaluable service in writing these reports. I have also relied on Bruce Meserve For reports. These reports will prove quite interesting, especially in shoving the breadth of interest and activity within the HPM structure. Read them and react to them—either to the contributors who are always gratified with responses to their work, or to this *Newsletter*. We are grateful to those who do the extra work after the conferences are over in order to keep the rest of us informed.

This issue of the *Newsletter* will be supported with supplements in Australia and the Americas. These will provide more details about regional meetings and activities. At present, we do not have supplements in place For Europe, Asia, and Africa. Until this is remedied, the *Newsletter* will continue to carry details of activities in these areas, when reported to the Editor. The supplements are being introduced in order to reduce the size of the *Newsletter* and save in costs. If it is successful, we hope to continue distributing the *Newsletter* at no cost.

I have been assisted in producing this issue by Wanda Back, a Ball State undergraduate majoring in mathematics (secondary) education. □

HPM Holds Session Prior to ICME-5

(George Booker; Bruce E. Meserve—) HPM held a 'pre-session' immediately preceding the ICME-5 Congress in Adelaide on Thursday and Friday, August 23rd and 24th. The pre-session was held on the Sturt Campus of the South Australian College of Advanced Education. The Thursday presentations were on the themes "History For Schools and For Teachers", with six speakers, and the Friday presentations Focused on "History in Tertiary Programs (excluding teacher education) and Relations Between History and Pedagogy", with four speakers. Some materials associated with the presentations were available For inspection, but the logistics of bringing them to Adelaide limited this aspect of the meeting. The sessions were ably chaired and kept on schedule, providing time For valuable discussions and comments, by David Wheeler.

•"How to Prepare Teaching Material on Historical Themes" *Otto Bekken, Agder College, Kristiansand, Norway*

This presentation Focussed on "themes from the history of algebra", using materials put together For the HPM meeting in San Francisco, April, 1984. Original source materials from Al-Khwarizmi, Euclid, Babylonian Tablet AO 8862 (Neugebauer), Ahmes, Omar Khayyam, Cardano, Descartes and Wessel were used to trace the development of algebra through three stages: the pre-symbolic (before 1600 AD), the numerical-symbolic (1600-1800), and the abstract-symbolic (after 1800). Since school algebra largely focuses on the second stage, where symbols represent numbers or quantities, the extracts were largely concerned with the pre-symbolic stage and the transition to the symbolic stage. A clear picture of the problems solved and understood at each point in time as well as those still requiring attention was provided.

•"History and Mathematics For Elementary Teachers" *John Berry, University of Winnipeg, Winnipeg, Canada*

A course in the history of mathematics For preservice elementary school teachers was described. Teachers K-9 were described as generalists whose mathematical preparation should include content (number systems and

geometry), methodology, and the historical development of elementary mathematics. His reasons for adding history included: (a) public ignorance about the nature of mathematics; (b) the traditional isolation of mathematics in school curricula; (c) teachers' perceptions of mathematics as unchanging, which result in vide resistance to curriculum change and misinterpretations of mandated changes; and (d) to illustrate the relevance of mathematics -- note that the stimuli that led to various advances in the development of mathematics were nearly all from applications outside of mathematics. The primary goal of the history course is to improve students' understanding of mathematics, as opposed to students' mastery of techniques. Active involvement of students is stressed through the use of worksheets, projects, and individual reports.

•"John Napier and the Discovery of Logarithms" *Arthur Gilks, Deakin University, Geelong, Australia*

This paper gave a short account of John Napier's contribution to the development of logarithms and a brief biographical sketch of his life largely drawn from papers presented in the *Napier Tercentenary Memorial Volume* (London: Longman, Green and Company, 1915). Napier's method for computing logarithms from an arithmetical progression and a geometric progression was illustrated and explained using a calculator in a manner accessible to senior high school students. The manner in which Napier was able to avoid tedious calculations in the constructions of his 'Deductio' and 'Constructio' tables by making use of the properties of logarithms that he discovered was also brought out.

•"Interplay between the History of Art and the History of Mathematics in Classroom Teaching" *Florence Fasanelli, The Sidwell Friends School, Washington, DC 20016, USA*

In this talk, Dr Fasanelli gave three different perspectives on the interplay between art and mathematics, illustrating each with slides of art work and providing a commentary expert in the history of both art and mathematics. Suggestions were given for classroom use of these ideas and sources were provided to assist those interested in implementing them. The first of these addressed the manner in which the study of mathematics affected the development of art, illustrated by a cultural history of the development of the fourth dimension as understood by mathematicians and artists. The source for much of this material was a recently published book, *The Fourth Dimension and Non-Euclidian Geometry in Modern Art*, by L.D. Henderson (Princeton University Press, 1983). The second topic followed from the first, discussing the manner in which artists broke the mathematical rules, creating a new art form. This was illustrated by reference to anamorphic art in the work of artists such as Leonardo da Vinci, Holbein and D'Arcy Thompson. References were made to an article by Martin Gardner "The Curious Magic of Anamorphic Art" (*Scientific American*, January, 1975) and to a book by F. Leeman, *Hidden Images, Games of Perception, Anamorphic Art, Illusion* (Abrams, 1975). The third aspect of the interplay between art and mathematics that was addressed was the use of mathematics to understand an art form, such as the use of symmetry groups to classify the ornamentation on Greek vases and Mexican pyramids. Sources were H. Weyl's *Symmetry* (Princeton University Press, 1952), J. Owen's *The Grammar of Ornament*, (1892) and the kit, "Mathematics of Islamic Art", published by the New York Metropolitan Museum of Art and available through the National Council of Teachers of Mathematics [1906 Association Drive, Reston, Virginia 22091, U.S.A.].

•"History of the Sciences and History of Education and their Relation to Research in the Didactics of Mathematics" *Hans-Georg Steiner, I.D.M., Bielefeld University, F.R.Germany*

This paper reported the results of a recent conference held at Bielefeld University on this theme. A summary of each paper was given along with evaluative comments and English translations of the titles. A full account of the conference is to be reported in *Historia Mathematica*. [See "Science, History and Mathematics. . ." elsewhere in this *Newsletter*.]

•"The Place of Ethnomathematics in History Courses" *Ubiratan D'Ambrosio, Unicamp, Brazil*

In his presentation, Professor D'Ambrosio discussed the nature of mathematics and of history of mathematics courses and suggested a broadened perspective on both. From its beginnings, mathematics has been perceived to have two different forms: scholarly mathematics studied as a discipline and practical mathematics used in everyday functioning. With industrialization and the advent of mass education, the distinction became blurred; scholarly mathematics encompassed practical applications and the methods of practical mathematics became more formalized, rigorous and scholarly. One form of mathematics, termed 'learned mathematics' by D'Ambrosio, has become the mathematics for both purposes. The study of the history of mathematics has been the study of this learned mathematics. Even when Chinese, Indian and primitive forms of mathematics are examined, it is to see how each matches this developed mathematics rather than how it functioned in its own society. Yet practical notions independent of a more formal mathematics have continued; these job- or situation-specific techniques D'Ambrosio terms ethno-mathematics. He argues that the evolution, uses and manner in which these techniques are understood and transferred should also be a part of the study of the history of mathematics, especially in anthropological-type studies. In particular, there should be an examination of the transition from ad hoc practices and solutions to the development of methods, the transition of methods to models and theories, and the extent to which theories have led to invention.

•"The Incompatibility of History and Mathematics" *Jack Gray, School of Mathematics, University of New South Wales, Sydney, Australia*

This discussion focussed on the mathematical tradition, drawing on evidence from graduate students at the University of New South Wales and on an analysis of the nature of the mathematician and historian. It was argued that history and mathematics are almost orthogonal, largely because mathematics viewed as a collection of symbols cannot have a history, but also because of the training, selection, psychology, values and intellectual temperament required and fostered by the two disciplines. Ramifications of this analysis and means of bringing about more compatibility were also considered.

•"On the Mystery of Geometrical Figures" *Yoshimasa Michiwaki, Gunma University, Japan*

In this paper, Professor Michiwaki used examples from a Japanese mathematical document from 1659 (Masashige Yamada: Kaisan-Ki) to affect strategies proposed by Polya on problem solving: emphasis was placed on induction and analogy and patterns of plausible inference. In addition to providing a geometrical dissection problem from this document, the author provided two extensions of the method to problems concerned with tesselations inspired by a visit to the Taj Mahal in India on the occasion of the 1500th anniversary of the birth of Aryabatha.

•"Mathematics Education as Meta-mathematics" *David Pimm, Faculty of Mathematics, Open University, Milton Keynes, U.K.*

This paper discussed the need for increased public discussions by mathematicians in the manner of Philip J. Hersh and Reuben Davis' book, *The Mathematical Experience* [Birkhäuser, 1981], and in the forum provided by the journal *The Mathematical Intelligencer* [Srpinger-Verlag]. It was argued that mathematicians are critically naive, lack a global perspective on their discipline, and as a consequence pass on to their students *Reader's Digest* compilations of mathematics rather than the original mathematics examined in context. Perhaps mathematics education can fulfill the task of providing a 'mathematical literary criticism' in the form of I. Lakatos' *Proofs and Refutations* [Cambridge, 1976], thereby providing a meta-mathematics superstructure to interpret and assess mathematical knowledge. Examples were given from the history of algebraic topology. (The presentation is to form the basis of an article to be published in 1985.)

•"Mathematical History, Historical Mathematics" *Jock McQualter, University of Newcastle, N.S.W., Australia*

This paper addressed the issue of how best to relate the history and pedagogy of mathematics, based on ten years involvement in the preparation and use of history of mathematics materials in teacher education. Effective use

of the history of mathematics must enable the teacher of mathematics to be an elementary mathematician, knowledgeable about mathematics, aware of not only the 'final format' but also the origins and development of mathematical ideas and the way in which they have been and are being used in society. Otherwise the history of mathematics is seen as somewhat irrelevant, used only to renew an acquaintance with school mathematics. McQualter also highlighted dangers in using historical material when the nature of historical research and scholarship are not made clear. A commonly encountered distortion was 'tunnel history' where all non-mathematical matters were ignored, an ahistorical perspective which purveyed an inaccurate, oversimplified and distorted view of change over time. In contrast, the historical study of a particular set of events is an attempt to explain how and why the events came about.

Dinner and drinks were organised in an Adelaide restaurant on Thursday night following the Executive Committee meeting. This allowed profitable discussions on the papers presented that day and on the future directions and activities of HPM, while old acquaintances were renewed and new friendships were forged. It was decided that the format of the subsequent ICME-5 sessions would include a daily social function at which informal discussions of the day's papers could occur. This proved a very valuable as well as enjoyable part of the program, especially when it continued on at dinner afterwards.

Tony Andrew provided the local liaison and logistics for the HPM pre-session, while George Booker co-ordinated the organization of the two days.

[This is a more complete report given only in outline in the previous issue of the Newsletter*—Ed.]* □

HPM Has Varied Program at ICME 5

> Its use is not just that history may give everyone its due and that others may look forward to similar praise, but also that the art of discovery may be promoted and its method known through illustrious examples. —Liebniz *(George Booker—)*

In contrast to these sentiments of Liebniz, mathematics is predominantly presented in a polished, logical form showing none of the difficulties, errors, guesses or stumbling that went into its creation. Such presentation is prepared from hindsight, ahistorically, largely as a consequence of a stress on symbolism and abstraction, and this has led to an emphasis of content over method—the justification of mathematical truths rather than analysis of the processes by which they were created. Yet there exist means of demystifying mathematics, of showing its origins as well as its results and applications, of revealing its history. HPM exists as an informal group to encourage colleagues throughout the world to use aspects of mathematical history in the teaching of mathematics to motivate interests, develop positive attitudes, and encourage appreciation of the nature and role of mathematics.

A knowledge of the history of the development of the mathematical curriculum can support its teaching by demonstrating how modern mathematics has its roots in the past, and revealing the improvements in mathematical rigour. Such calls to teach mathematics relative to its history also extend to investigations of national mathematical history, to the uses which might be made of this particular knowledge and to the light it might shed on prior assumptions, the manner of development, the persons involved and their roles.

Meetings held in conjunction with the International Commission on Mathematical Instruction through its ICME conferences and in regional conferences, such as meetings held in conjunction with the National Council of Teachers of Mathematics in the USA, form the major forums for discussing and disseminating these aims. During ICME 5, a series of four meetings were held with the dual intentions of introducing mathematics educators to the group and its aims, and of providing specific examples of such ideas in practice from a variety of countries and across all levels of education. The first session (presented by George Booker, Australia) provided a framework of suggestions for using the history of mathematics, supported by examples which showed the extent of such use in Australia. The second session (at which Rina Hershkoviz of Israel and Amy Dahan of France presented) examined uses which might be made of the history of mathematics for teachers and talented students. In the third session, historical documents suited to secondary school pupils were discussed (presentations by Amy Dahan and C. Borovcyz of France and Lucia Greuquetti of Italy), while the final session examined the interplay between the history of art and mathematics (presented by Florence Fasanelli of the USA) and concluded with a summary of the objectives being fostered by the HPM group and an account of the way these were being implemented in Canada (given by Israel Kleiner).

At the first session, two contrasting positions on the role of the history of mathematics in mathematics education were given and the discussions over the four days of the congress can be summarized in these terms. The first of these was that students evolve mathematics arising from a problem that interests them, thus focussing on the process of mathematical thinking rather than the end-product which are mathematical thoughts of earlier mathematicians. On the one hand this could involve problems and their solutions from the past such as those suggested in the recent book *Geometrical Investigations* by John Pottage of Melbourne University (Addison-Wesley, 1983). However there is also the point of view provocatively described by Professor Roland Stowasser as 'Ransacking the History for Teaching Mathematics' whereby critical incidents or examples are taken to illustrate a particular point or to generate a technique or method. One such instance presented in the second session examined the Arabic developments in mathematics as they took it from the orbit of Greek geometrical form into a theoretical discipline with its own methods and precise objects. Two currents were involved in this renovation; one from the geometrical construction of the roots of equations of degree higher than two, the other building from a deep dialectical movement between arithmetic and algebra. This led to the definition of the null power, $x^0 = 1$, and the use of a board

4	3	2	1	0	1	2	3	4
x	x	x	x	x	1	1	1	1
					x	x	x	x

to represent the multiplication of x^m by x^n (if $n > 0$) by taking n steps to the left from x^m. The practical basis that this invention has together with its origins in very real problems provides a valuable means of introducing this area of algebra which often seems very artificial to children and consequently a stumbling ground for their learning. Further details of this are available in Amy Dahan's recently published book *Routes et dédales* (1982). [See "Have You Read?" in *Newsletter* #7, October 1984.]

The second option proposed was to examine the development of mathematical ideas within an historical context, bringing out the relationship between mathematics and the rest of human thought and invention. In this way, students would gain a sense of perspective on mathematical discoveries and be able to tie mathematics to the problems from which it arose as well as to its discoverers. This setting of mathematics within the broader social and cultural fields has been addressed by R.L. Wilder in *Mathematics as a Cultural System* (Pergamon Press, 1983) and earlier works. It was also the theme of a paper presented by Professor d'Ambrosio of Brazil at the HPM pre-session preceding the ICME meeting [see report above], and in a keynote address to the congress where he raised the issue of 'ethnomathematics' as opposed to 'learned' mathematics.

Having determined these somewhat separate options, the discussion then focussed on ways in which they might be realized. The first suggestion was through an anecdotal approach, one which is frequently used in both elementary and high schools, when brief biographies of mathematicians are given or some context of the use of a particular system of numeration or computation is provided. While this would seem to be the easiest and most basic level of involvement of an historical orientation, it is necessary to bear in mind the distinction between history and simply story-telling. A person who would not present a mathematical theorem or statement without checking it, nevertheless would simply copy or rely on loose recall of history. This has given rise to

inaccuracies such as the integral was discovered by Riemann; common fractions can be ascribed to the Babylonians; Egyptian 'rope-stretchers' made use of a 3,4,5 triangle to determine right angles; Fermat was an amateur mathematician (presumably from his being described as 'the prince of amateurs'). Still, the provision of background material in this way can give some perspective on mathematical discoveries and inventions and can also provide interest and motivation to the subject.

Such an approach can also provide insightful, more intuitive or alternative teaching procedures. Examples can be found in the early Greek proofs that the series

$$1 + 1/2 + 1/4 + 1/8 + \ldots$$

sums to 2, and for Pythagoras' theorem.

A second way of using an historical approach has been through an application of the genetic principle in teaching mathematics: that effective learning requires each learner to retrace the main steps in the historical evolution of the subject under study. While this has been interpreted in the past in terms of 'a discovery approach', the frame of reference increasingly used to describe mathematics learning has been a constructivist one, and a similar approach to this use of the history of mathematics is warranted. That is, a focus on the process of reconstructing mathematics rather than rediscovering it. In contrast, the notion that effective mathematics learning requires rediscovery is derived from an emphasis on the completed form of mathematics rather than the process by which it is formed. A constructive perspective is likely to highlight more clearly inherent difficulties, such as the zero concept or the notions of algebra, the existence or formulation of which were long in gaining acceptance and understanding. It would also highlight the pedagogically more valuable order of presentation, which is from the intuitive to the formal, and increase motivation through students recognising the origin and developmental paths of problems, concepts and proofs. An historical approach of this form, then, would tend to lead to understanding rather than memorization.

One example of this approach is the recent publication *Una Historia Breve del Algebra* by Otto Bekken and published in Spanish by the Sociedad Matematica Peruana, Peru (1983) [see *Newsletter* #6, February 1984]. Themes from this history of algebra were presented in the pre-ICME conference of HPM, and at a similar HPM meeting prior to the 1984 NCTM convention in San Francisco. A source book of documents suitable for both children and teachers which is about to be published in France provides a second example. This has been developed through the IREMS, groups of mathematics teachers at both the secondary school and university levels, and addresses the broad issue of 'Object and Utility in Mathematics', as well as more subject specific areas such as arithmetic, algebra, analysis and astronomy. Another form of this approach has been taken in developing in-service programs for mathematics teachers in Israel at the Weizmann Institute under the direction of Professor M. Bruckheimer. Each topic is created as a series of worksheets within the format: (1) a brief introduction to set the historical scene; (2) a facsimile of an historical source; (3) a series of leading questions on the source material and on its mathematical consequences. Learning occurs through workshops and correspondence courses, supplemented with an extensive discussion leaflet with detailed solutions and further source material. Topics treated so far include negative numbers, irrational numbers and the story of equations.

One unfortunate outgrowth of the genetic principle was an emphasis on mathematical structures as a unifying idea for mathematics. Again, this evolved from the accent on the content of mathematics and its form rather than its processes of construction. The history of mathematics, because its focus has to be constructivist, is more likely to provide a basis of unification, showing the interdependence of the various parts of mathematics through their common origins, evolution or response to similar problems or cultural forces. Mathematics has grown in part because of professionals working in the subject, but men and women working in other fields have also contributed greatly to the subject; the philosopher Pascal, the clergyman Bolzano, the poet Omar Khayyam and the artist Dürer, for example. Indeed, the writings of Dürer as a theorist of art provided one of the first

mathematical textbooks in the German language, "The Teaching of Measurement with the Compass and Ruler" [*Underweysung der Messung mit dem Zirkel un Richtscheyt*] (1525), giving birth to German scientific prose and replacing traditional formulas with analysis and new creative constructions. Examining the interplay between subjects such as the history of art and the history of mathematics also allows another way to introduce mathematics topics which otherwise appear dry and devoid of practical application or interest.

An examination of the distinction between content and form provides a third way in which the history of mathematics can be utilised in its teaching. Content refers to the methods and results of mathematics while form refers to the symbolic notation in which these results are usually expressed and the chains of logical reasoning through which a proof is given. The two are inevitably, but not inextricably, linked for much of the content of mathematics would not have been discovered if it were not for advances in form. New results have often become possible because of a new mode of writing, such as the introduction of Hindu-Arabic numerals or the notation Liebniz provided for the calculus. Indeed, "An adequate notation reflects reality better than a poor one, and appears endoved with a life of its own which in turn creates new life" (Struik). Advances in form have often made it easier to learn mathematics, but they can also give rise to learning difficulties and bar the way to understanding. As Ginsburg (1977) notes, "Children's informal arithmetic is powerful, their understanding of written symbolism is weak." Valuable insights into mathematics could be provided by an examination of the evolution of the way in which particular results have been expressed or proved, of what other results have been derived from them, and which ideas have been inhibited by them. The treatment of the negative integers by the group in Israel provides an example of this approach.

A more explicit development of this approach is the notion of 'proofs and refutations' proposed by Lakatos (1976) [*Proofs and Refutations*]. While both content and form are examined, it is the limitations and the process by which mathematics is formalised that comes under examination. For Lakatos contends that it is formalisation that divorces mathematics from its history and only an awareness of the mathematical disputes and errors that went into the formulation of mathematics will provide real understanding of the content.

If these four frameworks for utilising the history of mathematics are so well known, what then is inhibiting their adoption and use in the classroom? The first reason must be the sparseness of teaching materials at the level of the students involved or for the content they are to be taught. The difficulties associated with the mathematical form used in the student texts or the teachers' own education also limit the capacity to see opportunities for incorporating historical material or methods. Teachers may feel inadequate with the different teaching methods needed to present and discuss history as opposed to mathematics. Changing emphases in teacher education can help, but the existence of these inhibiting factors is one of the major reasons for the establishment of the HPM. It provides a venue for sharing teaching ideas and materials, a network of support and contacts through the HPM *Newsletter*, and by its existence raises for discussion the issue of including an historical approach in all levels of mathematics education.

The meetings concluded with an HPM business meeting at which future activities were discussed and the following executive were selected: Chairpersons, Professor Ubiritan D'Ambrosio (UNICAMP, Brazil) and Professor Christian Houzel (Universite de Paris-Nord, France); members of the international committee: Otto Bekken (Norway), Sergei Demidov (USSR), Maassouma Kazim (Qatar), David Pimm (UK), Lee Peng Yee (Singapore), George Booker (Australia), Paulus Gerdes (Mozambique), Bruce Meserve (USA), Roland Stowasser (F.R.Germany), and David Wheeler (Canada).

Initial contact with HPM and requests to be placed on the mailing list for the *Newsletter* are best made direct to the editor [see address on page 1]. □

Reprints of Historical Material for Classroom Use

(George Booker—) Roland J.K. Stowasser has available several "Historical Miniatures for Talented Students and Historically Oriented Themes for Student Teachers". This was to be the title of Professor Stowasser's presentation at the recent ICME 5 meetings, in which he would have described these reprints. However time was too short in the session. Titles of these 'miniatures' are: "Stevin and Bolzano—practice and theory of real numbers"; "A textbook chapter from an idea of Pascal"; "Geometry and problem solving"; "Geometry reborn: a unifying idea for teaching geometry drawn from the history of mathematics" (with George Booker and Benno Mohry); "Some problems in the history of mathematics attractive for teaching recurrence methods"; "An excerpt from Dürer's 'Treatise on Mensuration with the Compasses and Ruler in Lines, Planes, and Whole Bodies' to be used for teaching"; "An idea from Jakob Bernoulli for the ordinary teaching of algebra—and as a challenge for the interested pupil" (with Trygve Breiteig). [A version of this last item appeared in the November 1984 issue of *For the Learning of Mathematics* ; see "Have You Read?".]

Mark your interest in writing to him at the Tecnische Universität Berlin, Strasse des 17.Juni 135, D-1000 Berlin 12, F.R. Germany. □

Two Latinamerican Publications Appear

(Alejandro Garciadiego—) A new journal with articles of interest to teachers and historians of mathematics began publication in 1984. *Quipu: Revista Latinoamericana de Historia de las Ciencias y la Tecnología* began publication with the Jan-Apr 84 issue. It will carry articles on the history and teaching of science, including mathematics [see "Have You Read?", this issue], and will publish documents (translations, reprints, etc.) and reviews. The editor is Dr. Juan José Saldaña, Facultad de Filosofía y Letras (Cubículo 8), Ciudad Universitaria, Delegación Coyoácan - 04510, México, D.F., México.

The Latinamerican Society of the History of Sciences and Technology is sponsoring a newsletter, *CEHMAL* (Commission of studies on the history of mathematics in Latin America), which will appear in September and March. The first issue appeared in September 1984. It will print news, comments on meetings, information, and the like, but no research articles or reviews. Correspondence should be addressed to CEHMAL, c/ o Dr. E.L. Ortiz, Imperial College, London SW7 2BZ, U.K. □

Recent Conferences in Mexico and Colombia

(Alejandro Garciadiego—) Meetings of interest to HPM were held in Latin America during the past year. The Latinamerican Society for the History of Science and Technology organized the Reunión Internacional de Historiadores Latinoamericanos de la Ciencia y la Tecnología (International Meeting of the Latinamerican Historians of Science and Technology) at the Faculty of Philosophy of the Universidad Nacional Autonoma de México (UNAM), on 2 and 3 May 1984. A paper on teaching and the history of mathematics was presented by Alejandro Garciadiego D.

The Instituto de Investigaciones Históricas (Institute of Historical Researches) organized a symposium on the history of science and technology entitled "Resistance to scientific and technological change". It took place on 18 - 20 June 1984. Two talks of particular interest were Mario H. Otero, "History of a case of philosophical adultery in the history of geometry", and Luis C. Arboleda, "Difficulties of the professionalization of mathematics in Colombia". [A detailed report of this conference is in the Americas Section Supplement to the *Newsletter*.]

There was a Latinamerican Seminary on the Alternatives for the Teaching of the History of the Sciences and Technology at Cali, Colombia, from 4th to 10th of November 1984. Among the talks was Eduardo Ortiz, "History and teaching of mathematics". (Contact: Comité Organizado, Sem. Lat. Alt. Hist. Cien. Tec., c/ o Fundación para la Educación Continua, Universidad del Valle, Apartado Aereo 2188, Cali—Colombia.)

An international seminary organized by the Latinamerican Society for the History of the Sciences and Technology on "Technical Problems of the Documentation of the History of the Sciences and Technology in Latin America" was held between 21 and 25 January 1985, at the Faculty of Philosophy, UNAM. Issues discussed ranged over archives, libraries, museums, documentation centres, and legislation. (Contact: Mtra. OFelia Escudero C.,Facultad de Filosofía y Letras (Cubículo 8), Ciudad Universitaria, Apartado Postal 21-873, México, D.F. 04000, México.)

[More detailed reports of these meetings may appear in future issues of the *Newsletter* Supplement, Americas Section. —Ed.] □

Conference Planned in Cuba

The Latinamerican Society of the History of the Sciences and Technology and the Cuban Academy of Sciences are organizing and sponsoring the First Latinamerican Congress of the History of the Sciences and Technology in conjunction with the Second General Assembly of the Latinamerican Society of the History of the Sciences and Technology. It will be held in Havana, from 21st to 25th of July 1985. Papers are invited. Send a short résumé in Spanish, Portuguese, English, or French by 31 March 1985. More complete details are available from Comité Organizador del Primer Congreso Latinoamericano de Historia de las Ciencias y la Tecnología, Palacio de las Convenciones, Apartado 16046, Zona 16, Habana, Cuba; Telex 511609 PALCO CUBA. (If it would help expedite your plans, the Editor will copy the brochure and send it upon request. Please include a self-addressed stamped envelope.) □

ICMI Accepts Invitation to Hold ICME 6 In Budapest

The sixth International Congress on Mathematical Education (ICME 6) will be held in Hungary in 1988. The following is a letter of acknowledgement from A. Hajnal, Secretary General of the János Bolyai Mathematical Society to ICMI.

> It was with great pleasure that the János Bolyai Mathematical Society, the Federation of Technical and Scientific Societies, the Hungarian Ministry of Education and Culture, and the Hungarian Academy of Sciences learned of ICMI's decisions to accept their invitation to hold ICME 6 in Budapest.
>
> Hungary has a long and proud tradition in mathematics and mathematics education and we are particularly pleased to act as hosts to so important an international meeting in this field.
>
> The Congress will be held in Budapest—a city renowned for its beauty, culture and architectural interest—and will take place at the Technical University from 27 July to 3rd August, 1988.
>
> We hope that the Congress will attract members from many countries; we extend a warm welcome to all and we hope that participants will take the opportunity not only to benefit from attending the Congress, but also to see more of Hungary and to become acquainted with its people, its natural attractions and its culture.

This letter is reprinted from the *Bulletin* of the International Commission on Mathematical Education, No. 17, December 1984. The ICMI Secretariat is located at the Centre for Mathematics Education, University of Southampton, Southampton, SO9 5NH, U.K. □

Science, History and Mathematics Education Conference Held in Germany

(H.G. Steiner—) A conference entitled "History of the Sciences and History of Education and their Relation to Research in Mathematics Education" was jointly arranged by the Institut für Didaktik der Mathematik (University of Bielefeld) and the Gesellschaft für Didaktik der Mathematik and held at Haus Ohrbeck (near Osnabrueck), May 7 - 12, 1984. The conference proceedings are available (in German): H.G. Steiner and H. Winter, Editors, *Mathematikdidaktik-Bildungsgeschichte-Wissenschaftsgeschichte* (1985, Koeln: Aulis Verlag Deubner). The speakers and the titles of their talks (translated into English) are:

H. Freudenthal (Utrecht) "Structures within sciences and the structure of the discipline and their relation to teaching"; H.G. Knapp (Graz), "Thoughts related to better Understanding the Language of Mathematics in a Philosophical-Historical Perspective";P. Verstappen (Enschede),"Mathematics and the 20th Century Image of Man"; H.N. Jahnke

(Bielefeld),"On the Historical Meaning of the Concept 'Indirect Application'"; B. Bekemeier (Bielefeld),"Pedagogically motivated Reflections on a New Foundation of Mathematics in Early 19th Century - The 'Complete Consequent System of Mathematics' by Martin Ohm, (1822)"; I. Lohmann (Bielefeld), "School-pedagogical and Curricular Aspects of the Reorganization of Mathematics into a Subject for General Education (in Prussia at the Beginning of the 19th Century)"; E. Papamastorakis (Berlin),"The Origins of the Calculus of Variation; the Brachystochrone Problem and its Treatment by Johann and Jacob Bernoulli"; J.M. Laborde (Grenoble),"Early Versions of the Concept of Uniform Continuity with Cauchy and possible Relations to Spontaneous Concepts with Pupils and Students"; E. Scholz (Wuppertal),"The Influence of Herbart on Bernhard Riemann"; P. Damerov (Berlin),"Aspects of Arithmetic Learning and Teaching in a Historical Study related to the Structure of Number Symbols in Economic Texts of the City of Uruk (ca. 3000 B.C.)"; D. Stoller (Lueneburg),"The Geometry of Visual Perception - From Craftsmen's Practice to Mathematical Theory"; C. Laborde (Grenoble), "Relations between the Presentation of the Concepts Point, Line, Line-Segment in 18th Century Textbooks and the Conception of these Concepts by Present Time Pupils"; H.J. Burscheid (Koeln),"An Example of the Emergence of a 'Scientific School' in the Didactics of Mathematics related to Mathematics Education in the German 'Gymnasium' at the End of the 19th Century"; S. Schmidt (Koeln),"On the Didactics of Teaching Arithmetic as a Training Component in Teacher Seminars in the Rhine-District in the 19th Century"; G. Schubring (Bielefeld),"Contrasts in the Conception of Mathematics among Amateurs and Professionals: A Case Study on 19th Century 'Circle - Quadraturalists' and its Implication for the Didactics of Mathematics"; H. Struve (Koeln), "On the Reconstruction of Historical Forms of Mathematical Concepts"; G. Hentschke (Berlin), "On a Project Documenting 19th Century Mathematical Textbooks"; I. Harding (Bielefeld), "The 1899 Foundation of the Chile Pedagogical Institute and the German Lecturers in Mathematics"; G. Schubring (Bielefeld), "On the Professional History of the Teacher of Mathematics-Systematic Main-Lines and Methodological Problems in the Content-specific History of Education"; H. Griesel (Kassel), "Strands of Guiding Ideas in the History of the Didactics of Arithmetic in Germany"; E.B. Wagemann (Giessen), "On the History of Didactics of Mathematics from 1945 to the End of the Reform in 1967 from a Personal View-Point"; R. Biehler (Bielefeld), "On the Historical Development of Theories on Statistical Testing - Mathematical Research Programs and their Changing Relation to Applications"; R. Richenhagen (Paderborn), "Applied Mathematics Overshadowed by Pure Mathematics - Carl Runge (1856-1927) and Numerical Analysis at the Turn of the Century"; G. Kaiser (Kassel), "The Place of Application in W. Lietzmann's Books on Didactics of Mathematics (1916-1932/50)"; G. Malle (Klagenfurt), "What can we Learn for Didactics from History? For Example: The Concept of Number"; H.G. Steiner (Bielefeld), "The Interest of Didactics of Mathematics in the History of the Sciences and the History of Education". □

HPM Will Meet at Berkeley In 1985

HPM will meet during the XVIIth International Congress of History of Science which will be held in Berkeley, California, 31 July - 8 August 1985. HPM Chair, Ubiratan D'Ambrosio, is organizing a panel discussion for the afternoon of August 8th, from 3 pm to 7 pm, immediately following the symposium organized by the International Commission on the History of Mathematics [see next item]. A small charge of $5.00 may be necessary to cover the cost of a room and coffee. Further details will appear in the next Newsletter, *if available. Suggestions would be welcomed by Professor D'Ambrosio (address on page 1).*

Information on hotels, registration, and the like for the XVIIth International Congress may be obtained from the conference secretariat, International Congress of History of Science, Office for History of Science and Technology, University of California at Berkeley, Berkeley, California 94720, U.S.A. □

Transmission of Mathematical Sciences Symposium at Berkeley, 1985

A symposium, "Transmission of Mathematical Sciences", sponsored by the International Commission on the History of Mathematics, has been organized by J. Dhombres (France) and C.J. Scriba (F.R. Germany) for the XVIIth International Congress of History of Science, to be held in Berkeley, July 31st to August 8th, 1985. There will be four sessions, a panel discussion, and a joint symposium on technical education. About forty presentations not to exceed twenty minutes each are scheduled. The four sessions are entitled "Communication Networks Among Mathematicians and Dissemination of Mathematics", "Cross-cultural Transmissions of Mathematical Sciences", "Methodological Problems About Transmission of Mathematics", and "Transmission of Greek Mathematics and the Rôle of the Arabic World". The panel discussion is "Transmission of Mathematical Knowledge to the Countries of the Third World". [Further information may be obtained from the chair of the International Commission on the History of Mathematics: Prof. Dr. C.J. Scriba, Institut für Geschichte der Naturwissenschaften, Mathematik und Technik, Bundesstraße 55, D-2000, Hamburg 13, F.R. Germany. A tentative program can be obtained from the Editor; send a self-addressed stamped envelope with your request.]

Information on hotels, registration, and the like for the XVIIth International Congress may be obtained from the conference secretariat, International Congress of History of Science, Office for History of Science and Technology, University of California at Berkeley, Berkeley, California 94720, U.S.A. □

HPM Plans to Meet With International Congress of Mathematicians in 1986

HPM Chair, Ubiratan D'Ambrosio, has announced plans to organize a meeting of HPM in conjunction with the International Congress of Mathematicians, which will be held August 3-11, 1986 (ICM-86) at the University of California, Berkeley. ICMI, the parent organization of HPM, will also meet during ICM-86. The HPM meeting will be scheduled at about the same time as the ICMI sessions.

Further information about ICM-86 may be obtained from Dr. Jill P. Mesirov, Executive Director, ICM-86, P.O. Box 6887, Providence, Rhode Island 02940, U.S.A. Details about HPM participation will be announced in this *Newsletter* as they become available. □

Brazilian Society Formed in 1983

The Sociedade Brasileira de História da Ciência (Brazilian Society for the History of Science, SBHC) was formed on December 16, 1983, at a meeting in the University of São Paulo. The Society plans special sessions on the history of mathematics and publishes a journal, *Boletim da Sociedade Brasileira de História da Ciência.* Address inquiries to SBCH, Caixa Postal 8105, 01000 - São Paulo - SP - Brasil. □

Have You Read?

Readers are encouraged to submit contributions to "Have You Read?". References need not deal exclusively or explicitly with history in the mathematics classroom, but should have the potential for motivating or enriching. N.B. Supply complete bibliographic information: names of author(s); complete titles of books or of both the article and journal; for journals include both the volume and date; for books, edition, copyright date, publisher and place of publication. Accuracy in spelling and wording is critical. Please provide concise annotations whenever possible. —Ed.

Arboleda, Luis Carlos 1984 "Historia y enseñanaza de las mathemáticas" *Quipu* 1, 167-94.

"History and teaching of mathematics". A new journal; see item in this *Newsletter.*

Beltran, Enrique 1984 "La historia de la ciencia en Latinamérica" *Quipu* 1, 7-23.

Bos, H.J.M. 1984 "Mathematics and its social context: a dialogue in the classroom with historical episodes" *For the Learning of Mathematics* 4:3 (Nov) 2-9.

Based on a lecture presented to a conference, "The use of historical topics in the teaching of mathematics," held in Vingsted, Denmark, January 1983.

Brody, Thomas L. 1984 "La historia de la ciencia en la enseñanza" *Quipu* 1, 185-204.

Bruins, Evert M 1981 "Egyptian arithmetic" *Janus* 68, 33-52.

Burton, David M. 1985 *The History of Mathematics. An Introduction.* Boston: Allyn and Bacon.
A general survey history text.

Byers, Victor 1982 "Why study the history of mathematics?" *International Journal of Mathematical Education in Science and Technology* 13, 59-66.

Cameron, Malcolm 1983 *Heritage Mathematics* North Melbourne, Australia: Hargeen Publishing (P.O. Box 4710)
History through great figures, suitable for secondary students. [A detailed and critical review of this book by John Pottage (HPS Dept, Univ of Melbourne) should be consulted.]

Campbell, William 1985 "A glimpse at Thales" *Bulletin* of Missouri Council of Teachers of Mathematics 10:1 (Jan) 4, 7.

———— 1985 "Archimedes of Syracuse" MCTM *Bulletin* 10:2 (Feb) 4-5.

Coughlin, Raymond, and David E. Zitarelli 1984 *The Ascent of Mathematics* New York: McGraw-Hill.
Written for a mathematics appreciation course; uses history to motivate and develop topics.

Faust, Don 1984 "The importance of teaching what is yet unknown" *New York State Math Teacher's Journal 34:3, 159-64.*
Uses historical problems to motivate discussion in secondary level mathematics.

Goldberg, Dorothy J 1984 "An international studies approach to the history of mathematics" International Journal of Mathematics Education in Science and Technology 15:2, 197-202.

Howson, Geoffrey 1982 *A History of Mathematics Education in England* Cambridge: Cambridge University Press.

International Congress on Mathematical Education 1983 *Proceedings of the fourth International Congress on Mathematical Education* Edited by Marilyn Zweng et al. Boston/Basel/Stuttgart: Birkhauser.
Contains several articles on using history in teaching mathematics.

Jahnke, Hans Niels 1981 *Wissenschaft und Bildung in der Mathematik des 19. Jahrhunderts* Bielefeld, Fed. Rep. of Germany: Universität Bielefeld (Institut für Didaktik der Mathematik).
Fifty-two page monograph tracing the influence on nineteenth century German education of the eighteenth century French Enlightenment.

———— 1982 "Zum Verhältnis von Bildung und wissenschaftlichem Denken am Beispiel der Mathematik. Eine Kontroverse um den mathematischen Lehrplan der preussischen Gymnasien 1829-30 und ihr methodologischer Kontext." In *Wissenschaft und Bildung im frühen 19. Jahrhundert* / Bielefeld: Institut für Didaktik der Mathematik der Universität Bielefeld.
Controversy over teaching analytic or synthetic mathematics in Prussian gymnasia.

Kimberling, Clark 1985 "Roots: half-interval search" *Mathematics Teacher* 78:2 (Feb) 120-23.
From the series "Microcomputer-assisted mathematics": a problem motivated by the fundamental theorem of algebra and Abel's proof that there is no algebraic formula for polynomials of degree higher than four.

Kost, Franklin C 1985 "Two solutions to a problem of Huygens" *Mathematics Teacher* 78:2 (Feb) 144-45.
Huygen's and J. Bernoulli's solutions to a probability problem.

May, Kenneth O. 1973 "Discovering and constructing logarithm tables and a slide rule by simple arithmetic" *International Journal of Mathematics Education in Science and Technology* 4, 137-41.
Historical development motivates the procedures described.

Metz, James R 1985 "The law of cosines as seen by Pythagoras" *Mathematics Teacher* 78:2 (Feb) 109-11.
Law of cosines shown as an extension of the Pythagorean theorem.

Mukherji, Viśvapriya 1982 "Introduction of the history and philosophy of mathematics into university education: an approach" *Ganita Bharati* 4, 44-49.

Nield, Don 1983 "University mathematics in Auckland: a historical essay" *Mathematical Chronicle* 12, 1-33.
Centenary history of a department.

Schubring, Gert 1983 *Die Entstehung des Mathematiklehrerberufs im 19. Jahrhundert* Studien und Materialien zum Prozeß der Professionalisierung in Preußen (1810-1870). Weinheim/Basel: Beltz-Verlag. [ISBN 3-407-58218-8]
First comprehensive study on the emergence of mathematics instruction as a main school subject and of mathematics teachers as a profession in Prussia during the 19th century; based on extensive archival findings.

Steiner, H.G., and H. Winter, Editors 1985 *Mathematikdidaktik-Bildungsgeschichte-Wissenschaftsgeschichte* Koeln (Cologne): Aulis Verlag Deubner.
Proceedings of the conference, "History of the sciences and history of education and their relation to research in mathematics education" held near Osnabrueck, F.R. Germany, 7-12 May 1984.

Stowasser, Roland, and Trygve Breiteig 1984 "An idea from Jakob Bernoulli for the teaching of algebra: a challenge for the interested pupil" *For the Learning of Mathematics* 4:3 (Nov) 30-38.
Uses Pascal's triangle. Translated from *Mathematiklehrer*, 1 (1983).

Swetz, Frank J. 1985 "Mathematics: a vehicle for better global understanding" *Mathematics Teacher* 78:3 (Mar) 207-15.
Thoughtful suggestions which could be supplemented with historical topics.

Teodorescu, Ion D 1980 [Founders of Romanian mathematical education] *Gazeta Matematica* (Bucharest) 85, 145-46.
In Romanian.

Wallace, Edward C and Joseph Wiener 1985 "A new look at some old Formulas" *Mathematics Teacher* 78:1 (Jan) 56-58.
Formulas for quadratic equation from Viète and Babylonians.

Whitt, Lee 1985 "Medical cozenage on Fermat's Last Theorem" *College Mathematics Journal* 16:1 (Jan) 55-56.
A spoof of why Fermat made his famous conjecture.

Yadav, S.P. & D.K. Sinha 1982 "Pedagogical implications of teaching of ancient mathematics up to 10th century AD" *Ganita Bharati* 4, 41-43.

Theses and Extended Research *This section of "Have You Read?" is a file of research on history and teaching. Send both old and new sources.* N.B. Supply complete bibliographic information. For theses, supply name of author; complete title; university; date; *Dissertation Abstracts International* abstract identifier and page number—thesis order number or similar data for theses not in DAI.

Delaney, Richard A. 1980 "An anecdotal and historical approach to mathematics" (New York University dissertation) *DIA* 41, 574-A; University MicroFilms No. 80-17493.

Caspar Wessel's Complex Plane (1798)

(Duane Deal—) The illustrations in this *Newsletter* are of the title page and page 9 of the first published usage of the complex plane to represent complex numbers. The author is a Norwegian surveyor and cartographer, Caspar Wessel. He was born in 1745 but went to Copenhagen at the age of 18 to further his education. He was soon hired by the Royal Academy of Sciences and Letters of Denmark to assist in preparing an up-to-date map of Denmark, and remained on the staff for 42 years. In March, 1797, he presented this paper, "On the analytical representation of direction: an attempt", which was then published in the *Memoirs* of the Academy in 1799. This earlier version, presently in the Rare Book Room of the University of Michigan Library, was published in 1798.

In the paper he appears to be thinking of vectors, but on page 9 we see, without a diagram, two perpendicular units which he calls +1 and ε, the units on the x- and y-axes. One

can see also the rules of multiplication and the conclusion, in the ninth and tenth lines, that ε actually equals $\sqrt{-1}$.

The paper was well received by the members of the Academy even though he was an employee and not a member, and though he was a self-taught mathematician. But the paper had little impact and remained unknown for a century. It finally appeared in a French translation in 1897, at which time it became clear that the 1806 publication of virtually the same ideas by the Swiss Jean-Robert Argand had pre-empted Wessel's work.

This is the second historical illustration included with the Newsletter. *It is one of the illustrations assembled for the Ann Arbor meeting of HPM, held in April, 1983. Cf. issue #7.* □

Om

Directionens analytiske Betegning,

et Forsøg,

anvendt fornemmelig

til

plane og sphæriske Polygoners Opløsning.

Af

Caspar Wessel,

Landmaaler.

Kiøbenhavn 1798.

Trykt hos Johan Rudolph Thiele.

§. 5.

Lad $+1$ betegne den positive rettlinede Unitet, og $+\varepsilon$ en vis anden Unitet, der er perpendikulär paa den positive, og har samme Begyndelsespunct: saa er Directionsvinkelen af $+1 = 0$, af $-1 = 180°$, af $+\varepsilon = 90°$, af $-\varepsilon = -90°$ eller $270°$; og i Følge den Regel, at Productets Directionsvinkel er Summen af Factorernes, bliver $(+1).(+1) = +1$, $(+1).(-1) = -1$, $(-1).(+1) = -1$, $(-1).(-1) = +1$, $(+1).(+\varepsilon) = +\varepsilon$, $(+1).(-\varepsilon) = -\varepsilon$, $(-1).(+\varepsilon) = -\varepsilon$, $(-1).(-\varepsilon) = +\varepsilon$, $(+\varepsilon).(+\varepsilon) = -1$, $(+\varepsilon).(-\varepsilon) = +1$, $(-\varepsilon).(-\varepsilon) = -1$.

Hvoraf sees at ε bliver $= \sqrt{-1}$, og Productets Afvigning bestemmes saaledes, at ei een eneste af de almindelige Operationsregler overtrædes.

§. 6.

Cosinus til en Cirkelbue, der begynder fra det sidste Punct af dens Radius $+1$, er det Stykke af samme, eller modsatte Radius, der begynder fra Centrum, og endes perpendicular ubfor Buens sidste Punct. Sinus til samme Bue drages perpendicular paa Cosinus fra samme sidste Punct til sidste af Buen.

I Følge §. 5 er altsaa Sinus til en ret Vinkel $= \sqrt{-1}$. Lad sættes $\sqrt{-1} = \varepsilon$; lad v betegne en Vinkel, hvilken som helst; og lad sin. v bemærke en ret Linie af samme Længde som Vinkelen v's Sinus, men positiv, naar Vinkelens Maal endes i første halve Omkreds, og negativ, naar det endes i den sidste halve: saa følger af §. 4 og 5, at ε sin. v udtrykker Vinkelen v's Sinus baade i Henseende til Direction og Længde.

§. 7.

I Overeensstemmelse med §. 1 og 6 er den Radius, som begynder fra Centrum, og afviger fra den absolute eller positive Unitet Vinkelen v, saa stor som cos. $v + \varepsilon$ sin. v. Men i Følge §. 4 skal Productet af to Factorer, hvoraf den ene afviger fra Uniteten Vinkelen v, og den anden Vinkelen u, afvige fra